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KEY

TO THE

STEREOMETRICON

OR

APPLICATION OF THE PRISMOIDAL FORMULA

TO

ALL SOLIDS

WITH TABLES OF SQUARES AND SQUARE ROOTS,

CIRCUMFERENCES AND AREAS OF CIRCLES TO EIGHTHS, TENTHS AND TWELFTHS OF UNITY.

SIDES OF EQUAL SQUARES, LENGTHS OF CIRCULAR ARCS, LENGHTS OF ELLIPTIC ARCS, AREAS OF SEGMENTS AND ZONES OF A CIRCLE,
SPECIFIC GRAVITIES.
MENSURATION OF AREAS, &c.

By CHS. BAILLAIRGE, M. A.

ARCHITECT, ENGINEER, SURVEYOR,

HONORARY MEMBER

OF THE SOCIETY FOR THE GENERALIZATION OF EDUCATION IN FRANCE, AND OF OTHER LITERARY AND SCIENTIFIC SOCIETIES

Recipient of 13 *medals and* 17 *diplomas and letters from France*
Russia Italy, Belgium Japan, &c.

QUEBEC:
C. DARVEAU, EDITOR AND PUBLISHER.
No. 82, Mountain Hill.
1876

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AL TABLEAU!

by one and the same rule.

rch, 1872.)

"The tableau," says this competent judge, "comprising "as it does a great variety of elementary models, will serve "admirably to educate the eye, and must greatly facilitate "the study of solid mensuration." "Again," says Mr. Wilkie, "the Government would confer a boon on schools "of the middle and higher class by affording access to so "suggestive a collection." There are others who, irrespective of considerations as to the comparative accuracy of the formula, or of its advantages, as applied to mere mensuration, are awake of the fact that the models are so much more suggestive to the pupil and the teacher than their mere representation on a blackboard or on paper, and who, in their written opinions, have alluded especially to this feature of the proposed system. M. Joly President of the Quebec Branch of the Montreal School of Arts and Design, in a letter on the subject to Mr. Weaver, the President of the Board, and after having himself witnessed its advantages on more than one occasion, says, in his expressive style, "the difference is enormous." Professor Toussaint, of the Normal School, Dufresne, of the Montmagny Academy, Boivin, of St. Hyacinthe, and many others, are of the same opinion; among them MM. R. S. M. Bouchette, O'Farrell, Fletcher, St. Aubin, Steckel, Juneau, Venner, Gallagher, Lafrance, and the late Brother Anthony, &c., &c. Neither will it be forgotten that the professors of the Laval University, after reading the enunciation of Mr. B.'s formula, as given in his treatise of 1866, expressed themselves thus: "Un doute involontaire "s'empare d'abord de l'esprit, lorsqu'on lit le No. 1521; "mais un examen attentif des paragraphes suivants, dissipe "bientôt ce doute et l'on reste étonné à la vue d'une "formule, si claire, si aisée à retenir et dont l'application "est si générale." Mr. Fletcher, of the Crown Lands Department, says: "I have compared, in the case of "several solids, the results obtained by your mode of "computation with those resulting from the ordinary and "more lengthy processes, and congratulate you sincerely "on your enunciation of a formula so brief and simple in "its character, and so precise and satisfactory in its "results." Mr. Baillairgé also took occasion during his lecture to allude, in other relations, to his treatise on geometry and mensuration, in which he showed he has introduced many important modifications in the usual mode of treating the subject of plane and spherical geometry and trigonometry. In conclusion, we must add that the Council of Public Instruction, at its last meeting, appointed a Committee, composed of the Lord Bishop of Quebec, and of Bishops Langevin and Larocque, to report to the Council at its next general meeting in June, and who, it may be taken for granted, after the many flattering testimonials in relation to the utility and many advantages of the stereometrical tableau for purposes of education, cannot but recommend and direct its adoption in all the schools of the Dominion.

CAL TABLEAU!

CATION IN FRANCE, ETC., ETC.

bodies, by one and the same rule.

ed in Europe.)

nder Lock and Key, so as to exclude dust while conceivable Elementary, Geometrical or other an be removed and replaced at pleasure, by the prismoidal formula

TABLEAU

D-STATES

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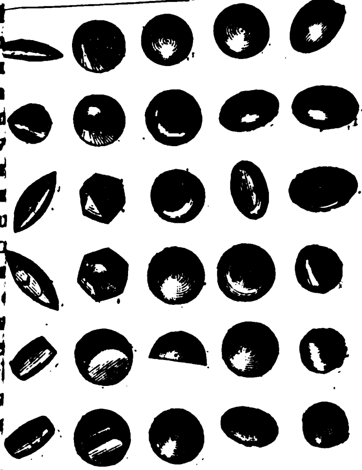
To find the solid content of any body.

RULE: To the sum of the parallel end areas, add four times the middle area, and multiply the whole by one sixth part of the height or length of the body.



Approved by the Council of Public Instruction of the Province of Quebec, and already adopted and ordered by many Educational and other Establishements in Canada and elsewhere. For information and testimonials apply to.

C. BAILLARGÉ,
QUEBEC,
CANADA.

Honorary Member of the Society for the Generalization of Education in France, etc., etc.

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The Archbishop of Quebec, the Bishop of Rimouski, the Bishop of Kingston, the Bishop of St. Hyacinthe, the Dominion Board of Works, the Schools of Art and Design, the Laval University, the Seminary, Q., the College at Ottawa, Nicolet, Rimouski, Montmagny, St. Michel, etc., l'Ecole Normale Laval, les Ecoles des Frères, the Commercial Academy, the Board of Land Surveyors, the Department of Education, New Brunswick, the Corporation of Quebec, R. Hamilton, Esq., F. N. Martin, and C. Roy, Civil Engineers, etc., La Société pour la vulgarisation de l'Enseignement du Peuple, France F. Peachy, J. Lepage, etc., Architects, N. Piton, T. Maguire, J. Marcotte, builders, the Council of Public Instruction, Q., the Jacques Cartier Normal School, M. Piton Manitoba, the Colleges of Aylmer, L'Assomption, Ste. Anne de la Pocatière, St. Hyacinthe, the High School, Q., the Morin College, Q., the Lafrance Academy, Q., Government Boards of Works, Q., the Ursulines Convent, the Convent of the Good Shepherd, Grey Nuns, Sœurs de la Congrégation, Sœurs de Jésus-Marie, Q., and M. S. W. Townsend, Hamilton, &c. &c. &c.

Etc. Etc. Etc.

nd Excise Officers, Professors of Geometry and nents, Schools of Art and Design, Mechanics, Ire.

KEY

TO

BAILLAIRGE'S

STEREOMETRICAL TABLEAU.

NEW SYSTEM OF MEASURING

ALL

Bodies,—Segments, Frusta and Ungulæ of Such bodies

BY ONE AND THE SAME RULE.

FOR THE USE OF

ARCHITECTS, ENGINEERS, SURVEYORS, PROFESSORS OF MATHEMATICS, GEOMETRY, DESIGN, DIRECTORS OF UNIVERSITIES, COLLEGES, SEMINARIES, CONVENTS AND OTHER EDUCATIONAL INSTITUTIONS, SCHOOLS OF ARTS, TRADES AND INDUSTRIAL DESIGN, MACHINISTS MEASURERS, GUAGERS, CUSTOM HOUSE AND EXCISE OFFICERS, SHIP BUILDERS, CONTRACTORS, ARTISANS AND OTHERS.

P. Florent

BY CHS. BAILLAIRGÉ, 1827-1906

ARCHITECT, ENGINEER, SURVEYOR,

HONORARY MEMBER

OF THE SOCIETY FOR THE GENERALIZATION OF EDUCATION IN FRANCE
AND OF OTHER LITERARY AND SCIENTIFIC SOCIETIES.

Recipient of seven Medals in Gold, Silver and Bronze, awarded him in Europe for his discovery and invention.

QUEBEC:
C. DARVEAU, EDITOR AND PUBLISHER,
No. 82, Mountain Hill.
1876.

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THE TABLEAU.

Registered, conformably to the act of Parliament of Canada, by C. P. F. Baillairgé, the 23rd February, 1871, in the Office of the Minister of Agriculture, at Ottawa.

Patented in Canada, the United States and Europe.

Registered according to the act of Parliament of Canada, in the year one thousand eight hundred and seventy four, (1874) by the author C. P. F. Baillairgé, Esq., in the Bureau of the Minister of Agriculture, at Ottawa.

READ

THE PREFACE.

The question may be asked "If the system be so simple, why so voluminous a "KEY"? Now, it will be immediately seen that the present work is in reality, for the most part, a mere "Mensuration of Areas" which might perhaps have been omitted, since there are already many works which treat on that subject, and that the mode of measuring the surface or area of any solid is supposed to be known before its cubical contents can be arrived at. It is however more satisfactory for Teachers in general, Professors and Students to find thus brought together in a single volume, all that they require, than to have to seek it elsewhere. The mensuration of areas is not at all superfluous, even in the "Key"; since, in point of fact the whole difficulty and labor of computing the solid contents of any body, consists in determining the areas of certain of its component faces and sections.

That which also contributes largely to swell the dimensions of the "Key", is the great number of examples, fully worked out, of the author's system as applied to the computation of the most intricate solids, and the numerous tables of which the great utility will become apparent, when, having to compute the capacity of any boiler tub, vat or cask—the volume of a cylinder, sphere, spheroid, conoid or of any segment, frustum or ungula of such bodies, the calculation will be found, so to say, fully worked out, since it will suffice to take out the requisite areas, add them and multiply their sum by the sixth part of the length or altitude of the body; after which a simple multiplication or division (as the case may be) of the units so obtained, will reduce them to inches, feet, mètres, gallons, litres, &c. or to any other units greater or less than the first.

AUG 1921

At page XXIX, however; that is, after the testimonials will be found an

ABRIDGED OR SYNOPTICAL KEY TO THE TABLEAU.

and, to any one who understands the nomenclature of solid forms and the mensuration of areas, this Abridged Key contains all that is essential to the full and entire intelligence of the author's system.

SUBSCRIBERS TO THE STEREOMETRICAL TABLEAU.

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The Bishopric of Kingston.
The Bishopric of St. Hyacinthe.
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The school of Arts & Design, Quebec.
The Board of Land Surveyors, Quebec.
The College, l'Assomption.
The Dept. of Public Works, Ottawa.
The College, Nicolet.
The College, St. Hyacinthe.
Dufresne's Academy, Montmagny.
The Academy, St. Michel.
J. F. Peachy, Archt., Quebec.
J. Lepage, Archt., Quebec.
The Education Office, New Brunswick.
R. Hamilton, Esq. Quebec, for a school.
The City Hall, Quebec, for the Dept. of Works.
Godin & Devarennes, builders.
N. Piton, builder.
F. N. Martin, Surveyor, Rimouski.
The Government, Province of Quebec, for Model Schools & Academies.
C. Roy, Surveyor & Engineer.
J. Maguire Plumber, &c., Quebec.
J. Marcotté, iron founder, Quebec.
The Jacques - Cartier Normal School, Montreal.
M. Piton, Manitoba.
The Ministry of Public Instruction, Belgium.
The Convent of the Good Shepherd, Quebec.
The Convent of the Sisters of Charity.
The Convent of Jesus-Marie, Cap Rouge.
The Dept. of Public Works, Quebec.
The Board of Arts & Trades, Montreal.
J. H. Clint, Lumber Mercht. Quebec.
P. Coté, builder, Quebec.
The College, Aylmer, Ottawa.
S. W. Townsend, Hamilton, C. W.
The Frères Schools, Canada.
The Frères Schools, France.
G. Bisset, iron founder, machinist, Quebec.
T. Archer, Lumber Merchant, Quebec.
R. Steckel, Civil Engineer, Ottawa.
V. Vannier, Paris & Canada.
The Commercial Academy, Quebec.
G. R. Baldwin, Civil Engr. Boston.
The Litterary & Hist. S., Quebec.
The Worcester Free Inst, Mass.
P. V. Du Tremblay, Surveyor & Engineer, Baie St. Paul.
C. Jobin, ship builder, Quebec,
J. Racine, iron smith, Quebec.
A. Réaume, Lumber Merc. Quebec.
Le College Melbrun, Haute Pyrenées, France.
A. Humbert, artist, Paris, France.
The Academy of Science, Paris, Fr.
The Conservatory of Arts and Trades,
&c., &c.. &c.

THE
STEREOMETRICAL TABLEAU.

TESTIMONIALS.

No. 2272-71.—*Ministry of Public Instruction,*

QUEBEC, this 18th September 1871.

C. BAILLAIRGÉ, Esq., Quebec,

SIR,—I am instructed by the Honorable, the Minister of Public Instruction to acknowledge receipt of your letter of the 8th instant, transmitting copy of the prospectus of your Stereometrical Tableau.

He will make it a duty to recommend its adoption in all educational establishments and in all schools, persuaded that he is of its practical utility. The tableau and accompanying formula reduce to an operation of the most simple kind the measurement of every description of solid, which required according to the old method a calculation long and often very difficult for persons especially who were not in the daily practice thereof.

I lave the honor to be, Sir, your obedient servant,

LOUIS GIARD, *Secretary.*

PARIS, 8 Juillet 1873.

MONSIEUR,—" Il est bien décidé que votre tableau aura le premier prix de la Société libre d'instruction et d'éducation populaire; vous en serez averti officiellement à la rentrée des vacances, fin Octobre, et vous serez en même temps convoqué pour la distribution solennelle des récompenses qui aura lieu dans le courant du mois de Mars prochain.

V. VANNIER.

C. BAILLAIRGE, Ecr.

Quebec Budget 12 *April* 1873.

THE URSULINE CONVENT.

We learn with satisfaction and legitimate pride that this admirable institution has ordered one of Mr. Baillairgé's Stereometrical Tableaus, and that this gentleman, during a single sitting of a few hours devotion, generously granted him by the Nuns, managed to render them thoroughly conversant with his system of nomenclature and mensuration. It seems almost incredible, and yet, we are ensured that the Reverend Superior (the talented Miss Cimon, of St. Paul's Bay) accompanied by Sister St. Croix and Sister St. Raphael, at once mastered and perfectly understood Mr. Baillairge's system in all its details. Paying comparatively little attention to the more ordinary forms of which the mode of measurement was to them apparent at a glance, they selected for their questions the more complex forms, such as the sections of the sphere and spheroids and the numerous and varied prismoids, to be found among the 200 models of the tableau. The education given by the Religious Ladies to their pupils comprises the geometry of lines and surfaces, and from this to the mensuration of solids by Mr. Baillairge's system, there is but one step, a simple addition of certain surfaces and the multiplication of their sum by one sixth part of the height or length of the body under consideration. The Nuns intend to commence immediately the teaching of this branch to their pupils, who will be examined on the tableau at the next examination which takes place in June. The noble example thus given by the Ursulines has been rapidly followed by another important educational establishment, the Convent of the Religious Ladies of Jesus Marie, on the Cap Rouge road, and we are, moreover, informed that the Sœurs de la Congregation de St. Roch, together with the Nuns of the Good Shepherd and the Sisters of Charity intend forthwith to add to the already varied programme of their tuition, this study of Stereotomy, which, up to the present time, could not be even dreamed of, but which Mr. Baillairge's system now renders possible by reducing, as it does, the study of a year to that of a day or two, so to say.

"Mr. Baillairgé next explained in detail his stereometrical tableau, which we hope to see soon introduced into all the schools of this Dominion. He showed how conducive il will be in shortening the time heretofore devoted to the study of solids and even to that of plane and convex superficies, spherical trigonometry, geometrical projection, perspective drawing, the development of surfaces, shades and shadows, and the like. Mr. Wilkie, so far as opportunity had been afforded him of proving the calculations, corroborated Mr. B.'s statement in relation to the immense saving in time, where many abstruse problems which generally required hours or days to solve, can now (if the rule be, as Mr. Baillargé asserts, so generally applicable, and, as been certified by so many persons in testimonials over their own signatures,) with the help of the new formula and tableau, be performed in as many minutes; to say nothing of the use the models are in imparting at a glance a knowledge of their nomenclature or names, and an acquaintanceship with their varied shapes and figures. He showed how, to the architect and engineer, the builder and mechanic, the models are suggestive of the forms and relative proportions of buildings, roofs, domes, piers and quays, cisterns and reservoirs, cauldrons, vats, casks, tubs and other vessels of capacity, earthworks of all kinds, comprising railroad and other cuttings and embankments, the shafts of the Greek and Roman column, square and waney timber, saw-logs, the camping tent, the square or splayed opening of a door or window, nich or loop-hole in a wall, the vault or arched ceiling of a church or hall, the billiard or the cannon-ball, or, on a larger scale, the moon, earth, sun and planets. Mr. Baillairgé, we may add, has received an order for a tableau from the Minister of Education of New-Brunswick, with the view of introducing it into all the schools of that Province; and Mr. Vannier, in writing to Mr. Baillairgé, from France, on the 10th of January last, to advise him of the granting of his letters-patent for that country, says that Messrs. Humbert & Noé, the President and secretary of the society for the generalization of education in France, have intimated their intention, at their next general meeting, of having some mark of distinction conferred on him for the benefit which his invention and discovery are likely to confer on education. Mr. Giard, in writing to Mr. Baillairgé, on the part of the Hon. Mr. Chauveau, Minister of Public Instruction, says: "Il se fera un devoir d'en recommander l'adop-" tion dans toutes les maisons d'éducation et dans toutes les écoles." From the Seminary and Laval University, Mr. Mainguy writes: "Plus on étudie, " plus on approfondit cette formule du cubage des corps, plus on est en-" chanté (the more one marvels at) de sa simplicité, de sa clarté et surtout de " sa grande généralité." Rev. Mr. McQuarries, B. A., "shall be delighted " to see the old and tedious processes superseded by a formula so simple and so exact." Newton, of Yale College, United States: "considers the " tableau a most useful arrangement for showing the variety and extent of " the applications of the formula." The College l'Assomption "will adopt " Mr. Baillairgé's system as part of their course of instruction." Mr. Wilkie has written to the author that "the rule is precise and simple, and will " greatly shorten the processes of calculation. The tableau," says this competent judge, "comprising as it does a great variety of elementary " models, will serve admirably to educate the eye, and must greatly facili-" tate the study of solid mensuration." "Again," says Mr. Wilkie, the " Government would confer a boon on schools of the middle and higher "class by affording access to so suggestive a collection." There are others who, irrespective of considerations as to the comparative accuracy of the formula, or of its advantages, as applied to mere mensuration, are awake to the fact that the models are so much more suggestive to the pupil and the teacher than their mere representation on a blackboard or on paper and who, in their written opinions, have alluded especially to this feature of the proposed system. Mr. Joly, President of the Quebec Branch of the Montreal School of Art and after having himself witnessed its advantages on more than one occasion, says, in his expressive style, " the difference is enormous. " Professeur Toussaint, of the Normal School, Dufresne, of the Montmagny Academy, Boivin, of St. Hyacinthe, and many others, are of the same opinion ; among them MM. R. S. M. Bouchette, O'Farrell, Fletcher, St. Aubin, Steckel, Juneau, Verner, Gallagher, Lafrance, and the late brother Anthony, &c., &c. Neither will it be forgotten that the professors of the Laval University, after reading the enunciation of Mr. B.'s formula, as given in his treatise of 1866, expressed themselves thus : " Un doute involontaire s'empare d'abord de l'esprit, lorsqu'on lit le No. 1521; mais un " examen attentif des paragraphes suivants, dissipe bientôt ce doute et l'on " reste étonné à la vue d'une formule, si claire, si aisée à retenir et dont l'ap- " plication est si générale." Mr. Fletcher, of the Crown Lands Department, says : "I have compared, in the case of several solids, the results obtained " by your mode of computation with those resulting from the ordinary and " more lengthy processes, and congratulate you sincerely on your enunciation " of a formula so brief and simple in its character and so precise and satisfac- " tory in its results." Mr. Baillairgé also took occasion during his lecture to allude, in other relations, to his treatise on geometry and mensuration, in which he showed he has introduced many important modifications in the usual mode of treating the subject of plane and spherical geometry and trigonometry. In conclusion, we must add that the Council of Public Instruction, at its last meeting, appointed a Committee, composed of the Lord Bishop of Quebec, and of Bishops Langevin and Larocque, to report to the Council at its next general meeting in June, and who, it may be taken for granted, after the many flattering testimonials in relation to the utility and many advantages of the stereometrical tableau for purposes of education, cannot but recommend and direct its adoption in all the schools of the Dominion. —From the Quebec " *Daily Mercury* " march 26 1872.

Our progress and success are of course attributable, before all, to your perfect mastery of the subjects of which you treat, and to the clear and concise manner in which you enunciated and demonstrated the several propositions introduced to our notice ; but neither must we omit to say that our intelligence of the problems and demonstrations has been greatly facilitated by our access and reference to the models of your " *Tableau.* "

We deem it not out of place to remark that in our opinion the word "*Stereometrical*" which you have prefixed, as qualitative of the uses that your "*Tableau*" can be applied to, is not suggestive enough of the many advantages which such a varied collection of models presents; for, not only is it of paramount importance and utility, as illustrative of your system of mensuration, by one and the same invariable formula, implied in the title at the head of the board; but, we hesitate not to say that to the use of the "*Tableau*" we are indebted in an eminent degree for the singularly rapid progress we have been enabled to make since the 4th of January last (only 30 lessons) not only in Geometry proper and in the Mensuration of surfaces and solids, both plane and spherical; but also in the study of geometrical projection and perspective, shades and shadows, the development of surfaces and the lines of penetration of divers solids, &c., &c.—From an address to Chs. Baillairgé, professor of the school of Arts.

Québec, 6 Septembre 1872.

MONSIEUR,—J'ai le plaisir de vous annoncer que le Conseil de l'Instruction Publique vient d'approuver votre Tableau et suis heureux de vous en féliciter.

Bien sincèrement, Votre tout dévoué,

C. BAILLAIRGÉ, ECR. P. J. O. CHAUVEAU.

Quebec, 9th January 1872.

G. W. WEAVER, ESQ.

President of Board of Arts & Manufactures.

MY DEAR SIR,—Our evening class began last week. I am happy to say every thing looks promising and we have been fortunate enough, to secure again Mr. C. Baillairgé's invaluable services, as teacher, for this winter.

We have procured from him, for our school, the "*Stereometrical Tableau*" which is his invention, and I am so delighted with it, that I send you a photographic representation of it, and a number of letters and other documents printed which will serve to explain the tableau, and show at the same time how useful it is considered by the most eminent authorities in this country.

You ought to get the tableau for your schools at Montreal. You showed me last year when I visited your schools, several wooden models of geometrical figures; I was struck with their usefulness at the time, and thought of procuring some for our schools, but there are only a few of them and their price is very high. Mr. Baillairgé's Tableau costs only fifty dollars, and it contains *two hundred* geométrical figures. I fancy the collection embraces every variety of figure that can ever be required for practical use.

They are solid figures made of wood, each fixed on a nail, so that they can be taken off by the teacher for demonstration and handed to the pupils; who are enabled to understand and master their divers shapes and forms with much greater ease, than if they saw them drawn on a black board, or in a book; the difference is enormus.

In addition to the great help they afford, for the study of geometry, these figures are very useful as models for earthworks, piers, reservoirs, castings, roofs, domes, columns, cauldrones, &c., &c., &c.

The tableau is most useful too for the working out of that wonderfully simple rule, which has been applied by Mr. Baillairgé, for the first time, to the measurement of the solid contents of all bodies. It was known previous to his discovery to apply to a certain number of bodies, but he has found out that it applied *to all without exception.* You will find that rule in the papers I send you, and in his treatise on geometry. I will soon let you know, what progress the school is making and remain, my dear Sir.

Yours truly,

(Signed,) H. G. JOLY.

President Quebec School of Art.

................Cette formule est véritablement curieuse par sa généralité et mérite d'être recommandée.

....Enfin j'ai accompli cette tâche, et voici le résultat: —L'auteur démontre sa formule comme rigoureusement exacte pour un grand nombre des corps énoncés, et comme aussi approximative qu'on voudra pour ceux auxquels elle ne s'applique pas d'une manière absolument rigoureuse. J'ai vérifié soigneusement la démonstration de la formule pour les 1ers corps, ceux auxquels elle s'applique rigoureusement. La proposition et la démonstration sont *exactes et vraies* dans tous ces cas.

Quand aux autres corps, il est vrai que plus on multipliera les sections, suivant le besoin, plus l'approximation sera proche de la vérité.

................Je finis en disant que j'approuve et recommande la méthode de Monsieur Baillairgé telle que proposée par lui.—Lettre du Rév. L. Billion professeur de Math. Séminaire de St. Sulpice, Montréal, à Mgr. Larocque Evêque de St. Hyacinthe.

"*Montreal Gazette*" 13 November 1873.

EDUCATIONAL.—Last evening, Mr. C. Baillairgé, C. E., and Mr. J. Carrel, both identified with the Quebec School of Art, visited the rooms of our Art School in St. James street. Mr. Baillairgé there exhibited his stereometrical tableau, and explained his system of teaching solid geometry by a method so simple as to bring this hitherto difficult subject within the grasp of ordinary students The tableau cannot fail to make easy and popular the study of solid forms and the mode of measuring their surfaces and their solidities and volumes. This tableau or board, which is made up of some two hundred models, comprises almost all the elementary forms which it is possible to conceive. To compute the volumes or cubical contents of the solids, Mr. Baillairgé has found that it is necessary only to apply the following rule: To the sum of the parallel end areas add four times the middle area, and multiply the whole by one sixth part of the height or length of the solid. The whole difficulty is, therefore, reduced to measuring the areas of the opposite bases and middle section, the remainder of the work being a mere multiplication. This system, it appears, has attracted considerable attention among mathematicians and educational authorities both in this country and in Europe, and its introduction into several of the colleges and schools in Quebec has led its author to seek its adoption in similar institutions in Montreal. The stereometrical tableau is well worthy of the attention of those engaged in educational work.

....The extraordinary progress made by the pupils, in the short space of three months, in stereometry or the mensuration of solids, is attributable to the grand and important discovery by their professor, Mr. Baillairgé, of a rule, one and the same, applicable to every known form, from a pyramid to a sphere, from a stick of timber to a vessel or other body of any shape or dimensions.

.............We have seen them in a very few minutes by the help of Mr. Baillairgé's new and beautifully simple and accurate rule arrive at the number of gallons in a cask of any size or shape. We have seen them determine by the same rule the exact weight of a shell, the true contents and weight of a hollow cast-iron colum, the size and weight of a pontoon and its draught of water.—From the Saturday *Budget* of May 6, 1871.

................In the instances which I have subjected to critical analysis, I have found the rule to work most admirably—combining comprehensiveness, utility with simplicity and great exactness. It will render a study heretofore charged with difficulty and abstruseness at once easy and acceptable—modernizing that which was ancient and which from its multitudinous formulæ had become an isolated branch of Mathematics. Believing it to be of universal use, I shall heartily lend myself to the introduction of your system.

Horatio R. N. Bigelow,
M. A.

Quebec decr. 26, 1871.

I have compared, in the case of several solids, the results obtained by your mode of computation with those resulting from the ordinary and more lenthy processes, and congratulate you sincerly on your enunciation of a formula so brief and simple in its character and so precise and satisfactory in its results.

E. T. Fletcher,
Inspector of Surveys Dept. of Crown Lands.

Québec 27 decr. 1871.

J'ai lu attentivement les appréciations que nombre d'hommes compétents ont faites de votre donnée réellement merveilleuse, dans les solutions Stéréométriques quelconques, et j'y donne mon plein assentiment.

J'ai eu occasion d'en remarquer la justesse, il y a un an, en préparant quelques élèves pour l'étude de l'arpentage, et je puis dire que toutes les application que j'en ai faites ont été des plus satisfaisantes.

Veuillez bien me considérer comme votre souscripteur.

Cde. Dufresne.
Professeur de Mathématiques, etc.

Collège de Montmagny 4 Janvier 1872.

..............I shall be delighted to see the old tedious processes superseded by a formula so simple and so exact.

A. N. McQuarrie, B. A.
Professor of Mathematics, etc., at the Morin College.

Quebec 31st Jany. 1872.

................What pleases me most is your *Stereometrical Tableau* as, under the direction of the Revd. Superior, I am now engaged in certain theoretical studies relating to this formula which your are about to render famous.

The more one examines, the more deeply one looks into this formula of the " cubing of bodies, " the more one marvels (plus on est enchanté) at its simplicity, its clearness and especially its great generality.

Nor can I do otherwise than hope it may, *as well in theory as in practice*, assume the place which it is entitled to and that thus your efforts may be fully crowned with success.

L. F. N. Maingui, Ptre.
Professor of Mathematics,

Seminary of Quebec 25th November 1871.

................Je vous déclare donc, monsieur, que je recommande vivement l'adoption de votre Tableau Stéréométrique, et vous prie de m'en faire connaître le coût probable.

T. Boivin, Ptre.

Collège de St. Hyacinthe 8 Janvier 1872.

Please to accept my sincere thanks for the copy of your excellent Treatise on *Geometry, Trigonometry*, &c., which you kindly sent me some time ago. I have perused its pages with deep interest. In my humble opinion it is a complete success ; whether we consider its admirable arrangement, the clearness and conciseness of its demonstrations, or the amount of valuable knowledge it contains. Your Book on Mensuration is a boon to every practical mathematician, and your *remarkable Theorem* for finding the contents of any solid, is sufficient, of itself, to immortalize its originator. In a word, your work merits a distinguished rank among the ablest productions on the exact sciences. The flattering and well-merited praise lavished on it by the press, both English and French, is sufficient guarantee that it meets the approbation of the public.

Your Treatise is used as a book of reference by the pupils of our Academy.

Bro. Anthony.

Commercial Academy, 25 Feb. 1868.

The January Session of the Board of Examiners for Land Surveyors for the Province of Quebec. 1872.—*Extract from the minutes.*

Moved by the President, Adolphe Larue, Esq., and seconded by E. T. Fletcher, Esq., and resolved :

" That the Board of Examiners for Land Surveyors, having taken into consideration the Stereometrical Tableau of Charles Baillairgé, Esq., Civil Engineer, and the very neat and precise formula connected therewith, desire to record their opinion of the utility and importance of this formula, and coincide wholly with the opinions expressed by those to whom it has been already submitted, and further they would recommend that the Board be provided with one of these Tableaux.

Alexander Sewell,

Quebec, January 2nd, 1872. Secretary of the Board of Surveyors.

Paris, 10 Janvier 1872.

Monsieur Charles Baillairgé, à Québec,

Cher Monsieur,—Aujourd'hui seulement, j'ai pu déposer votre demande de brevêt et je m'empresse de vous en envoyer le bulletin de dépôt qui vous donne le droit de brevêt à partir de ce jour, 10 Janvier.

Envoyez-moi maintenant votre tableau-modèle le plustôt possible, j'en ai parlé à plusieurs membres de l'académie, entre autres à Littré qui désire beaucoup le voir.

Messieurs Humbert et Noé, président et secrétaire de la société pour la généralisation de l'instruction en France sont très-simpathiques à votre œuvre et se proposent de vous faire récompenser à leur prochaine assemblée générale.

V. VANNIER.
No. 94. rue de Lévis, PARIS.

Ser. 2, No. 255. *Education Office, Province of New Brunswick,*
FREDERICTON, JANUARY 25TH 1872.

C. BAILLAIRGÉ, ESQ., Quebec.

Dear Sir,—I am instructed by the Board of Education for this Province to apply to you for a set of your *Stereometric Tableau* and your text-book on Practical Mathematics. The Board desire these articles for inspection, with a view of prescribing them for general use in all the Schools of this Province, should they be deemed suitable for the purpose. Should there be any charge for these articles, the same will be met by this Department.

Your Obdt. Servt.,
THEODORE H. RAUD.

Mr. Baillairgé's Stereometrical Tableau seems to me to be a very useful arrangement for showing the variety and extent of the applications of the *Prismoidal Formula.* Where demonstrations are given in the study of Mensuration of Solids, it will aid a teacher in illustrating the rules, but it would probably be much more valuable to those who try to teach that study without introducing démonstrations of the rules.

H. A. NEWTON,
Yale College, Feb. 5th 1872. Prof. of Math. in Y. College.

No. 13567. Subj. 995. Ref. 20814. *Department of Public Works,*
OTTAWA, FEBY. 7th 1872.

Sir,—In reply to your letter of the 26 ulto., I am directed by the Minister to request you to furnish the Department with one of your "Tableau Stéréométrique" at the price of Fifty dollars, together with your account for the same.

I have the honor to be, Sir your obt. servant,

Chs. Baillairgé, Esq.
Architect, etc., Quebec.

F. BRAUN,
Secretary.

NEW HAVEN, FEB. 7th 1872.

CHS. BAILLAIRGÉ, ESQ.,

Dear Sir,—I have been much interested in looking over the papers descriptive of your useful, valuable and (as it very plainly appears) universal application of a rule for the mensuration of solids. I sincerely congratulate you on the success which your discovery has met with in all quarters in which it has at present been introduced. It must have been a great labour to work it out to its present state of perfection and you have the satisfaction of knowing that you are a benefactor and staunch pilot in that sea of difficulty : Geometry.

Yours very sincerely,
E. B. BARBER.

HIGH SCHOOL, QUEBEC, 7th FEB. 1782.

CHS. BAILLAIRGÉ, ESQ.,

My Dear Sir,—I beg to acknowledge with many thanks the receipt of a number of papers explanatory of your new formula for finding the contents of solid bodies.

The rule is precise and simple, and being applicable to almost any variety of solid, will greatly shorten the processes of calculation. I have proved its accuracy as applied to several bodies.

The Tableau comprising a great variety of elementary models will serve admirably to educate the eye and must greatly facilitate the study of solid mensuration.

The Government would confer a boon on schools of the middle and higher class by affording access to so suggestive a collection.

I have the honor to be, my dear sir, your obedient servant,

D. WILKIE,
Rector.

.................Votre travail est d'une utilité supérieure. Votre formule est destinée, ce me semble, à simplifier de beaucoup les opérations dans le toisé des corps, et à rendre, par là, des services signalés à l'enseignement comme à l'application de cette partie importante des mathématiques.

Aussi mon plus grand désir est-il de voir adopter votre formule et votre tableau par nos Maisons d'éducation.

En finissant, j'ai l'honneur de vous informer que nous adopterons votre système, comme partie de notre enseignement.

J. C. CAISSE, PTRE.,
Préfet des Etudes au Collège de L'Assomption.

Collège L'Assomption, 27 Janvier 1872.

...............From Formulæ evolved by the Calculus, I find tha *your Theorem holds good* " FOR ALL SOLIDS *produced by the* REVOLUTION "*of any* STRAIGHT LINED FIGURE, or of *any* CURVED LINED FIGURE of the "*second degree,* AROUND ANY AXIS within or without the figure in either case." As to its application to all regular Polyhedrons, you give in your Treatise a clear Demonstration of the fact. I have reason, moreover, to suspect that your Theorem holds good, with mathematical exactness, in more cases than you give it credit for. I hope soon to be able to present you with an analytical Demonstration of your theorem as applied to solids of Revolution.

Needless to say that your Stereometrical Tableau should be in use in every school where Mensuration is taught within the Dominion.

In conclusion, let me congratulate you on the highly remarkable, and deeply useful discovery you have made.

Quebec, 26 decr. 1871. J. O'FARRELL.

Il n'y a pas besoin d'une longue dissertation pour faire voir quelle immense utilité offre un pareil tableau. Tous les professeurs qui ont enseigné la *Géométrie dans l'espace* et la *Géométrie descriptive* savent que, dans les classes les mieux composées, il y a toujours un certain nombre d'élèves, très intelligents d'ailleurs, qui éprouvent des difficultés souvent insurmontables, à s'imaginer exactement, d'après des lignes tracées sur un tableau noir ou sur le papier, la forme exacte d'un solide. Le tableau de M. B. supprime totalement ces difficultés. Quand on voit il faut bien croire, et dans toutes les classes ou l'on emploiera ce tableau, tous les élèves seront à même de suivre aisément et de comprendre les explications du professeur.

Il serait trop long de détailler ici les avantages qu'offre le même tableau pour calculer le volume d'un corps quelconque. Ce corps fut-il de la forme la plus bizarre, on trouvera, dans le tableau de M. B., une ou plusieurs figures qui représentent approximativement ce corps ou les parties dans lesquelles on peut la décomposer, et il deviendra dès lors facile de calculer, du moins avec une erreur très faible et inappréciable dans la pratique, le volume d'un corps de forme quelconque.

Ottawa, le 27 x bre 1871 E. B. DE ST. AUBIN,

(Extrait du *Courrier du Canada*, 1er Octobre 1873.)

Nous apprenons avec plaisir qu'une nouvelle médaille d'honneur vient d'être décernée à un canadien par une société française. M. Chs. Baillairgé de cette ville, a reçu cet honneur de la " Société pour la généralisation de l'instruction en France, " et il est en même temps nommé membre honoraire de cette société. Nos félicitations à M. Baillairgé qui par son travail contribue à faire connaître son pays à l'étranger.

"*Stereometrical Tableau.*"—In a large number of schools in Germany the pupils are taught, from the time they commence the alphabet, to judge of color and geometrical form by the regular use and comparison of colored slips and small blocks of wood representing the elementary principles which will, in after years, be called into study. The advantages of this early training are very manifest, as all are aware of the superiority of this nationality in that branch of science. An exceedingly ingenius device for the study of forms has been invented by Mr. C. Baillairgé, a native of Quebec, and patented in the United States, Canada, and Europe. It consists of a board about five feet long, and three feet wide, on which are placed some two hundred models, comprising, so to say, all the elementary forms, their segments and sections and numerous other solids, both simple and compound. The instruction conveyed by this tableau, appealing as it does to the uneducated eye and mind, is, the inventor thinks, destined to be of great use in developing the intelligence of the beginner and the untaught masses of mankind, M. Baillairgé is in possession of a mass of printed testimonials from high officials and other distinguished men in Canada and Europe, together with reports from educational institutions, all highly complimentary to him and the invention. A specimen of this stereometrieal tableau may be seen at No. 7 Park street, of this city, in the possession of Mr. S. W. Townsend; and we would recommend our teachers to call and inspect it, believing as we do that this method of training should be considered as a subject of great importance.—Hamilton *Daily Spectator* 19 Sept. 1873.

BAILLAIRGÉ'S STEREOMETRICAL TABLEAU.

Our engraving is a perspective view of the above named educational device, which has been patented for its inventor, Mr. C. Baillairgé, of Quebec, in the United States, Canada and Europe. It consists of a board, about five feet long and three feet wide, with some two hundred wooden models, comprising, so to say, all the elementary forms, their segments, and sections, and numerous other solids, simple and compound.

The tableau is set in an appropriate frame, with glass covering, so as to exhibit the models while excluding the dust. The front can be opened at pleasure so as to afford access to the models, each of which is merely supported on the board by a round nail or wire, which admits of its easy removal and replacement by teacher or pupil. The instruction conveyed by this tableau, appealling, as it does, to the uneducated eye and mind, is, the inventor thinks, destined to be of great use in developing the intelligence of the untaught masses of mankind. He expects to introduce it into all the educational institutions of the United States and elsewhere, as it is now being disseminated in Canada; and he has no doubt that the tableau will also find its place in the studio of the engineer and architect, to whom the models will be suggestive of various forms and relative proportions which cannot fail to aid them in their pursuits. The rapid success attained by a school in Quebec, in mensuration of all kinds of surfaces and yet higher mathematics, including conic sections, was attributed to the use of this tableau. Every tableau is inscribed with a rule for finding the solid contents of any body, called " the prismoidal formula." This formula has been shown, by Mr. Baillairgé in his treatise on geometry and mensuration published in 1866, to be less restrictive than supposed, and he has added to the known solids, measurable thereby, a long list of others discovered by him, the whole of which are given in the tableau. Each tableau is also accompanied by a printed treatise, explanatory of every use to which the models can be put. Mr. Baillairgé is in possession of a mass of testimonials, from high officials and other distinguished men, both in Canada and Europe together with reports of various educational and other institutions, all highly complimentary to him and his invention.

Dr. Wilkie, of Quebec, thinks " the government would confer a boon on schools of the middle and higher classes by affording access to so suggestive a collection;" and Professor Newton, of Yale College, considers the tableau " of great use for showing the variety and extent of applications of the prismoidal formula. "—*Scientific American* June 1st 1872.

WORCESTER FREE INSTITUTE.

Worcester, Mass., 24 *July*, 1873.

This is to certify that I have carefully examined Baillairgé's models as applied by him to the teaching of mensuration by the prismoidal formula and I consider them eminently useful in all schools where mensuration is taught.

(Signed,) C. O. THOMPSON,
Principal Wr. Free Inst.

—*Geometry, Mensuration and Stereometrical Tableau*, by CHARLES BAILLAIRGÉ, civil engineer, &c.; *Middleton et Dawson, éditeur, Québec*, 1872.

M. E. Blain de St. Aubin, donne l'appréciation suivante du travail de M. Baillairgé:

……………………………………………………………

—On sait quelle série interminable de règles ou formules, dont plusieurs très-compliquées, les anciens traités de géométrie donnent pour le mesurage des solides. M. B. n'en a qu'une qu'il énonce comme suit et démontre clairement être applicable à toute espèce de solides, si bizarres

que puissent être leurs formes,—" A la somme des surfaces des bases parallèles du solide à évaluer, ajouter 4 fois la surface au centre et multiplier le tout par la sixième partie de la hauteur ou longueur du solide."

C'est dans le but de populariser l'usage de cette règle que M. B. a eu recours à son *Tableau Stéréométrique.* " Ce *tableau,* dit M. Baillairgé, est un cadre où sont placés environ 200 modèles différents de solides; chaque modèle peut être déplacé à volonté, en sorte qu'on peut le mettre entre les mains de l'élève pour qu'il l'examine. Le *tableau* comprend toutes les formes élémentaires imagilables de solides, depuis le prisme ordinaire jusqu'au cône concave, etc......, etc .... " Sur chaque modèle,—dit plus loin M. B.—est tracée une ligne qui indique la nature et les dimensions de la section du milieu......................"

On conçoit aisément les avantages que présente l'emploi de ce *Tableau.* L'élève doit apprendre en fort peu de temps la manière d'appliquer sûrement *l'unique* formule, énoncée tout à l'heure, au calcul du volume de chacun des 200 solides contenus dans le *Tableau;* et. plus tard, dans la pratique, il s'habituera vite à décomposer un solide quelconque en parties se rapprochant, par la forme, des modèles qu'il a ainsi étudiés.

Quant aux solides de formes comparativement régulières, tels que pièces de bois, blocs de marbre ou de pierre, réservoirs et chaudières dans les usines à vapeur, les distilleries, etc., l'application de la formule de M. B. offre des facilités et des avantages qui défient tout concurrence, et nul doute qu'elle se répandra universellement au grand avantage de tous les praticiens. Telle est, du reste, la prédiction que n'ont point hésité à faire plusieurs savants étrangers qui ont eu connaissance de la découverte de M. Baillairgé; et nos meilleurs professeurs canadiens sont tous du même avis...
—Journal de l'Instruction Puplique, Nov. 1873.

Je me réserve de revenir bientôt sur le tableau stéréométrique de M. Baillairgé, ou nouveau système de toiser tous les corps, segments, troncs et onglets de ces corps par une seule et même règle. Ce travail, d'une importance majeure, comme tout ce qu'a produit M. Baillairgé, est marqué au cachet de l'utilité pratique. C'est un esprit progressiste, un jugement rare et une forte conception, qui ont présidé à son exécution. En le couronnant d'un premier prix et d'un diplôme, le juré a su rendre témoignage au mérite, et nul doute que nos colléges et autres institutions de première classe ratifieront cette appréciation en l'introduisant dans leurs classes pour aider l'enseignement ardu des mathématiques.—*L'Opinion Publique.*

Sept. A. N. MONTPETIT.

que puissent être leurs formes.—[illegible]
lèles du solide à évaluer, [illegible]
tout par la sixième partie de la [illegible]

C'est dans le but de [illegible]
recours à son *Tableau* [illegible]
est un cadre où sont [illegible]
chaque modèle peut être [illegible]
entre les mains de l'élève pour [illegible]
toutes les formes élémentaires [illegible]
dinaire jusqu'au cône [illegible]
chaque modèle,—dit plus loin M. B.—[illegible]
nature et les dimensions de la section [illegible]

On conçoit aisément les avantages qu[illegible]
L'élève doit apprendre en fort peu de temps [illegible]
ment *l'unique* formule, énoncée tout à l'heure [illegible]
chacun des 200 solides contenus dans le Tableau. [illegible]
pratique, il s'habituera vite à décomposer [illegible]
se rapprochant, par la forme, des modèles qu[illegible] [illegible] révé-

Quant aux solides de formes [illegible] [illegible] [illegible]lles
pièces de bois, [illegible] [illegible] main du
les usines à vapeur, les distilleries [illegible] [illegible]grès sur ce
B. offre des facilités et des avantages [illegible]
doute qu'elle se répandra [illegible] [illegible]démique, ne craint
praticiens. Telle est, du reste, [illegible] [illegible] stéréométrique, il
plusieurs savants étrangers [illegible] [illegible], à un élève, ayant
Baillairgé; et nos meilleurs [illegible] [illegible] de solides avec plus de
avis.............................. [illegible], et même en dix-huit mois,
—*Journal de l'Instruction* [illegible] [illegible] C'est si simple, si clair, dit-il,

[illegible] amitié avec plusieurs instituteurs

Je me réserve de [illegible] [illegible]dèle, je connais leur opinion et sais
Baillairgé, ou nouveau [illegible] de cette nomenclature. Tous s'accor-
et onglets de ces corps [illegible] [illegible]aillairgé répandra nécessairement le goût
portance majeure, [illegible] province. Ce serait donc un bien grand
au cachet de l'utilité [illegible] la jeunesse canadienne, que de distribuer cet
rare et une forte [illegible] grande importance.
ronnant d'un [illegible] des instituteurs sont peu rénumérés et que leurs
au mérite [illegible] sont très-limitées, nous ne pouvons même pas dési-
cla[illegible] [illegible] de sacrifice achètent, de leurs propres deniers, un
[illegible]tudes et le temps qu'il a coûtés, est audessus de leurs

[illegible]rnement du Nouveau-Brunswick a répandu ce tableau
[illegible]les de cette province. L'Honorable M. Chauveau, mi-

No. 2272-71. ***Ministère de l'Instruction Publique.***

QUÉBEC, ce Sept. 1872.

C. BAILLAIRGÉ, Ecuyer, Québec.

Monsieur.—J'ai l'honneur de vous transmettre, sur l'autre feuillet, copie de la résolution adoptée par le Conseil de l'Instruction Publique, approuvant votre "Tableau stéréométrique pour toiser tous les corps, segments, troncs et onglets de ces corps," ainsi que votre "Nouveau traité de géométrie et de trigonométrie rectiligne et sphérique, suivi du "Toisé des surfaces et des volumes."

LOUIS GIARD. · Secrétaire-Archiviste.

Journal de l'Instruction Publique, Nov. 1872.

TABLEAU STÉRÉOMÉTRIQUE DE M. BAILLAIRGÉ.

Nous avons déjà eu occasion de parler de ce tableau, et de l'impulsion extraordinaire qu'il doit donner l'étude du toisé. L'auteur a, depuis obtenu les certificats les plus flatteurs de tous les hommes compétents sur cette matière. Le tableau, avec la formule qui l'accompagne, est appelé, au dire de tous, à faire une véritable révolution dans les méthodes de mesurage pour les solides. Le conseil de l'instruction publique, à sa dernière séance l'a approuvé, avec le "Traité de géométrie" du même auteur. Ce tableau, de cinq pieds par trois contient deux cents modèles en bois, comprenant toutes les variétés de formes, depuis les corps les plus simples jusqu'aux corps les plus bizarres et les plus difficiles à toiser. Ces modèles sont mobiles et ne sont fixés au tableau que par une petite tige en fer, de sorte que les élèves peuvent les examiner et les étudier de main en main. L'aûteur espère que son œuvre, tout en simplifiant et en facilitant les calculs du savant, aura surtout pour résultat de mettre à la portée de tous, une science demeurée jusqu'ici, par ses difficultés presque insurmontables, en dehors des atteintes du plus grand nombre. Tous les colléges et les écoles trouveront dans le "Tableau stéréométrique" un puissant moyen de progrès sûrs et rapides.

Nous en publions ci-dessous une gravure, et nous renvoyons le lecteur, pour de plus amples détails, à notre bulletin bibliographique, où un homme expert en cette matière, M. Blain de St. Aubin, en fait une excellente appréciation, dans le compte-rendu qu'il donne d'une conférence lue par M. Baillairgé, devant la Société historique de Québec.

Extrait du Courrier du Canada du 20 *Août* 1873.

TABLEAU STÉRÉOMÉTRIQUE DE CHS. BAILLAIRGÉ, ECUYER.

Monsieur le Rédacteur,

M'accorderiez vous dans les colonnes de votre bienveillant journal, quelques lignes touchant un object bien paisible, malgré que nous soyions dans un temps de *grande agitation?* Je désire appeler l'attention du gouvernement local sur un sujet qui concerne l'enseignement public : je veux parler du tableau stéréométrique de M. Chs. Baillairgé et exprimer mon humble opinion sur les résultats pratiques que peut avoir cette étude pour les professeurs et les élèves.

Si l'on en juge par les témoignages partout obtenus, ce tableau est destiné à une grande renommée. D'ailleurs cette renommée s'étend aujourd'hui non-seulement dans les différentes Provinces de la Puissance, mais même aux Etats-Unis, en Europe, en France surtout, d'où sont venus à M. Baillairgé les éloges et les recommandations des plus hautes autorités compétentes.

Plusieurs de nos maisons d'éducation l'ont mis en pratique, et en ont obtenu des résultats qui dépassent toutes prévisions. Les dames Ursulines de Québec, les premières, ont mis le tableau et la formule de M. Baillairgé en usage, avec tant de bonheur, que toutes les autres communautés religieuses ont décidé de suivre cet exemple, convaincues qu'elles sont de l'utilité et des grands avantages qu'offre ce précieux travail de notre distingué mathématicien. Outre les Révérendes dames Ursulines, les Révérendes sœurs de la Congrégation de St. Roch, les sœurs de la Charité, celles du Bon Pasteur, et les dames du couvent de Jésus-Marie sur le chemin du Cap-Rouge, ont toutes décidé que leurs élèves seraient interrogées sur ce tableau aux prochains examens.

Un de mes amis, instituteur diplômé d'Ecole académique, ne craint pas d'affirmer qu'à l'aide de la formule et du tableau stéréométrique, il ferait comprendre en quelques leçons seulement, à un élève, ayant des aptitudes ordinaires, les toisé de toutes espèces de solides avec plus de succès qu'il ne l'a jamais pu obtenir, en un an, et même en dix-huit mois, avec les formules suivies jusqu'à présent. " C'est si simple, si clair, dit-il, que ça saute aux yeux même des enfants."

Quant à moi, M. le Rédacteur, lié d'amitié avec plusieurs instituteurs possédant leurs diplômes d'Ecole Modèle, je connais leur opinion et sais leur désir d'enseigner, par le moyen de cette nomenclature. Tous s'accordent à dire que l'ouvrage de M. Baillairgé répandra nécessairement le goût des mathématiques dans cette province. Ce serait donc un bien grand service à rendre au pays, à la jeunesse canadienne, que de distribuer cet ouvrage reconnu d'une si grande importance.

Comme la plupart des instituteurs sont peu rénumérés et que leurs ressources pécuniaires sont très-limitées, nous ne pouvons même pas désirer que ces hommes de sacrifice achètent, de leurs propres deniers, un tableau qui, par les études et le temps qu'il a coûtés, est audessus de leurs moyens.

Déjà le gouvernement du Nouveau-Brunswick a répandu ce tableau dans toutes les écoles de cette province. L'Honorable M. Chauveau, ministre de l'Instruction Publique, en en reconnaissant toute la valeur, s'était engagé envers l'auteur à le faire répandre dans nos écoles et nos maisons d'éducation, et le bureau des Travaux Publics en a distribué un grand nombre d'exemplairès, pour guider ses employés dans tous les mesurages.

Il est donc à espérer que le gouvernement local prendra en considération le désir unanime des hommes qui s'occupent de l'instruction publique, et pourvoiera libéralement les instituteurs d'écoles académiques et d'écoles modèles de ce tableau si précieux et destiné à toute une heureuse révolution dans l'enseignement des sciences mathématiques. Ce sera en même temps un bienfait pour la jeunesse studieuse et un honneur pour notre gouvernement, qui le premier aura encouragé ce mode d'enseignement simple, pratique, et fructueux.

UN AMI DE L'ÉDUCATION ET DU PROGRÈS.

(*Extrait d'une lettre de Aug. Humbert à V. Vannier.*)

PARIS, le 4 Août 1872.

J'ai fait recevoir M. Baillairgé Membre Titulaire de la Société de Vulgarisation pour l'enseignement du Peuple.

A défaut de son tableau que nous n'avons pas, j'en ai exposé la photographie à Paris et à Lyon.

Son tableau appelle tous les regards à l'exposition, on m'interroge, on me demande des explications que j'ai données de mon mieux, et cela intéresse beaucoup. J'ai donc tout lieu de compter sur un succès.

....

Extrait d'un Journal de Paris du 16 *Août* 1873.

TABLEAU STÉRÉOMÉTRIQUE

Nouveau système de toiser tous les corps, quelles que soient leurs formes, par une seule et même règle : par C. BAILLAIRGE, Architecte, etc., à Québec (Canada.)

M. C. Baillairgé, architecte du gouvernement à Québec (Canada), nous a adressé un tableau qu'il appelle *stéréométrique*.

Ce tableau, espèce d'armoire, contient deux cents petits solides en bois, affectant deux cents formes différentes, sphères, demi-sphères, segments, cônes, troncs de cônes, pyramides, troncs de pyramides, polyèdres les plus variés, onglets, etc. ;—enfin figurant toutes les formes élémentaires de solides qu'on peut rencontrer dans les arts, la construction et la nature.

M. C. Baillairgé donne, inscrite en tête de ce tableau, pour trouver le volume d'un de ces deux cents corps, une formule qui pourra s'appliquer à un corps quelconque de la nature, car son volume pourra toujours se décomposer en un de ceux compris dans le tableau ou s'en rapprocher infiniment.

Voici cette règle :

A la somme des surfaces des extrémités parallèles ajouter quatre fois la surface au centre, et multiplier le tout par la sixième partie de la hauteur ou longueur du solide.

Cette formule est évidemment d'une simplicité extrême et abrégera considérablement les calculs pour les hommes qui s'occupent spécialement de mesurage et de métrage,—tels que les toiseurs-vérificateurs, les mesureurs, jâugeurs des contributions indirectes et des douanes, les arpenteurs, géomètres, constructeurs, architectes et ingénieurs.

Quant aux élèves des maisons d'éducation, des écoles d'arts-et-métiers, il est bon de leur faire connaître une méthode abrégée et se rapprochant infiniment de l'exactitude. Mais nous pensons qu'il convient dc leur enseigner comment on arrive par les données de la science à formuler cette règle universelle.

Si l'on doit éviter de donner trop de temps aux études théoriques, il faut bien prendre garde à l'excès contraire. Le jeune homme dont toute la science consiste en une mémoire bien farcie de formules dont il ne sait pas retrouver les éléments est toujours un praticien bien embarrassé, lorsque les conditions du travail qu'il a à remplir sortent des données ordinaires.

Ce sera un bon caboteur, si vous le voulez, qui manœuvre bien son navire en vue des côtes ; mais, une fois en pleine mer, il sera désorienté, ne sachant ni faire le point, ni déterminer sa position par le calcul.

M. Baillairgé, du reste, paraît être complètement de notre avis, car, antérieurement au tableau dont nous parlons, il a publié un excellent ouvrage sur la géométrie et la trigonométrie, dans lequel il a développé une foule de questions pratiques, de théorèmes et de formules remarquables par leur nouveauté, parmi lesquelles se trouve celle que nous venons d'exposer concernant la mesure des solides de forme quelconque.

Cet ouvrage a attiré à M. Baillairgé les éloges mérités des hommes compétents du Canada.

Nous regrettons vivement que l'ouvrage n'ait pas accompagné le tableau qu'on nous a adressé ; le compte-rendu que nous avons entrepris y aurait gagné en intérêt et en clarté. Réduits aux éléments ordinaires de la science, nous sommes entraînés à des calculs trop longs, et probablement par des sentiers moins directs que ceux indiqués par l'auteur, pour que nous vous invitions à vérifier avec nous l'exactitude de la règle donnée par M. Baillairgé. A ceux à qui la trigonométrie et la géométrie descriptive sont familières, nous dirons : Faites comme nous, cherchez et vous trouverez.—A ceux qui travaillent pratiquement, nous conseillerons d'appliquer la règle que nous indiquons à un solide quelconque, dont ils savent mesurer le volume par une autre méthode, et ils se rendront compte de l'exactitude de la formule par la similitude des deux résultats ; d'où ils concluront que ce qui est vrai pour un cas le sera pour l'autre.

On reconnaîtra ainsi que la formule est mathématiquement exacte, pour les prismes et prismoïdes, cylindres et cylindroïdes, droits ou inclinés, pyramides régulières, et irrégulières, et les troncs de ces solides entre bases parallèles, pour le cône droit ou oblique et son tronc entre bases parallèles, pour la sphère ou le sphéroïde et tout segment ou tronc de ce corps séparé du solide entier par un plan incliné d'une manière quelconque aux axes ou diamètres, ou compris entre deux plans parallèles quelconques, pour les conoïdes paraboliques et hyperboliques droits ou inclinés et les troncs de ces solides compris entre plans parallèles ; et ces divers solides constituent dans leur ensemble la presque totalité des solides élémentaires que l'on puisse être appelé à évaluer.

Il n'y a que pour les fuseaux seuls et les onglets que la formule n'est pas mathématiquemeut exacte, mais encore se rapproche-t-on sensiblement de la vérité.

Du reste, il est toujours facile, quand un corps affecte une forme par trop fantaisiste, de le décomposer eu plusieurs solides se rapprochant des formes plus géométriques et auxquels on peut appliquer la formule en toute sécurité. Le tableau construit par M. Baillairgé donne précisément les formes les plus diverses qui permettent, en rapprochant les différents solides, d'en construire de nouveaux de formes composées. On fait ainsi l'analyse et la synthèse de la méthode, d'une part, en appliquant la méthode au volume total composé des différents solides ; de l'autre, en calculant le volume de chaque morceau séparé et en additionnant les résultats ; l'addition de cette seconde manière d'opérer doit donner le même chiffre que l'application de la règle au volume total.

Est-il nécessaire d'indiquer les applications qu'on peut faire de cette méthode dans la pratique ? Cela nous paraît inutile, nous remplirions toutes les colonnes de la *Revue ;* les hommes pratiques sauront bien le moment de s'en servir.

Quant aux élèves, je laisse aux professeurs le soin de leur faire toucher du doigt ces applications, multiples et si journalières.

Pour me résumer, je dirai que la méthode de M. C. Baillairgé est d'une simplicité et d'une exactitude remarquables, qu'elle est appelée à rendre de très-grands services aux praticiens, en leur évitant des pertes considérables de temps et en leur permettant d'obtenir les résultats cherchés aussi exactement que par les anciennes formules mathématiques; c'est donc un devoir pour tous de propager ce système; et pour moi un plaisir d'en féliciter l'auteur. J. **Morand.**

SECRETARIAT GENERAL
RUE TRUFFAUT, 14,
Paris.

AFFRANCHIR

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Membres titulaires.

	F.
Droits d'entrée	1 "
Cotisation annuelle	2 "

Paris, le 1er Août 1872.

A
Monsieur Baillairgé,
Architecte, etc.
A Québec, (Canada).

Monsieur,

J'ai l'honneur de vous donner avis que le Conseil Supérieur vient de vous admettre à faire partie de la Société de Vulgarisation pour l'Enseignement du peuple à titre de Membre Titulaire.

Nous sommes heureux d'une décision qui assure à notre Œuvre votre précieux concours et nous espérons les meilleurs effets de votre propagande active en faveur de l'Instruction et de l'Education populaires.

Veuillez agréer,
Monsieur et très honoré Collègue,
l'assurance de mes sentiments de haute considération.

Le Secrétaire Général fondateur,
(Signé,) **Aug. Humbert.**

Extract from a letter of Brother Basilian of St. Thomas, to Brother Paulian of the Lasalle Institute, New York.

........................Mr. Baillairgé's system has also received the general approbation of the principal Houses of Education in the Dominion of Canada, and in foreign land. It surpasses, by its facility and its simplicity, all that ever appeared of the like on the continent.

I am very anxious to have you examine Mr. B's system, as no one is better able to judge of its merits than yourself, being, as every one knows, so distinguished a mathematician.

Extrait d'une lettre du Rév. Frère Basilian, ci-devant de Montréal, au Frère Ligouri, Visiteur des Frères des Ecoles Chrétiennes, à Londres (Angleterre) et au Rév. Frère Patrick, à Paris.

................................Mr. Baillairgé est reçu membre honoraire de la Société d'Education de France. Honneur que lui a mérité son nouveau système de stéréométrie et pour lequel il est mandé à Paris, par le Président de la Société, pour y recevoir le premier prix de concours.

Ce système a aussi reçu l'approbation des principaux établissements d'éducation de la Puissance du Canada et à l'étranger ; il surpasse par son extrême simplicité tout ce qui a jamais paru en ce genre, comme vous pourrez vous en convaincre par ce que vous en dira ce monsieur.

Je suis très aise, très cher Frère visiteur, que vous preniez connaissance de ce système, par ce que je ne connais personne qui puisse mieux que vous apprécier le mérite de cette nouvelle découverte.

OTTAWA, 17 Décembre 1873.

.............................J'ai enseigné, moi-même, depuis de longues années, cette spécialité d'évaluation des surfaces et solides, et, vu l'extrême complication que comporte cette étude, j'applaudis vraiment au mode *sommaire* que vous venez de nous révéler, lequel paraît évidemment destiné à ***supplanter*** le système suivi jusqu'ici.

Sans être révolutionnaire, j'aime ces petits bouleversements tendant à réformer certains régimes parfois trop conservateurs, lorsque surtout ces benins *cataclysmes*, n'ont d'autre effet que de vulgariser une branche d'enseignement qui devrait être accessible à tous.

Votre dévoué,

Révd. Frère des Ecoles Chrétiennes. F. ANDRÉ.

(*Translation*).

St. Thomas, December, 15 1873.

Sir,

I beg to acknowledge receipt of your letter of the 4th instant informing me that the first prize had been awarded you, in France, for your Stereometrical Tableau. This I consider a happy presage of its future success.

It pleases me as it must all lovers of science, that, notwithstanding the prejudices of the age, you have known how to triumph over the difficulties which naturally presented themselves in the research of the fundamental principles of such a system. Indeed, with the grace of God, your efforts will soon be crowned with complete success; and I hope you will be rendered the glorious testimony of being a benefactor of the human race, by the discovery of a new system which will give a fresh impetus to popular education, and be the means of giving to the less educated an opportunity of understanding its commercial value. It is to be hoped that the promoters of science will recognise the immense benefit you are conferring on Society at large and will render you full justice.

Had I remained in Montreal, I should have labored to introduce your system in our classes there. But be assured, Sir, that very soon, I hope, you will have the satisfaction of seeing your new system introduced to the greater number of our institutions in Canada. It must be acknowledged that until recently it was not much known, but owing to your kindness its reception in our institutions will soon make it so.

It cannot be otherwise than that, after so many testimonials have been awarded you from the principal educational establishements of this Province, approving of the superiority of your system—not only for its facility but for its simplicity—you will soon, I say, have the satisfaction of seeing it in use in all educational institutions, even the most elementary, throughout the Province.

I herewith send you a letter of introduction to our venerable Brother Patrick, in Paris, lately a visitor to the christian schools in the United States, a man of great merit and a very great lover of Science. The second letter is for the Rev. Brother Ligouri, visitor to England and residing in London (french) and who recently visited Canada. When here he saw your work on Geometry, and passed high encomiums on it, I have no doubt but that he will most willingly adopt your system. The third letter is for the Rev. Brother Paulian, visitor to New-York, a mathematician of the first order.

I stopped at Quebec with the intention of seeing you, but was informed that you were out of Town. I think that the Rev. Brother Aphraates, Director of Quebec, who is so well known in France, will do himself the pleasure of giving you several letters of introduction.

I have the honor to be, Sir, Your very humble Servant

(Signed) Frère Basilian.

(Translation).

STE. ANNE DE LA PÉRADE, 23d December 1873.

Sir,—I have the honor to acknowledge the receipt of your interesting letter of the 22d instant.

With you, I should very much like to see your Stereometrical Tableau figuring in my School, and if the Government will but lend a helping hand there is but little doubt it will be adopted by all our Academies and Model Schools. Believe me, Sir, as the friend of youth, to whose instruction I have devoted myself for 28 years past, no exertion will be wanting on my part, to secure for them the advantages to be desired from your important discovery.

I authorize you to add my name to those petitioning the Honorable The Minister of Public Instruction to subscribe for your Tableau and introduce it in our Schools, Academies and Model Schools. I am of the opinion that the youth of the Country will derive great advantage from it and that it will contribute not slightly to disseminate among the people a taste for the mechanical arts, a taste which, so to say, only begins to have existence among us.

I have no doubt but that the government will give you such encouragement as will, at least in part, indemnify you for your great labor and the expenses you have been at in the advancement of useful science throughout the Country. The flattering testimonials conferred upon you by France and the United States should impress upon those at the head of our affairs the justice of coming to your aid by asking of the Legislature to vote a sum which would renumerate you for your invaluable labor and reimburse you in the amount you have expended.

I am therefore prepared to sign any petition that will have for its object the introduction of your excellent Tableau in all our Schools, or of demanding of the Legislature to indemnify you for your expenditure and labor.

I will write to Mr. Letourneau for a copy of the petition being signed and think I can assure you of obtaining the signatures of a good many persons of influence in this locality.

With consideration, I remain, Sir

Chs. Baillairgé Esq. Your devoted Servant
Architect &c. Quebec. (Signed) D. N. ST. CYR.

QUEBEC, Sept. 27th 1873.

The undersigned who have witnessed the many advantages of the Stereometrical Tableau as applied in the teaching of Geometry and Mensuration, ect., to the pupils of the Quebec School of Arts and Manufactures.

Would recommend that the Montreal School be also provided with one or more of these useful adjuncts.

(Signed,) Hon. E. CHINIC,
J. WOODLEY,
L. J. BOIVIN,
Revd. O. AUDET.

Members of the Board of Arts & Manufactures for the Province of Quebec.

SYNOPTICAL OR ABRIDGED
KEY
TO THE AUTHOR'S NEW SYSTEM
OF MEASURING ANY SOLID,
SEGMENT, FRUSTUM OR UNGULA OF SUCH SOLID,
BY ONE AND THE SAME RULE.

(**1**.) *To the sum of the areas of the opposite and parallel ends or bases of the body to be measured, add four times the area of a section thereof parallel to these bases and equidistant from each of them, and multiply the whole by the sixth part of the height or length of the solid.*

(**2**) To be brief, we will call "*intermediate or half-way section*" the section in question in the formula; or again, and at will; "*centre section*" "*middle section*," and we shall always designate this section by the letter M, initial letter of the word *middle* as we designate by B and B′ the opposite bases or ends of the solid, and by L or H its length or height.

(**3**) The *length* or *height* of the solid under consideration, shall always be the distance between its parallel bases or ends, that is the perpendicular drawn from one of these bases to the other or to the plane of this base, produced if necessary.

Then the formula will write:

$$\text{Volume} = (\text{area B} + 4 \text{ area M} + \text{area B}') \times \tfrac{1}{6} \text{ L or H.}$$

or:

$$\text{V.} = (\text{B} + 4\,\text{M} + \text{B}')\,\tfrac{1}{6} \text{ L or H.}$$

or,

to dispose the areas so as to facilitate their addition:

$$\text{V} = \left\{ \begin{array}{l} + \ \text{area B} \\ +4 \ \text{area M} \\ + \ \text{area B} \end{array} \right\} \times \tfrac{1}{6} \text{ L or H,} \qquad \text{or} \quad \left\{ \begin{array}{l} + \ \text{B} \\ + \ 4\text{M} \\ + \ \text{B}' \end{array} \right\}$$

Sum of the areas
× ⅙ L or H.

: Use prismoidal formulae for contents of a sphere

Nature and value of the bases B, B'.

(**4**) Sometimes one of the ends or bases of the solid, as with the pyramid, cone, conoid, segment or ungula of a sphere, spheroid, or spindle, &c., will be but a point and its area, consequently, null or equal to zero (0). Sometimes, each of the bases will be null as to area or $=0$, as in the case of the sphere and spheroid; now, one of the bases will be a simple line, as for the wedge and certain prismoids and ungulæ, and its surface again null; at other times, each of the bases, as with certain prismoids, will be but a line and the surfaces null, as before; but in all cases, the author advises the pupil to maintain entire the formula and to write, as the case may be:

$$\text{V.} = \overset{\text{areas}}{\left\{\begin{array}{l} +\ B \\ +\ 4M \\ +\ B' \end{array}\right\}} \qquad \text{or, V.} = \overset{\text{areas}}{\left\{\begin{array}{l} +\ 0 \\ +\ 4M \\ +\ B' \end{array}\right\}}$$

$$\text{Sum} \times \tfrac{1}{6}\,\text{L or H.} \qquad \text{Sum} \times \tfrac{1}{6}\,\text{L or H.}$$

$$\text{or, V.} = \overset{\text{areas}}{\left\{\begin{array}{l} +\ B \\ +\ 4M \\ +\ 0 \end{array}\right\}} \qquad \text{or, V.} = \overset{\text{areas}}{\left\{\begin{array}{l} +\ 0 \\ +\ 4M \\ +\ 0 \end{array}\right\}}$$

$$\text{Sum} \times \tfrac{1}{6}\,\text{L or H.} \qquad \text{Sum} \times \tfrac{1}{6}\,\text{L or H.}$$

(**5**) **REM.** It is clear from what precedes that the respective surfaces in question are all plane surfaces, or must be considered as such, and that, with the author's system, every surface is null, to which a plane surface or a plane can touch but in one point, as in the sphere, spheroid and conoid; which does not prevent one from measuring in the same manner by the formula, and with the same accuracy, a spherical cone or pyramid, or any frustum of such a body comprised between paralled or concentric bases, one of which is consequently concave and the other convex.

(**6**) These enunciations would be quite sufficient to give a perfect understanding of the author's system, but some observations concerning more particularly, if not each of the solids of the tableau, at least every category or class thereof will perhaps not be useless.

(**7**) We say "class" or "category" and in fact it is proper to observe that the solids are disposed, on the tableau, by groups or families, each in one or several vertical rows. These rows are 20 in number and the horizontal rows 10 in number, forming 200 pieces.

The first row to the right (it would be indifferent to reverse the order and begin at the left) comprises the prism under some of its varied forms.

(**8**) The four following ranges offer the prismoid, under several diversified aspects (see introduction, page 6) including the regular or platonic solids, (dodecahedron, icosahedron, &c.,) and certain ungulæ of prisms.

(**9**) The sixth row, still going towards the left, is the pyramid and the frustum of that solid.

(**10**) Rows 7 and 8 show the right, inclined, truncated cylinder, and the numerous ungulæ, and frusta of ungulæ, of this solid, with also some cylindroids.

(**11**) 9 and 10 are the right and inclined cones, their frusta and ungulæ.

(**12**) 11 is the concave cone with its varieties and sections. 12 and 13 are the right and inclined parabolic and hyperbolic conoids, with their frusta, ungulæ and truncated ungulæ.

(**13**) 14, 15 and 16, the flattened and elongated spindles with their decomposed parts and varieties.

(**14**) 17 and 18 are the sphere and its segments, frusta, ungulæ, &c., spherical cone and pyramid and frusta of these bodies between parallel bases. These solids offer also to the appreciation the spherical, tri-rectangular, tri-acutangular, tri-obtusangular, &c., triangle, and facilitate to the pupil, the understanding of spherical geometry and trigonometry, and to the professor, the teaching of these sciences.

(**15**) 19 and 20, finally, are the flattened and elongated spheroid with the decomposed parts of these bodies.

See again on this subject "The Introduction" page 7.

Let us first consider the

PRISM OR CYLINDER,

Right, Inclined, Twisted.[1]

(**16**) The prism is a body whose breadth or size is every where equal or uniform; it is, in other terms, a solid which throughout its whole length or height is of invariable diameter or thickness, and the opposite and parallel bases or ends of which, as well as each section parallel to these bases, are consequently, similar and equal plane figures; these figures may indifferently be rectilinear, curvilinear or mixtilinear.

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[1] See the Introduction, page 11, last paragraph, letter of the Revd. M. Billion mathematician of the St. Sulpice Seminary, Montreal.

We will then obtain the solidity or volume by making

$$\begin{array}{c}V.=\left\{\begin{array}{l}+\ \ \ \text{area } B\\ +\ 4 \text{ area } M\\ +\ \ \ \text{area } B'\end{array}\right\}\\ \hline \text{Sum of areas} \times \tfrac{1}{6}\text{ L or H.}\end{array} \qquad \begin{array}{c}\text{and, supposing}\\ \text{the base}\\ =1,\end{array} \qquad \begin{array}{c}=\left\{\begin{array}{l}+1\\ +4\\ +1\end{array}\right\}\\ \hline \text{Sum of areas} \times \tfrac{1}{6}\text{ L. or H.}\end{array}$$

$$=\left\{\begin{array}{r}6 \text{ a. } B\\ \text{or } 6 \text{ a. } M\\ \text{or } 6 \text{ a. } B'\end{array}\right\} \times \tfrac{1}{6}\text{ L. or H} = \left\{\begin{array}{r}\text{a. } B\\ \text{or a. } M\\ \text{or a. } B'\end{array}\right\} \times \text{L. or H.}$$

(**17**) That is: in the case of the prism, the general formula is reduced to the simplified expression: B or B′ or M × L; but we advise the pupil not to endeavour to remember this formula, simplified though it be, since he will always (see the introduction, page 9) return to it of himself; for one soon sees that it is the same thing to multiply any number by another number, or to multiply 6 times the first by the sixth part of the second.

PRISMOID

Right, Inclined, Twisted.

(**18**) The prismoid of which we treat at length, from page 161 to 167 of this work, has for its opposite and parallel bases or ends, any plane figures, equal or unequal, similar or dissimilar, rectilineal, curvilineal, or mixtilineal, and one of which, as in the case of the pyramid or the wedge, may be a simple point or a line, or each of the bases a mere line as already stated (**4**).

We must then write, according to the case:

$$V=\begin{array}{c}\left\{\begin{array}{l}+\ \ \ \text{a. } B\\ +4 \text{ a. } M\\ +\ \ \ \text{a. } B'\end{array}\right\}\\ \text{Sum of the a.}\\ \times\tfrac{1}{6}\text{ L or H.}\end{array} \quad \text{or} \begin{array}{c}\left\{\begin{array}{l}+\ \ \ O\\ +4\ M\\ +\ \ \ B'\end{array}\right\}\\ \text{Sum of the a.}\\ \times\tfrac{1}{6}\text{ L or H.}\end{array} \quad \text{or} \begin{array}{c}\left\{\begin{array}{l}+\ \ \ B\\ +4\ M\\ +\ \ \ O\end{array}\right\}\\ \text{Sum of the a.}\\ \times\tfrac{1}{6}\text{ L or H.}\end{array} \quad \text{or} \begin{array}{c}\left\{\begin{array}{l}+\ \ \ 0\\ +\ 4M\\ +\ \ \ 0\end{array}\right\}\\ \text{Sum of the a.}\\ \times\tfrac{1}{6}\text{ L or H.}\end{array}$$

PYRAMID, CONE

Regular, Irregular. Right, Inclined.

(**19**) In the pyramid, the base, or one of the ends is any plane figure and the intermediate section a figure similar to the base and equal in area to the fourth part of the base (**95, T.**).

The section of the cone, as of the pyramid, by a plane passing through its axis and apex, is a triangle, and the breadth of this triangle, taken at the half of its altitude is (page 85, rem.) half that of the base. Now, this same half-way breadth of the triangle furnishes the corresponding diameter of the pyramid or of the cone; that is, the diameter of the half way section of the solid by a plane parallel to the plane of its base. The cone, if right, has for base a circle; if inclined, an ellipse, and for its middle section parallel to the base, a circle or ellipse similar to this base and equal in surface to the fourth part of it; the other base or end, of the cone or pyramid, is a mere point, and its area in consequence is null or = 0.

Which gives us:

$$\text{V.} = \frac{\left\{\begin{array}{l} + \ \text{a. B,} \\ +4\ \text{a. M} \\ + \ \text{a. B} \end{array}\right\}}{\text{Sum of the areas } \times \frac{1}{6} \text{ L or H.}} = \frac{\left\{\begin{array}{r} + \quad 0 \\ + \ 4\text{M} \\ + \quad \text{B} \end{array}\right\}}{\text{Sum of the areas } \times \frac{1}{6} \text{ L or H.}}$$

$$\begin{array}{c}\text{And suppos.} \\ \text{the base} \\ =1,\end{array} = \frac{\left\{\begin{array}{l} + \qquad 0 \\ + \ 4 \times \frac{1}{4} \\ + \qquad 1 \end{array}\right\}}{\text{S. of the a. } \times \frac{1}{6} \text{ L or H.}} = \frac{\left\{\begin{array}{l} +0 \\ +1 \\ +1 \end{array}\right\}}{\begin{array}{c}\text{S. of the a. } \times \\ \frac{1}{6} \text{ L or H.}\end{array}} = \frac{\left\{\begin{array}{c} 1 \times \frac{1}{3}\text{L, H} \\ \text{or} \\ \text{B} \times \frac{1}{3}\text{L, H} \end{array}\right\}}{}$$

That is : for the pyramid, the cone, the formula reduces to multiplying the surface or area of the base by the $\frac{1}{3}$ of the height.

PARABOLIC, HYPERBOLIC CONOID

Right, Inclined.

(**20**) Here the base is a circle or an ellipse, according as the solid is right or inclined, and the half-way section between the base and the apex or the opposite ends, is, as any other section parallel to the base, a figure similar to such base and in the paraboloid, equal (**7**) in area to the half of it; or, which is the same thing, the diameter of this section is equal to the square root (see the tables) of half the square of the corresponding diameter of the base. The other base or end of the solid is but a point, since we have agreed to consider as such every curved surface which a plane surface or a plane can touch, at a time, but on an infinitely small extent; that is, a point.

Whence:

$$\text{V.} = \frac{\left\{\begin{array}{l} + \quad \text{area B}' \\ + \ 4\ \text{area M} \\ + \quad \text{area B} \end{array}\right\}}{\begin{array}{c}\text{Sum of the areas} \\ \times \frac{1}{6} \text{ L or H.}\end{array}} = \frac{\left\{\begin{array}{r} + \quad 0 \\ + \ 4\ \text{M} \\ + \quad \text{B} \end{array}\right\}}{\begin{array}{c}\text{Sum of the areas} \\ \times \frac{1}{6} \text{ L or H.}\end{array}}$$

$$\text{And suppos. the base} = 1, \quad = \frac{\left\{\begin{matrix} + & 0 \\ +4 \times & \frac{1}{2} \\ + & 1 \end{matrix}\right\}}{\times \frac{1}{6} \text{ L or H}} = \frac{\left\{\begin{matrix} +0 \\ +2 \\ +1 \end{matrix}\right\}}{\times \frac{1}{6} \text{ L or H}} = \left\{\begin{matrix} 3 \times \frac{1}{6} \text{ H or L} \\ \text{or} \\ 1 \times \frac{1}{2} \text{ H or L} \end{matrix}\right\}$$

(**21**) So that for the paraboloid, the general formula is reduced to that of multiplying the area of the base by half the height; but as this expression, simplified though it be, differs from the general formula and may confuse the memory, (Introduction page 9) the pupil will do well not to endeavour to retain it; but instead of that, and to remove all doubt concerning the simplified formula, to resort immediately—although, it is true, with a few additional figures—to the sole and universal formula of the author; for, it cannot be denied that a longer process under a less tension of the mind, is less toilsome, and causes less anxiety as to the accuracy of the result, than a shorter but more arduous operation.

SPHERE, SPHEROID

Flattened, Elongated.

(**22**) In the sphere and spheroid, the only area to be computed is that of the central or half-way section, each of the two other areas being, as that for the top of the conoid, null or = 0. The central section is either a circle or an ellipsis.

Thence :

$$V = \frac{\left\{\begin{matrix} + & \text{area B}' \\ +4 & \text{area M} \\ + & \text{area B} \end{matrix}\right\}}{\text{Sum of the areas} \times \frac{1}{6} \text{ diam. or H. or L.}} = \frac{\left\{\begin{matrix} + & 0 \\ + & 4\text{M} \\ + & 0 \end{matrix}\right\}}{\text{Sum of the areas.} \times \frac{1}{6} \text{ D or } \frac{1}{3} \text{ R.}} = \left\{\begin{matrix} \text{four great circles} \\ \text{or ellipses as the} \\ \text{case may be.} \end{matrix}\right\} \times \frac{1}{3} \text{ R.}$$

(**23**) **REM.** As for the spheroid or ellipsoid, it is indifferent under which aspect it be considered, respecting its half-way section and its height, length or diameter: but as it is more simple to find, either by calculation or from the tables at the end of this treatise, the area of a circle than that of an ellipse, matters can be managed so that its central section be a circle, which will be done by performing the imaginary section of the solid by a plane perpendicular to the fixed axis. The solid would equally be measured in an inclined position (**174, R.**) being attentive however, as has been said (**3**) to take for the height or length a perpendicular to the plane of section and terminated on both sides by planes parallel to such a section and both of them on opposite sides tangential to the solid under consideration.

SEGMENT

of Sphere, Spheroid.[1]

(**24**) The segment having but one computible base, the formula to measure it does not differ in any way from that of the cone or conoid, except however that the relation between the area of its base and that of its intermediate section varies with the height of the segment. The radius of this section in the segment of a sphere "small circle of the sphere" is equal (**374**, **G.**) to the square root (see the table) of the product of the half-versed sine (height) of the segment, by the remainder of the diameter of the sphere of which the segment is a part, and when necessary this diameter is obtained by dividing the square (see the tables) of the radius of the base of the segment, by its height, to get the remainder of the diameter.

$$V=\left\{\begin{array}{l} +\quad \text{area } B' \\ +\ 4 \text{ area } M \\ +\quad \text{area } B \end{array}\right\} \qquad =\left\{\begin{array}{l} +0 \\ +4M \\ +\ B \end{array}\right\}$$

Sum of the areas × $\frac{1}{6}$ H. Sum of the areas × $\frac{1}{6}$ H.

FRUSTUM

of Pyramid, Cone, Conoid, Sphere, Spheroid.

(**25**) In all these solids with two parallel bases, the bases and half-way section are similar figures : circles, if the frustum be that of a right cone or conoíd, sphere or spheroid cut by planes perpendicular to its fixed axis; similar regular polygons, if the frustum is a part of a regular pyramid of the same name; and, similar rectilineal, mixtilineal or curvilineal figures, if the pyramid is irregular.

(**26**) In each of these cases, the vertical section of the solid by a plane parallel to its axis, presents a trapezium. Now, the mean breadth of the trapezium is obtained by taking the half-sum of its paralled sides, that is, their arithmetical mean; and this mean is precisely the diameter of the frustum at half-height between its two bases; whence it is easy to arrive at the factors of the half-way section of the solid, and consequently at the area of such a section (see the tables.)

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[1] We do not add: "segment of pyramid, cone and conoid" simply because all such segments, that is, all such parts cut off from the apices of these solids by a plane parallel or not to the base, is still a pyramid, a cone, a conoid and its volume subject to the formula already given.

$$V=\frac{\left\{\begin{array}{l} +\quad \text{area } B' \\ +\ 4\ \text{area } M \\ +\quad \text{area } B \end{array}\right\}}{\text{Sum of the areas} + \frac{1}{6}\,H.}$$

UNGULA

of Sphere, Spheroid, comprised between planes of section passing in any direction through the centre of the solid.

(**27**) In each of these solids, the opposite bases or ends are null as to area or$=0$; the central section alone has any value and this section, in the sphere, is a sector of a circle (a part of a circle comprised between an arc and two radii) whilst in the spheroid, the same section is circular, if the planes of section have their common intersection in the fixed axis of the solid, in the other case it is elliptical.

Whence, the cubic content is :

$$V=\frac{\left\{\begin{array}{l} +\quad \text{area } B' \\ +4\ \text{area } M \\ +\quad \text{area } B \end{array}\right\}}{\text{Sum of areas } \times \frac{1}{6}\,H. \text{ or } L.} \qquad =\frac{\left\{\begin{array}{r} 0 \\ +4\ \text{area } M \\ 0 \end{array}\right\}}{\text{Sum of areas } \times \frac{1}{6}\,H. \text{ or } L.}$$

(**28**) **REM.** In practice, the length of the arc of the sector may be directly measured, by means of a metallic ribbon or the like, or of a thin rod that can be fitted to the curve of the solid, to determine its circular or elliptical circumference, or any part of such circumference.

UNGULA

of any Prism, Prismoid, Cylinder, Cylindroid, Pyramid, Cone, Conoid, comprised between planes of section having their common intersection in the axis of the solid.

(**29**) It is clear that the ungula of a prism or prismoid, cylinder or cylindroid is nothing else itself but a solid of the same name ; that the ungula of a pyramid or of a cone is simply a pyramid having for base, in the case of the pyramid, any plane figure, and in the case of the cone, a circular or elliptical sector, according as the cone of which the ungula forms part, is right or oblique. As for the ungula of a conoid, it will be considered, with respect to its measurement, as the segment of an ungula of a sphere or spheroid (see the following paragraph). It is clear that the apex or one of the bases of the ungula is but a line or point, as the case may be, and that

In all such cases the formula is:

$$\begin{array}{c}\text{V.} = \left\{\begin{matrix} \text{area B}' \\ +\,4\ \text{area M} \\ +\ \text{area B} \end{matrix}\right\} \\ \hline \text{Sum of areas} \\ \times\,\tfrac{1}{6}\ \text{H. or L.}\end{array} \begin{matrix}\text{that is, as} \\ \text{the case} \\ \text{may be.}\end{matrix} \begin{array}{c}\text{V} = \left\{\begin{matrix} \text{O} \\ +\,4\ \text{M} \\ +\ \text{B} \end{matrix}\right\} \\ \hline \text{Sum of areas} \\ \times\,\tfrac{1}{6}\ \text{H. or L.}\end{array} \text{ or } \begin{array}{c}\text{V.} = \left\{\begin{matrix} \text{O} \\ +\,4\ \text{M} \\ +\ \text{O} \end{matrix}\right\} \\ \hline \text{Sum of areas} \\ \times\,\tfrac{1}{6}\ \text{L. or H.}\end{array}$$

SEGMENT, FRUSTUM OF AN UNGULA

in the conditions of the enunciation, par. 127, of the treatise; that is, in the conditions enumerated in the two last paragraphs (27 and 29).

(30) It is plain that if the segment in question be that of an ungula of a sphere or spheroid, this segment will have but one base of any value, the other base being a mere point. The base will be a circular or elliptical sector and the section at half-height and parallel to the base, will be a sector similar to the base. We will then have for the expression of the volume of the proposed segment:

$$\begin{array}{c}\text{V.} = \left\{\begin{matrix} +\ \text{area B}' \\ +\ 4\ \text{area M} \\ +\ \text{area B} \end{matrix}\right\} \\ \hline \text{Sum of the areas} \\ \times\,\tfrac{1}{6}\ \text{H. or L.}\end{array} \qquad \begin{array}{c} = \left\{\begin{matrix} +\ \text{O} \\ +\ 4\,\text{M} \\ +\ \text{B} \end{matrix}\right\} \\ \hline \text{Sum of the areas} \\ \times\,\tfrac{1}{6}\ \text{H. or L.}\end{array}$$

(31) If it be a **frustum of an ungula, sphere or spheroid between parallel bases,** the expression will be:

$$\begin{array}{c}\text{V} = \left\{\begin{matrix} +\ \text{area B}' \\ +\ 4\ \text{area M} \\ +\ \text{area B} \end{matrix}\right\} \\ \hline \text{Sum of the areas } +\tfrac{1}{6}\ \text{H. or L.}\end{array}$$

(32) Finally if it be a **frustum of an ungula of a prism or prismoid, pyramid, cone or conoid** (for the segment of an ungula of a pyramid, cone or conoid, is evidently a solid of the same

name as that of which the ungula forms part) the formula will be, as always:

$$V=\underline{\left\{\begin{array}{l}+\ \ \text{area } B'\\ +4\ \text{area } M\\ +\ \ \text{area } B\end{array}\right\}}$$

Sum of areas $\times \frac{1}{6}$ H. or L.

SPHERICAL CONE OR SECTOR, SPHERICAL PYRAMID.

(**33**) To arrive at the volume of these bodies, we must do precisely as for the ordinary cone and pyramid, save that the base and middle section will be convex or concave surfaces which will be measured according to the rules found (**165, 167**), the volume being always:

$$V.=\underline{\left\{\begin{array}{r}\text{area } B'\\ 4\ \text{area } M\\ \text{area } B\end{array}\right\}}\qquad =\underline{\left\{\begin{array}{r}0\\ +4M\\ +\ B\end{array}\right\}}$$

Sum of the areas $\times \frac{1}{6}$ H. Sum of the areas $\times \frac{1}{6}$ H.

FRUSTUM

of a spherical cone or pyramid between parallel bases.

(**34**) Will be expressed as the frustum of the ordinary cone and pyramid by:

$$V.=\underline{\left\{\begin{array}{l}\ \ \ \ \ \text{area } B'\\ +4\ \text{area } M\\ +\ \ \text{area } B\end{array}\right\}}$$

Sum of the areas $\times \frac{1}{6}$ H.

FRUSTUM OF A TRIANGULAR PRISM

that is, having its opposite bases or ends not parallel to one another.

(**35**) The frustum of a triangular prism, considering any of its lateral faces as one of its bases, and the edge or opposed side as the other base, is nothing else but a prismoid; such is the wedge when the edge of that solid is of unequal breadth with the head. Under this view, the edge or side in question being but a mere line and consequently null as to area, we will have as an expression of the volume:

$$V=\underline{\left\{\begin{array}{l}\ \ \ \ \ \text{area } B'\\ +4\ \text{area } M\\ +\ \ \text{area } B\end{array}\right\}}\qquad \text{that is } V.=\underline{\left\{\begin{array}{r}0\\ +\ 4M\\ +\ \ B\end{array}\right\}}$$

Sum of the areas $\times \frac{1}{6}$ H. Sum of the areas $\times \frac{1}{6}$ H.

If the **frustum of the triangular prism** (of the last paragraph) is itself **truncated** by a plane parallel to one of its lateral faces, we will still have a prismoid whose volume will be:

$$\text{V.}=\begin{array}{c}\underline{\left\{\begin{array}{r}\text{area B}\\ +\ 4\text{ area M}\\ +\ \text{area B}\end{array}\right\}}\\ \text{Sum of areas}\times\frac{1}{6}\text{ H.}\end{array}$$

SPHEROID WITH THREE AXES.

(**36**) This solid, as also any segment, frustum, or ungula thereof, segment or frustum of such ungula, is exactly measured by the formula, whatever the direction of the planes of section may be. Therefore, as the case may be:

$$\text{V.}=\begin{array}{c}\underline{\left\{\begin{array}{r}+\quad\text{O}\\ 4\text{ a. M}\\ +\ \text{a. O}\end{array}\right\}}\\ \text{Sum of the a.}\\ \times\frac{1}{6}\text{ H or L.}\end{array}\quad\text{or}\quad\begin{array}{c}\underline{\left\{\begin{array}{r}\text{O}\\ +4\text{ M}\\ +\quad\text{B}\end{array}\right\}}\\ \text{Sum of the a.}\\ \times\frac{1}{6}\text{ L or H.}\end{array}\quad\text{or}\quad\begin{array}{c}\underline{\left\{\begin{array}{r}\text{B}\\ +4\text{M}\\ +\quad\text{B}\end{array}\right\}}\\ \text{Sum of the a.}\\ \times\frac{1}{6}\text{ L or H.}\end{array}$$

COMPOUND BODIES.

(**37**) The tableau presents a certain number of these bodies; for instance a cylinder terminated at one end by a segment of a sphere or spheroid (such would be a mortar); a frustum of a cone ending in the same way (a gun for instance); a cylinder or frustum of a cone crowned with a cone (a hay-stack or circular tower with a conical roof); a cone ending at its base by a segment of a sphere or spheroid, like certain kinds of buoys. It is plain that to measure these compound bodies or any other forms that can be decomposed into elements of the kind already treated on, the composing parts thereof must be separately computed, in order to make up afterwards the sum of such parts, according to the rules which have just been given

APPROXIMATELY.

(See the general expression, par. 127).

(**38**) And very nearly, say generally at .005 or at about ($\frac{1}{2}$) one half per cent, more or less, often (see the detailed problems of the treatise) with perfect accuracy or very near an exact result; is the volume obtained of

ANY FRUSTUM

of a Prism or Prismoid, Cylinder or Cylindroid, Pyramid, Cone or Conoid, Sphere or Spheroid, comprised between non parallel bases.

(**39**) By decomposing it, by an imaginary section parallel to one of its bases and passing through the nearest point of the other base into a frustum with parallel bases (the exact volume of which is obtained by the rules already given) and an ungula.

ANY UNGULA

of a Prism or Prismoid, Cylinder or Cylindroid, Pyramid, Cone or Conoid, Sphere or Spheroid.

(**40**) In this solid, as in the regular ungula of paragraphs (**27** and **29**) the apex or one of the bases or ends, is but a mere line or point, and its volume is very nearly.

(See the detailed ungulæ of the treatise).

$$V. = \left\{ \begin{array}{r} \text{area } B' \\ +4 \text{ area } M \\ + \text{ area } B \end{array} \right\} \qquad \text{That is } V. = \left\{ \begin{array}{r} O \\ 4\ M \\ +\ B \end{array} \right\}$$

Sum of the areas $\times \frac{1}{6}$ H. Sum of the areas $\times \frac{1}{6}$ H.

REM. As will be seen (**120**) if the base of the cylindrical ungula be not truncated, that is, if this base is a circle or an ellipsis, the formula gives the exact volume of the solid, and in the same manner under the same conditions, the exact volume of an ungula of a prism will be arrived at.

FRUSTUM OF AN UNGULA.

If the ungula of the last paragraph is cut off by a plane parallel to its base, of which the tableau offers examples, the volume will not the less be, as usual:

$$V. = \left\{ \begin{array}{r} \text{area } B' \\ +\ 4 \text{ area } M \\ +\ \text{area } B \end{array} \right\}$$

Sum of the areas $\times \frac{1}{6}$ H.

ELONGATED SPINDLE, FLATTENED SPINDLE.

(**41**) The spindle considered, as a whole, is not a usual solid; it has little importance, and to be convinced that it cannnot be measured at once, as with the elongated or flattened spheroid, it is sufficient to compare it in one's mind to an exact spheroid having the same axes or diameters. It is then seen how much its volume is less than that of the corresponding spheroid which is more swollen towards the ends of its axis in the elongated spheroid, and in the opposite direction, if it be a flattened spheroid.

(**42**) But if it be impossible to arrive at once at the volume of the spindle, one succeeds almost immediately, by measuring the half of this solid, and aferwards doubling the result, since then, by taking its section half-way between the centre of the spindle and its apex or end, the very element which contributes especially to make its volume vary, is considered, and this process applied to the flattened spindle, will give the exact volume if the perimeter of a section of the half-spindle by a plane passing through its fixed axis, is an arc of a conic section, as will generally be the case, the flattened spindle being then considered as two equal segments of a sphere or spheroid, united by their bases or planes perpendicular to the fixed axis of the solid, of which the composing segments of the spindle form part.

It will be seen (prob. LI) that it is sufficient to divide the half-spindle into two parts which will be measured separately and the sum of which will be afterwards taken, to arrive at a result which shall differ from the truth but by the 9th part to the quarter of one per cent.

CENTRAL FRUSTUM OF ELONGATED SPINDLE.

(Cask).

(**43**) This solid which gives its form to the thousand and one varieties and dimensions of casks, throughout the whole world, is, respecting the measurement of its capacity or volume, of great importance, on account of the generally high price of the contents. Well! as will be seen (prob. LII), it is sufficient to measure at once the half-cask to arrive at the exact volume, within the quarter to the fortieth part of one per cent; maximum error of one quart on a hundred gallons or of one litre on 400 litres and which does not exceed generally $\frac{1}{10}$ to $\frac{1}{20}$ of a gallon or litre for every 100 gallons or 100 litres, and can, besides, by that itself, be rectified, that the error is known to be always in excess and that consequently the result may be diminished by so much, if required.

$$V.=\left\{\begin{array}{r}\text{area } B \\ +\ 4\,\text{area } M \\ +\ \text{area } B'\end{array}\right\}$$

Sum of the areas × $\frac{1}{6}$ L or H.

CONCAVE CONE.

(**44**) The concave cone is analogous, as to its volume, to the elongated half-spindle, which may also be called a convex cone; and in the same way as we very nearly arrive at the volume of the half-spindle, by measuring it in two slices; so, if the hollow or convex cone is decomposed into two parts, by a plane parallel to that of its base, to measure separately each of these parts and add them together afterwards, the volume will be obtained at less than one half per cent loss.[1]

FRUSTUM OF CONCAVE CONE

between parallel bases.

(**45**) A great many vessels of capacity assume the form of this solid and as the hollow or concave cone is analogous to the half-spindle or convex cone, so the frustum of the concave cone may be considered as analogous to the central half-frustum of the elongated spindle or half-cask. Then by measuring it at once, provided its curve be uniform throughout its height and especially if this curve is not considerable, the volume or desired content will be very nearly arrived at, and if this curvature of the lateral face of the solid or vessel of capacity in question, is considerable or of unequal radii in various parts of the height of the frustum, a nearly perfect accuracy can be secured, by decomposing it mentally with a view to its measurement, into two or at most into three parts or slices by planes of section parallel to the bases.

The volume of each of the component slices will be:

$$V. = \left\{\begin{array}{r} \text{area } B \\ + \ 4\,\text{area } M \\ \underline{+ \ \ \text{area } B'} \end{array}\right\}$$

Sum of the areas × $\frac{1}{6}$ H or L.

COMPOUND BODIES.

(**46**) These bodies may assume many varied forms. The tableau presents some of them: for instance, a central or eccentric frustum of a sphere or spheroid surmounted by a concave cone (kind of

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[1] For forms with concave sides the volume is less; as for convex bodies the volume is more.

dome or minaret); segment of a sphere or spheroid surmounted by a segment of an elongated spindle or convex cone, or of a hollow or concave cone; two frusta of right cones united by their broader bases; two others, by their smaller bases; two frusta of concave cones and two others in the same conditions. And it is clear that other forms may be conceived, in almost infinite variety, but of which the rules already given are sufficient to determine the respective and composing volumes.

SUNDRY.

(47) Besides the solids which have just been enumerated, it is proper to say a word of certain forms which the tableau presents and of which the origin or the whole solid of which the body under consideration forms a part, might not at once suggest themselves to the mind. Thus, the **eccentric solid ring** may be considered as the central frustum of a very elongated spindle bent on itself. Then will it be measured by adding to the sum of the areas of both its less and greater sections, 4 times the area of the half-way section between these first, to multiply the whole by the length of the half-circumference used as the imaginary axis of the ring.

(48) The **bent cone or half-spindle** in form of the horn of an ox is measured like the inclined cone, considering as its height, the perpendicular drawn from its apex to the plane of its base.

(49) There is the **eccentric frustum of an elongated spindle**, which may represent the shaft of the roman column, swollen as it is, about the third part of its height, and the volume of which may be had by taking separately that of each of its composing half-frusta.

(50) The **regular polyhedrons**, as it is seen, may be decomposed into as many regular and equal pyramids as the solid has faces and be easily measured in this manner, each pyramid having for base one of the faces of the polyhedron and for height the half-height of the polyhedron, that is the half-diameter or radius of the inscribed sphere.

(51) The **decomposed parts of the flattened and elongated spindles and of certain other solids** furnish the idea and in consequence the manner of measuring, or gauging any sailing vessel, steamer or other, by decomposing it, if necessary, into elements of the kind already treated of.

(52) **REM.** The regular polyhedrons could equally be at once measured, by taking the trouble of finding the area of the central section of each of them. All these solids have two parallel bases, one of the bases being, for the tetrahedon, a point—for the tetrahedron is nothing but a pyramid.—The octahedron may be considered as a double pyramid or a compound of two pyramids, base to base, and be measured in this manner. As to the dodecahedron, it will be seen that while each of its parallel bases is a regular pentagon, its half-way section between these bases, is a regular decagon or a ten sided regular polygon, and each side of which is equal in length to the half-diagonal of the pentagon. As to the middle section of the exahedron, if it be taken paralled to two opposite and paralled sides of the solid, it will be a twelve sided regular polygon, the perimeter of which it would be too long to determine here. If on the contrary the half-way section is supposed parallel to or equidistant from two opposite vertices of the solid, that is, perpendicular to the axis or diameter uniting two opposite points or extremities of the solid, this section will be a regular decagon each side of which will be equal to the half-side of the triangle forming the face of the polyhedron. Finally, if any two opposite sides or edges be taken for the parallel bases of the icosahedron, the half-way section parallel to these edges and perpendicular to the plane which unites them will be a six sided symmetrical polygon, two opposite and paralled sides of which, each equal to the side of the triangle forming the face of the polyhedron and being one of these sides, and the other sides of the exagon, parallel, two and two, and respectively equal to the height or right radius of the said component face.

REDUCED TABLEAU.

(53) **REM.** It is hardly necessary to say that, in this treatise and in the abridged key, every thing which relates to the so called tableau of 200 models, is equalled applicable to the reduced tableau of 105 models which the author is preparing for elementary schools and with a view to reduce the price of it in order to place it within the means of persons less capable of ordering it.

In the reduced tableau the models will be 105 in number, disposed in 15 vertical rows and on 7 horizontal rows (15 × 7 = 105). Then beginning for instance by the left, the

1st vertical row would be the prism, its frustums and ungulas.
2nd, 3rd, 4th row, the prismoid and various frusta and ungulæ.
5th row, the pyramid, its frusta and ungulæ.
6th row, the cylinder, its frusta and ungulæ.
7th row, the cone, its frusta and ungulæ.
8th row, the conoid, its frusta and ungulæ.
9th row, the flattened spindle, its segments, frusta and ungulæ.
10th row, the elongated spindle, its segments, frusta and ungulæ.
11th, 12th and 13th row, the sphere, its segments, frusta and ungulæ.
14th row, the flattened spheroid, its segments, frusta and ungulæ.
15th row, the elongated spheroid, segments, frusta and ungulæ.

And if in the tableau any segment, frustum or ungula is wanting to complete the number of those included in the nomenclature of the solids to which the formula relates, it can easily be mentally supplied in the same way, as, if required, any compound solid may equally be decomposed by imaginary planes of section, into elementary forms, to submit its volume to the required computation.

KEY TO THE
BAILLAIRGÉ
STEREOMETRICAL TABLEAU.

INTRODUCTION
TO THE
NEW METHOD OF MEASURING
ALL
BODIES—SEGMENTS, FRUSTUMS AND UNGULAS OF THESE BODIES,
BY ONE AND THE SAME RULE.

The author of the new method has the honor to inform his colleague—Architects, Engineers, Surveyors, Professors of Geometry and Mathematics, the Directors of Universities, Colleges, Seminaries, Convents and other Educational Establishments, the Professors and Pupils of Schools of Art and design, Mechanics, Measurers, Guagers, Custom House and Excise Officers, Ship-Builders, Contractors, Artisans and others of Canada and Elsewhere.

That he has perfected his " Stereometrical Tableau " in order to reduce to practice his universal formula for finding the solid content of a body of any form or dimensions.

ADVERTISEMENT.—With the view of protecting his discovery and invention and of guarding against any infringement of his rights, either here or elsewhere, the author has taken out Letters Patent of his tableau, as well in Canada as in the United States and in Europe.

On the subject of this formula the Quebec Seminary after having submitted it to *competent persons,* expresses itself thus: "Of the theorems and formulae remarkable for their novelty, the most striking is the general rule for the content of any known solid. An involuntary doubt at first takes possession of the mind; but a careful examination soon dispels this boubt and *one remains astounded at sight of a formula so clear, so easy to retain, and of which the application is so general.*"

An article in the "Journal d'Education" speaks of the *sudden impulse* which this discovery imparts to science.

His method of explanation, says another journal, is new, and so simple as to be within the grasp of every mind.

The tableau, the dimensions of which are about 3 feet in height by 5 feet in length, is of hard wood, dyed or painted of suitable colour. Fixed to the board are some 200 models of hard wood, polished, oiled or varnished as required. Each little model is adjusted to the tableau by means of a peg or pin of wire attached to the board so that the model may be easily removed and replaced at pleasure.

The models comprise almost all the elementary forms that it is possible to conceive, or to which any compound body, by division or decomposition, could be reduced. Among them will be found all the prisms and prismoids, cylinders and cylindroids right and oblique, and the frustums and ungulas of these bodies; pyramids, cones and conoids, right and oblique, with their frustums and ungulas; the sphere with its subdivisions into hemisphere, quarter sphere, half-quarter or tri-rectangular spherical pyramid, segments, zones, pyramids, frustums and ungulas; the prolate and oblate spheroid and ellipsoid, with their segments, half, quarter, frustums, etc.; in fine, the spindles and their frustums, etc., including casks of all sorts; the regular polyhedrons, concentric and eccentric rings, and a number of other varied practical forms too many to enumerate here.

The new rule or formula dispenses with all consideration, all preliminary calculation as to the nature, form or dimensions of the entire solid of which the body to be estimated forms a portion. Thus, when it is required to find by ordinary rules, the cubical contents of a segment, frustum or zone of a spheroid, for instance, it is first necessary to find the axes of the solid so as to take them into account; but by the new system we proceed immediately to estimate the solidity required, by applying directly to it the formula $(B' + B + 4\,M)\,\frac{1}{6}\,H$. In like manner, if one supposes he has to deal with the frustum of a pyramid for example, the fist thing to ascertain is that it is one, in order to apply to it the ordinary rules of mensuration; for this purpose, it is required to measure the respective lengths of the edges or sides of the upper and lower bases, so as to establish the proportion between them, and be assured thereby that they are, or are not proportional, without which the solid to be estimated is not the frustum of a pyramid; whilst, on the contrary, by the new formula, it is sufficient to be certain of the parallelism of the sides or edges (which is seen at a glance and without any measurement) to proceed directly to the application of the rule; for, from the authors point of view, the frustum of a pyramid, reputed such, is regarded as a prismoid, and subject, on that account, to the general formula, in the same way as the whole pyramid which is also a prismoid.

But, setting aside all preliminary consideration as to the nature of the solid to be measured, and of the long and abstruse calculations which must be made for that purpose when the ordinary rules are followed—it is to be remarked that the calculations which remain to be made to find the content of the solid or liquid in question are laborious, difficult, long, and sometimes neverending, as for instance, when they relate to the guaging of a cask, or the measurement of the frustum of a spindle. There are very frequently algebrac formulae, differential and integral calculus, and a simple mis-print in the formula of which the mind can not penetrate or follow the mysterious intricacies,—an error which may not be detected and would consequently remain uncorrected—a simple error of this sort is enough to render the whole calculation useless, and necessitate its being gone over again; instead of which, by the system now proposed, as there is but one rule, one simple formula to learn and retain, and that of the simplest nature, the mind can follow it step by step, and the eye itself perceive immediately it there be error.

At present, there are as many different rules as there are solids: one for the prism or cylinder; one for the pyramid or cone; another for the frustum of the cone or pyramid; a third for the sphere; then three others for the segment, zone and ungula of this body; another for the spheroid, with additional formulae and in equal number for the segment, frustum and ungula, according as the intersecting plane is parallel or inclined to the small or great axis or to any diameter whatever of the solid. How many other formulae differing always from one another, and each of them from all those already enumerated, when it is necessary to arrive at the content of a parabolic or hyperbolic conoid, right or oblique, of a segment of a circular, elliptical or other spindle, of an ungula of a cylinder, cone, conoid or spindle. Well! we can now lay aside all these rules, all these various formulae which it is impossible for the memory to retain, and for which we always need a book at our disposal; we can set aside all these multifarious formulae with the books containing them, and armed with the new system, take hold of any solid whatever to find its content by the aid of a quite simple rule that may by remembered like the Lord's prayer, namely: " to the sum of the parallel end areas, add four times the middle area, and multiply the whole by one sixth part of the height or length of the solid."

The calculation is thus reduced in every case, by the authors system, to that of the areas of the opposite bases and of the middle or intermediate section, to attain which end is precisely the object of the tableau in question, where one may immediately see the form of the solid, the nature of the surfaces which form its bases, and, by means of a stroke or line, the nature and dimensions of the section, surface or base half-way between the bases or opposite extremities of the solid to be estimated. In this manner it will be seen that in the pyramid, the cone, the conoid, the segment of a sphere or spheroid, the upper surface is reduced to zero or is null, whilst in the sphere, the spheroid, and certain prismoids, each of the opposite surfaces or areas becomes nothing, which reduces the calculation to multiplying 4 times the middle section by the sixth part of the height of the solid.

The formula is mathematically exact for prisms and prismoids, cylinders and cylindroids right or oblique, regular or irregular pyramids and the frustums of these solids between parallel bases; for the right or oblique cone and its frustum between parallel bases, the sphere, the spherical segment and zone, the spheroid and every segment, zone or frustum of this body cut from the whole solid by a plane inclined in any manner whatever to the axes or diameters or comprised between any two parallel planes; for right or oblique parabolic or hyperbolic conoids and the frustums of these bodies comprised between parallel planes; all of which various solids constitute nearly the total number of the elementary forms thats one can be called on to estimate.

It is for spindles only, and certain ungulas of these bodies, and of other solids, that the formula is not mathematically correct, and yet even in these latter cases can we obtain results more correct, more satisfactory by means of the general formula than can be done by any of the other means to which we commonly have recourse in practice. It suffices for this to divide the half spindle or frustum of this solid into two, or at most into three parts, by sections or planes parallel to the bases, and apply the formula separately to each of these parts, to arrive at a result approaching almost to perfect accuracy, and so of the ungulas of prisms and prismoids, of pyramids, cones, conoïds and spindles, of spheres and spheroids and the frustums of these ungulas between parallel bases, to divide the solid into 2, 3, or 5 parts at most, to apply next the formula to each of the component segments separately, as it is besides evidently necessary to do so for the cubical contents of compound solids such as ships, schooners, yachts, boats, &c., and other similar forms, composed as they are in every case of some of the elementary forms which will be found among the models of the tableau.

And in the majority of cases, even this subdivision of the solid into 2, 3, component segments will be useless since, for the ungulas of prisms, cylinders, pyramids, and cones, the maximum error between the true content and that obtained by the proposed formula does not exceed .005 or the half of one per cent.

Let us consider also the immense advantage of such a tableau for the mere nomenclature of bodies. By it, the least advanced pupils in Colleges, Convents, and other schools, are enabled to prosecute a study from which they have hitherto been debarred. In fact, for the mind to seize and understand, on paper, the graphic representation of a body, it needed a previous knowledge of drawing and perspective which is only acquired generally by advanced pupils and in the last years of college; whilst to-day the youngest, the least advanced may detach the model from the tableau, take it, hold it, and handle it; the master or professor, the nun or mistress will tell him or her the name of it and point out an example in the thousand and one objects met with in our every day pursuits and requirements.

What the several models of the tableau may represent.

The segment of an erect or reversed cone will, according to circumstances, represent a tower, lighthouse a tumbler, salting tub, butter firkin, bucket, an ordinary tub or vat or of such as are seen in breweries and elsewhere; the flat or depressed cone will suggest the idea of the cover or bottom of a cauldron, of a roof, etc., the pyramid will be the image of a pinnacle or of the spire of a steeple etc. ; the right conoid, the segment of a sphere or spheroid will be a dome more or less elevated or depressed, whilst the same solid, by reversing it, will present to the imagination a basin, a reservoir, a boiler such as is found in the sugar hut, the distillery or other manufactory.

The inclined and reversed conoid, the segment of a spheroid equally inclined and reversed, will represent the space occupied by any liquor, liquid, or fluid whatever at the bottom of a vessel which from one cause or another has become inclined to the horizon. The prism, the prismoid will likewise be the model of those thousand and one plain or elaborate roofs which crown our domestic habitations, our public buildings, our palaces. The roof of the common dormer-window, if it be sloped, will be a triangular prism oblique or inclined ; if it be not so, it will be the frustum of a prism, and the body of the dormer will be, indifferently, according to the aspect under which it is viewed, a right triangular prism, or the ungula of a quadrangular prism. Amongst the prismoids will also be found the stick of timber or saw log, the trunk of a tree, the railroad cutting and embankment, the reservoir, quay, pillar, camping tent, the splayed or square opening for a window door, nich or loop-hole in a wall. The quarter of a sphere or spheroid, the half segment will be the vault of the apsis of a chuch or of a hall terminated in the same manner. The whole sphere, the spheroid will be the billiard ball, the ball of a steeple, the earth we inhabit, the moon, the sun, the planets. In a word the model of each of the elementary forms which it is possible to conceive will be found on the tableau, and can be handled at pleasure so as to examine it under every possible aspect.

The bases, lateral faces, central or intermediate sections of the models of the tableau, also offer to the pupil. in the previous study of the mensuration of surfaces, the representation of each of the plane figures, and of every sort of convex or concave surface of either single or double curvature.

The square, rectangle, lozenge, parallelogram, trapezium—the equilateral, isosceles, scalene, right-angled, acute-angled, obtuse-angled triangle;—the regular, irregular polygon — the circle, semi-circle, quadrant, sector, segment, zone, lune, concentric and excentric ring—the ellipse, semi ellipse, segment of ellipse less or greater than the half—the other conic sections, parabola, hyperbola —the equilateral, tri-rectangular, tri-obtuse-angular spherical triangle,—the spherical segment, zone, lune,—segments and zones, &c., of prolate and oblate spheroids or ellipsoids, &c.

The tableau will have the effect of interesting the pupil and of making attractive a study, heretofore dry and almost impossible. By means of the tableau and the general formula, stereotomy and stereometry, that is, the nomenclature, properties and measurement of bodies can be taught in even elementary schools where sufficient geometry has first been taught to enable the pupils to determine the area of any plane figure whatever ; since, in reality, the proposed system reduces the mensuration of bodies or volumes to this—the remainder of the work being merely an addition of the areas thus found and the multiplication of their sum by the sixth part of the height of the solid.

THE FORMULA.

The prismoidal formula proper, is, of course not new, since it has been sometimes used in the computation of earthworks on railways and otherwise ; but it does not appear to have suggested itself in the case of the frustum of a cone or of a pyramid, an inclined conoid or segment of a spheroid, the frustum of an ellipsoid contained between parallel planes inclined in any way to the axes of the solid and in general to most of the solids of my Tableau, and even, had the idea suggested itself so to do, still would the formula never have become of general application either in practice or in the teaching of Stereometry without the help of the Tableau, any more than steam without the steam engine, or electricity without the telegraph.

The author must share with others the merit of the discovery as applied to certain solids to which its applicability has been shewn, though in a varied and more or less indirect way; but herein he is not singular, and perhaps it is as well that it should be so, as it but adduces additional prof in support of his scheme ; nor should he take it to heart any more than Leverrier and Adams in relation to Neptune, Newton or Leibnitz concerning the discovery of *fluxions*.

" It is plain, says the Rvd. Mr. Billion, in his letter to Bishop " Larocque, that the author does not require that one should make

" use of his formula in many cases where a more simple expres-
" sion may replace it."

True, there are, not "many cases," but some three or four cases, where the general formula may be replaced by one of a more simple nature; but the simplified expressions flow directly and without effort from the general formula. Few persons; as yet, have in this respect understood the authors object in regard to the necessity, the advantage of one and only one and the same formula for all possible solids. A perusal of the following letter from professor Lafrance will however show the wisdom of his views.

Quebec 11 *decr.* 1871.

To M. C. BAILLAIRGE.

" Sir,— ....

" As to the correctness of the formula, none can doubt it, since when promulgating it, as you have done in your treatise 1866, you have yourself given all the necessary proofs to bear it out.

" With respect to the prism or cylinder, to begin with, (Article 1254 of your treatise) if objected to, that for these solids, at least, your formula far from offering any advantage, but complicates the calculation; I should reply that it is not so, since in this case the formula is immediately reduced to the ordinary rule. Thus the pupil who has already learned that every section of a prism or of a cylinder by a plane parallel to its base is a figure similar and equal to the base, will say to himself at once " But the sum of the bases, plus 4 times the intermediate section, is equal to 6 times the base, and 6 time the base multiplified by the sixth of the height is simply the same as multiplying once base by the whole height;" and in fact this is the case, but by applying the formula as well to the prism and cylinder, its use becomes general for all solids and obviates the necessity of learning or remembering one single additional rule, for in fine therein is the immense advantage of the system you propose " one and the same formula " for all possible solids, and the moment you introduce a second, aye even a third for the pyramid and cone, a fourth for the paraboloid, to cube which by the ordinary rules would appear more simple than by yours, you then introduce confusion into the mind of the pupil, of the measurer; from that moment there is danger of confounding these rules, of mistaking the one for the othe, of forgetting all of them or not remembering them and of the necessity, for thal very reason, of a book of reference, a theing which your system entirely dispenses with.

Again, for the pyramid or the cone, as you give it (1525 of your Geometry) the surface of the intermediate section is a quarter of that of the base; now $\frac{1}{4} \times 4 = 1$, and $1 + 1 = 2$, and it is directly seen that the 6th part of 2 is the same thing as the third part of 1; whence it follows that the general formula brings one back immediately to the ordinary rule, and that, without the necessity of knowing this rule beforehand.

" Again, let the prism, the cylinder, the pyramid, the cone, as happens in practice, be ever so little convex or concave; where should we be with the ordinary rules, whilst on the contrary in this case your formula is the only one which can correctly cube the body in question; and for the spindle (1534) the paraboloïd (1564) be its lateral surface the least in the world too much or too little convex or bulged, the ordinary rule which consists in multiplying its base by its height and taking half the product, will give either too great or too little a volume. How them in this case arrive at the truth, if not by your formula which, as in the case of the cask, introduces into the calculation the versed sine (or very nearly) of the greater or lesser convexity of the body to be cubed, that is to say the very element which tends to vary its cubical contents.

" I have just pointed out the articles or paragraphs of your treatise containing the proofs of the correctness of the formula for the above mentioned bodies; paragraph 1561 also proves it in the case of the prolate or oblate spheroid or ellipsoid;—1562, for the segment of this solid,—1566, for the hyperboloid, and 1581 to 1592 for the prismoid in general.

" The fact is that too few persons yet have taken the trouble to examine your books, so great is the tendency with us to remain always in the rut of the old routine. In fact, it has taken us 20 years to substitute for pounds, shillings and pence the infinitely more simple and expeditious decimal calculation of dollars and cents, where the mere shifting of a point works wonders: it will take 10 more yet to appreciate your work, to introduce your formula, your tableau indispensible as it is, into the general education of this country.

" I firmly believe, however (and it is too often the case, so little is a prophet heeded in his own country) that you will be immediately appreciated abroad, and I do not doubt but that your tableau, once known in the United States and Europe where, I am told, you have had your invention patented: I do not doubt, I say, but that as soon as you have published your prospectus in a foreign country, so soon will you have orders and in great numbers for the introduction of the tableau in all the Universities, Schools and other institutions devoted to the teaching, not only of youth, but also of maturer years among all nations.

" C. J. L.-LAFRANCE.
Professor.

The following letter from a distinguished mathematician of Alsace, France, sets forth very concisely and generalizes the nomenclature of solids to which the Stereometrical formula applies.

" WESTMORELAND POINT, N. B., 4th Nov. 1871.

" My Dear Sir.—I see that you are bent on doing all you can to render your rule for finding the solidity of any body whatever really useful and practical. I consider that a Stereometrical Tableau such as that of which you send me the prospectus is the indispensible complement of the rule for the majority of persons who may require to use it, and in colleges, etc., this will also be a great help to the pupils who are studying analytical geometry of three dimensions and spherical trigonometry.

" Having had occasion to verify the exactness of your formula relatively to different kinds of bodies, I have found in fact, as you say in other terms, that it is strictly applicable to all polyhedrons without exception, the same of all solids generated by the revolution of curves of the second order around one or other of their principal axes as well as to segments of these solids, whatever be the direction of the plane of section. In this respect, it seems to me you might add to your prospectus, that in general the rule is applicable in a manner rigorously exact:

" 1° To all solids generated by the revolution of a straight line around two planes parallel to one another limited in extent by continuous outlines of any form whatever and irrespective of the variations of the relative velocities of the two extremities of the moving line.

" 2° To every solid generated by a plane of definite outline moving with uniform velocity in a straight line from one point to another, whilst its surface varies in a corresponding manner as the square of the generating cord of a zone of any conic section.

" As to solids generated by regular curves of an order higher than the conic sections, it is generally easy enough to subdivide them, with an exactness more than sufficient for all practical purposes,—so that the resultant partial solid may be classed in one or other of the two categories of solids which I have just described.

" Moreover, apart from certain kinds of ungulas of elementary solids, I see few bodies of regular forms which may be met with in practical measurement of guaging or be in use in the arts and trades, for which a subdivision repeated beyond two or three times is necessary; and as for irregularly shaped ungulas it will be very easy for you to refer to them in your Key to the tableau.

" I found it very convenient, lately to apply the rule (proceeding by subdivision) to a trunk of a hyperbolic conoïd generated by the revolution of a hyperbola of the 5th order around one of its asymptotes, with the view of veryfying the degree of precision brought to bear on the construction of a converging copper tube having that particular shape which I have employed in a hydraulic experiment.

" R. STECKEL."

Mr. Steckel, after having proved the mathematical correctness of the formula for all bodies mentioned by the author in his prospectus, has made long and laborious analyses respecting the application of the formula to the ungulas of the cylinder, cone, conoïds, and spheroids, &c., &c., and has demonstrated that in fact, as the author says, the maximum error for these solids which are moreover rarely met with in practice, does not generally exceed .005 or the half of one per cent, taking the whole body at one measurement, and that this error, little as it is, is easily eliminated and is reduced to nothing, so to say, by subdividing the ungula, the half frustum of spindle, &c., into two or at most into three parts, and applying the formula to each of them and afterwards taking the sum of the component parts.

The Revd. Mr. Billion (Mathematician of the seminary of St. Sulpice, Montreal,) says that " the formula is applicable to a whole series of other bodies which the author has not spoken of " and I here mean bodies to which it applies exactly. In fact, let us suppose any body whatever mentioned by the author, for example the frustum of a pyramid. This body may be considered as formed by the juxtaposition of an infinity of planes parallel to two bases. Now, let us suppose all these planes strung from one base to the other by any straight or curved line whatever, and susceptible of being curved or bent in any manner whatever. By inclining, bending, curving or twisting this kind of dirextrix, every section of the said body is similarly affected and a new solid results, of the same bases and same height as the first, perfectly equivalent in volume, but curved to the arc of a circlė, parabola or other curve under any law whatever, or bent of any angle whatever, the sections remaining always in the same plane. If the direction is changed into spiral and that the base is a circle, we have a wreathed pillar, &c., &c. There then are a multitude of bodies to which the same formula applies, for the reason that they are equivalent to the first ones.

" The author demonstrates his formula to be strictly correct for a great number of bodies enumerated and as approximate as could be desired for those to which it does not apply in a measure absolutely exact. The proposition and the demonstration, in every case, are *exact and true*.

"As for other bodies, it is true that the more numerous the subdivisions, if need be, the closer the approximation to the true answer."

"The works published up to the present time, says Mr. Blain, contain quite a number of different rules for finding the contents of solids formed by the revolution of a curve of the second order around its axis. These rules which give rise to as many formulas must be remembered; that is, we must overload the memory, without being certain of its readiness and faithfulness at the moment we need any particular formula. Mr. Baillairgé's formula is *general* and dispenses with this somewhat painful effort.

"Moreover, this formula is applicable when a solid is generated by a curve revolving around an axis not its own, and that is a case which has rarely, if ever been foreseen by the authors who have written on the subject and whose habitual fault, without intending to lessen their profound knowledge, has been to have always kept theory too much in view to the great detriment of practice.

"I have established with Mr. Steckel, that in the greater number of works on mensuration no mention is made of a spheroid cut by a plane in any direction oblique (inclined) to its axes, a case provided for by Mr. Baillairgé, No. 1560 of his treatise. No more is anything said of a paraboloid in the same conditions, (1,564) and still less of a hyperboloid, (1,566).

"Guagers, particularly, may derive immense assistance from Mr. Baillairgé's formula, since the great majority, it might almost be said, the whole of the tuns, puncheons, barrels, boilers, kettles, reservoirs, &c., and all vessels generally used for holding liquids, are nothing but segments or frusta of circular, parabolic, hyperbolic, or elliptical spindles, spheroids or frusta thereof, spherical hyperbolic (calottes) domes, frusta of cones or of conoids with concave or convex surfaces, &c.

"Mr. Baillairgé's formula also dispenses with the difficulty of classifying the solid to be measured, an operation subject to many errors in practice.

"In fine, the same formula supplies the means of finding the quantity of liquid in any vessel only partly filled, in any position the vessel may be and without being obliged to change this position, (1,577, 1,578, &c.).

"I should far exceed the limits of a newspaper article, if I wished to enumerate all the advantages of the *discovery* made by Mr. Baillairgé, for it is really a discovery of great importance.

I say frankly, I at first doubted the exactness of Mr. Baillairgé's calculations, and before expressing an opinion, I made the calculations myself, then consulted clever men and well versed in practice. I but record here their opinion which will be confirmed later by all those who shall employ the new system of measurement.

Now as to the correctness of the results given by the formula in the case of frusta of spindles, for instance, and of vessels of capacity of this kind compared with those by the ordinary rules, the author could not do better than publish at length the pertinent remarks of professor Gallagher on this subject.

" That this formula is mathematically correct, as applied to al the solids enumerated by you in your prospectus, there is of coursel no doubt. You have fully demonstrated this in your valuable work on Geometry and Mensuration published in 1866. Mr. Steckel has not been slow in showing this in a most concise manner in his letter to you on the subject, to say nothing of the letters of the RR. MM. Méthot and Maingui on part of the professors of mathematics of the Quebec Seminary and Laval University, where the expressions " étonné " and " enchanté " sufficiently show the high estimation in which your discovery is held by these competent judges ; but, in my opinion, you do not sufficiently insist on the great value, the many and manifest advantages of your rule as applied to spindles, the middle frusta of which are met with every day and in every part of the civilized world under the thousand and one forms of casks of every conceivable size and variety, and the necessity of measuring which with promptness, on account of their number, and with accuracy, on account of the generally valuable nature of their contents, renders some simple, easy and commodious rule, like the one now proposed by you, of the first importance to all mankind.

Now, Sir, that your rule embraces these valuable requisites, let me compare it, in its working and in its results, with the rules laid down by some of our best mathematicians and authors such as Bonnycastle for instance, see Rev. E. C. Tyson's edition of his mensuration, page 147.

Problem XXVII (for example).

" To find the solidity of the middle frustum of an elliptic spindle ;
" its length, its diameters at the middle and end being given ; also
" the diameter which is half way between the middle and end dia-
" meters being known."

Rule, " 1° From the sum of three times the square of the middle
" diameter, and the square of the end diameter, take four times the
" square of the diameter between the middle and end, and from four
" times the last diameter take the sum of the least diameter and
" three times that of the middle, and ½ of the quotient arising from

" dividing the former difference by the latter will give the *central*
" *distance.*

" 2° Find the axes of the ellipse by Problem II, and the area
" of the elliptical segment, whose cord is the length of the frustum,
" by Problem V.

" 3° Divide three times the area thus found by the length of
" the frustum, and from the quotient subtract the difference between
" the middle diameter and that of the end, and multiply the re-
" mainder by eight times the central distance.

" 4° Then from the sum of the square of the least diameter,
" and twice the square of that in the middle, take the product last
" found, and this difference multiplied by the length and the pro-
" duct again by .261799, &c., will give the solidity required."

Here, the mind is absolutely bewildered at the mere recital of the multifarious operations to be performed (not less than 27 in number) and the mere results of each of these operations, irrespective of the details of the multiplications, divisions and other computations necessary to arrive at them, take up two whole pages of the book.

Applied, say to a cask of 28 inches in length, bung diameter 24 inches, head diameter 21.6 inches, and diameter half way between head and bung 23.40909 inches, the result, as fully worked out at page 148, 149 of said book, gives 11,854¾ cubic inches, very nearly, or 51 gallons and 5 half-pints.

Now, the same example, Sir, by your formula, brings out 11,855.2 cubic in., which differs from the last result by only .0000045 or less than half an inch on nearly 12000 inches, or the 240th part of one per cent in excess, the 14th part of a gill.

Not only then, is your formula in this case to be considered in every respect as accurate as that of Bonnycastle, but it is really ormos in practice; for, even if the error in excess attained the maximum of .005 or ½ of one per cent, where is the practical measurer or gauger who, for the sake of a quart on a 50 gallon keg or half a gallon on a hoghshead, would, could devote hours of his time to calculate by the old method what can be done with greater accuracy and in less than 2 minutes by the new; for, every merchant will tell you that in practical cask gauging there is generally an error in excess or in defect of from one to two gallons on a hogshead.

And even this comparative accuracy of the old rule, can only be arrived at by taking in all the decimals, which no one would be likely to do, on account of the immense labour of the computations; whereas, by the new formula, by reason of its great simplicity and conciseness, all the decimals may easily be taken in and no harm can result at some of the last decimals being neglected, since the result as shown above is, and, for convex forms, always is, though ever so slightly, in excess of the true content.

I am wrong however in assuming that the maximum error in cask gauging by your rule is .005 or the half of one per cent, neither do you say so in your prospectus, and on the contrary you show most satisfactorily at page 708, 709 of your said treatise, in the numerous examples given by you and fully worked out and compared in each case with the results given by Bonnycastle's rules, that the maximum error in excess does, in your first and 2nd examples, not exceed $\frac{1}{4}$ of one per cent or one quart on a hogshead; in ex. 8 it is $\frac{1}{5}$ of 1 per cent; Ex. 10 gives the maximum error as $\frac{1}{6}$ of 1 per cent; 5 gives $\frac{1}{7}$ of 1 per cent; Ex. 4 and 12 (2), $\frac{1}{10}$ of 1 per cent; Ex. 9, $\frac{1}{16}$ of 1 per cent; Ex. 2 and 12 (1 and 3), $\frac{1}{20}$ of 1 per cent; and ex. 7, $\frac{1}{46}$ of 1 per cent, and these examples cover all varieties and sizes of circular, elliptic, and parabolic casks, that is of the three varieties generally met with in practice.

But in dwelling on the formula, I find I have as yet said nothing of the all important "Stereometrical Tableau" without which, as you pertinently remark, the rule would be almost as useless in teaching mensuration in schools, if not in the practice of it, as steam without the steam engine or electricity without the telegraph.

There are many other advantages, apart from the mere mensuration of bodies, which your tableau possesses, as enumerated by you in your prospectus and which it is useless for me to dwell upon, as I fully concur in all that you claim for it; though I think you might have further insisted on the advantage of such a tableau in the studio of the apprentice, nay even of the professional architect, who will, among the models, find that of almost every conceivable shape or proportion of roof, dome, &c., which he may be called upon to design; the Civil Engineer, every description of prismoïd to be met with in the cuttings or embankments for railroads, canals, docks, &c., or in the piers or abutments of bridges or other structures; the mechanical engineer, every variety of boiler, copper or other vessel and the component parts of all sorts of machinery.

Quebec, 9th December 1871.

Extract from a Discussion of Baillairgé's Stereometrical formula by the Rev. N. Maingui, professor of Mathematics at the Laval University.

It is easy to conclude that, in practice, unless it be known beforehand that it is required to cube a sphere, an ellipsoid or segments of those bodies, it would be extremely long and difficult to establish

1° the kind of curve to which the 2 directrices belong and

2° the position of their axes.

Therefore it is much simpler to suppose the body to be cubed divided into a certain number of sections so that the curved side may be sensibly a straight line. These sections, like truncated cones, are cubed very readily by the stereometrical formula.

This is moreover, the only resort for all solids that the stereometrical formula can not cube at once. The same remark applies *a fortiori* to solids terminated laterally, partly by planes and partly by a curved surface.

Because, in practice, the stereometrical formula cannot give, at one attempt, the exact contents of certain bodies, we must not hence deduce an argument against this formula; and for the very simple reason that, in these cases, the measurement of the *body as a whole* is impossible. And if in some exceedingly rare cases, certain very complicated formulas exist, *practically* they will give a result less exact than the stereometrical formula.

Up to the present time, a certain number of bodies were worked by easy rules; others by very complicated ones; others again had to be divided mentally into different parts or were reduced by approximation; now the stereometrical formula applies with advantage to all these cases

1° It is as easy to apply as any one whatever of the old rules.

2° In application it is much simpler than a number of others.

3° It can compare very advantageously with all the others on account of its great exactness, according to the case in hand: the preceding demonstration and discussion being intended to show the conditions in which the result is rigourously correct in order the better to point out what course to follow for resolving certain problems in a satisfactory manner.

The object of the accompanying formula is and the result of its employment will be to popularize and generalize the study of solid geometry. The author is free to admit that in Universities and other establishments of higher education, the advantage of this one and only one undeviating formula for the solid contents of all bodies may not at first sight appear to offer the same advantages as in elementary and other institutions of the kind; for in the first, other rules are taught and the study of algebra and the higher calculus afford facilities not possessed by the second, of arriving at satisfactory conclusions; but, once out of college, all this algebra and calculus, all these ever varying formulas are soon, very soon forgotten, and the professional, the highly educated man will lose all trace of his mensuration of solid forms; while the mere artizan, the mechanic, the engineer and architect, the measurer, guager and practical man of every grade, who has never learnt aught but to compute areas of all kinds by simple rules not easily forgotten, need only have recourse to simple addition, multiplication and division to work out the contents of the most complicated solid, or of a vessel of capacity of any kind whatever. His less fortunate employer, the learned professional, will appear to a disadvantage in the eyes of the world at large from his incapacity to do the same, unless he also shall learn how to use the formula in every case where college and university rules and formulas have been long ago forgotten. There are now a days too many other sciences to learn and college life is too short to allow of devoting years or even months to a study which can be mastered in a day, if old fogyism and conservatism will but stand aside.

KEY

TO

BAILLAIRGE'S

STEREOMETRICAL TABLEAU.

NEW SYSTEM OF MEASURING

BODIES—SEGMENTS, FRUSTA AND UNGULÆ OF THESE BODIES

BY ONE AND THE SAME RULE.

(1) It is useful to gather and present under a concise form the various formulæ or rules which relate to the calculation of the superficies and volumes of the various bodies and figures previously spoken of.[1] A synopsis of this kind will allow one to refer more easily to those rules, in order to find at a glance the one required, concerning the problem to be solved; and some practical examples of the various cases, will better teach the pupil the course to be followed to arrive at the result required.

(2) To determine an area or a volume is, as has been seen, (**333** and **1014 G.**[2]) to find the number of times that this area or volume contains another area or volume, which is taken as the measuring unit (**21 G.**). Thus, when it is said that a square toise contains 36 square feet, it must be understood that the measuring unit is the square foot and that this unit is contained 36 times in the square toise, the lineal toise being 6 feet, and 6 × 6 =36. Likewise, if the cubic toise, contains 216 cubic feet, it is that the cubic foot is in that case the unit of measure, and that this unit is contained 216 times in the toise, which being 6 lineal feet, its volume is (**1018 G.**) 6 × 6 × 6=216; and if the cubic metre contains 1000 cubic deci-metres, it is that the measuring unit is the deci-metre and that 10 × 10 × 10=1000.

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[1] (New Treatise on rectilineal and spherical geometry and trigonometry, &c., by the same author).

[2] REMARK—The numbers in black print and in parentheses, as (333 and 1014 G.), (24 G.), (1018 G.), &c., refer to the "new treatise on rectilineal and spherical geometry and trigonometry, &c.," by the same author, and the numbers also in black print and followed by a T., refer to the present treatise in which the corresponding numbers of the paragraphs or articles relate to the definition, demonstration or solution, as the case may be, of the thing enunciated in the treatise in question.

(**3**) The measuring unit which it is proper to employ is usually the square or the cube (as the case may be) whose side is (**333** and **1014 G.**) the lineal unit which served to establish the lineal dimensions of the figure to be measured; but it is clear that nothing prevents one from computing in square metres or yards the surface of a figure whose dimensions are expressed in feet or inches, &c.; and in the same way it will be indifferent to express in cubic feet, in metres or toises, &c,, the content of a body or solid, whose lineal dimensions might be given in yards, feet, or inches, &c.; paying attention only to the reductions necessary to change the given elements into elements of another name, that is, of a different value.

(**4**) The formula of the author, to find at once, or by decomposition, the volume of any body, is as follows :

" *To the sum of the bases or opposite and parallel end areas of*
" *the body to be measured, or of any one of its component slices, add*
" *four-times the area of a section parallel to these bases and situated*
" *half-way between them, and multiply the sum of these areas by the sixth*
" *part of the height or length of the solid.*"

(**5**) The new system then requires but the simple measurement of certain surfaces and sections of the body under consideration, since what remains to be done to arrive at the proposed volume is but a simple addition of these areas and the multiplication of their sum by the height or length of the solid, to take afterwards the sixth part of the result.

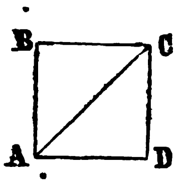
(**6**) But there is often a superficies or area to be measured independant of every consideration relating to the volume of the body of which such area forms the total or partial surface.

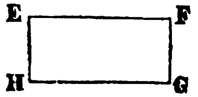
For these various reasons, it is then proper to treat first of the

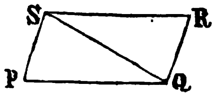
MENSURATION OF SURFACES.

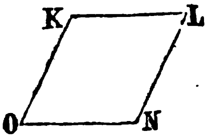
PROBLEM I.

To determine the area of any square, rectangle, lozenge, rhomb or parallelogram. (1)









(7) **RULE. I** *Multiply the base* (**182 G.**) *by the height* (**180 G.**) *and the product will be the required area* (**333** *and* **341 G.**).

Ex. 1. What is the area of a square whose side measures 204.3 feet ? **Ans.** 41738.49 square feet.

2. What is the number of *squares* (the *square* is 10 × 10=100 square feet) in a rectangular floor, ceiling, partition, wainscot,roof, &c., whose length=60 feet and breadth 35 feet ? **Ans.** 21.

3. What is the area of a parallelogram whose base is 12.25 and height 8.5 ? **Ans.** 104.125.

4. How many square yards of painting, in a rectangle whose base is 66.3 feet and height 33.3 feet ? **Ans.** 245.31.

5. To determine the area of a rectangular floor whose length is 12½ feet, and breadth 9 inches ? **Ans.** 9⅜ sq. inches.

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(1) See the component faces and sections of the prisms and other models of the TABLEAU. These figures are met with every where in the practice of the measurer, geometer, surveyor, &c.; thus, the floor or ceiling, or one of the walls of a room or apart ment will be generally a square or a rectangle. The same for a door orwindow, part of which at least will be rectangular, and this figure will be met again in the developed surface of a door, or any other opening which would be arched without being splayed, as well as in the development of the circumference of any apartment the plan of which were a circle or any other curvilineal figure and of which it will always be easy to obtain with sufficient accuracy the curvilineal dimensions with the aid of a ribbon, if the surface to be measured bs convex, or by means of a rod, thin enough to be fitted to the concave surface to be measured. As for the oblique-angled parallelogram, such surfaces will often be met with where two superposed courses of stairs of the same inclination occur. The subdivisions of territories into cantons, lots and parts generally affect for the most part figures of this kind.

6. Required the number of square yards of tapestry necessary to cover a parallelogram, whose base is 37 feet, and height 5 feet 3 inches? **Ans.** $21\frac{7}{12}$.

7. How many square feet of glazing in a rectangular window being 75 inches high by $37\frac{1}{2}$ inches broad? **Ans.** $75 \times 37\frac{1}{2} \div 144 = 19$ square feet $76\frac{1}{2}$ square inches $= 19\frac{76.5}{144} = 19.53125$ feet, or $6'.3'' \times 3'.1\frac{1}{2}'' = 19.6\frac{3}{8} = 19\frac{6.375}{12} = 19\frac{51}{96} = 19.53$ or about $19\frac{1}{2}$ sq. feet.

8. How many square inches of gilding are required to cover a surface whose length is 3 feet 3 inches and developed breadth or perimeter 13 inches? **Ans.** 507.

9. What is the number of superficial feet in all the mouldings of a stone, wooden or plaster cornice, &c., whose length is 60 feet 7 inches high and developed breadth or contour 3 feet $3\frac{1}{4}$ inches?

Ans. $199\frac{5}{12}$ (very nearly) sq. feet.

REMARK. These developed breadths, contours or perimeters are obtained by means of a thread or ribbon which is bent round the various mouldings, in a direction perpendicular (**996,998, G.**) to their length.

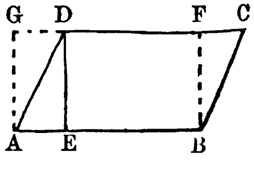
10. Required the number of square yards of varnish on a door whose height is $7\frac{1}{2}$ feet and developed breadth (measured around all the mouldings, &c.) 3 feet 11 inches? **Ans.** 3 sq, y. $2\frac{4.5}{12}$ s. f. = 3 sq. y. 2.375 sq. f. $= 3\frac{2.375}{9}$ sq. y. $= 3.2639$ sq. y. say nearly $3\frac{1}{4}$ sq. y.

11. How many square metres in a piece of ground being 113.75 metres long by 10.5 metres broad? **Ans.** 1194.375.

12. Determine in square arpents and perches, the area of a farm 40 arpents 5 perches deep or long, by 3 arpents $7\frac{1}{2}$ perches front or broad (10 lineal perches forming a lineal arpent and consequently 10×10 or 100 square perches, a square arpent.).

Ans. 151 arpents $87\frac{1}{2}$ perches.

(8) RULE II. *Find the product of the two adjacent sides of the parallelogram, and multiply this product by the natural sine of the included angle.*



It has been seen (**1231, 1° G.**) that when R=1 the perpendicular D E of the right angled triangle AED is equal to the product of the hypothenuse AD by the sine of the angle A; but DE is the height of the parallelogram AC, and since area AC=AB × DE and that DE=AD × sin. A, it is clear then that area AC=AB × AD × sin. A.

Ex. 1. What is the area of a rhomb or lozenge whose side is 25 chains and included angle 57° 33′. **Ans.** 25 × 25=625, and 625 × .84386 (nat. sin. of 57°33)=527.4125 sq. ch.

(9) **To solve this same problem by logarithms**[1] where R=10, we have (**1229** 1° **G**) R.: sin. A :: AD : DE; whence, $DE=\frac{AD \times \text{sin. } A}{R}$; but, area AG=AB × DE and by substituting to DE, its value $\frac{AD \times \text{sin. } A}{R}$, we obtain for area AG, the expession $AB \times \frac{AD \times \text{sin. } A}{S}$, or which is the same thing, area $AG=\frac{AB \times AD \times \text{sin. } A}{R}$; that is, *we must add together the logarithms of the two adjacent sides and the logarithmic sine of the included angle ; this sum, diminished by the logarithm of the radius, will be the logarithm of the required area.*

$$\text{Log. area AG}=\left\{\begin{array}{lll} +\text{log. AB} & 25\ldots\ldots\ldots\ldots & 1.397940 \\ +\text{log. AD} & 25\ldots\ldots\ldots\ldots & 1.397940 \\ +\text{log. sin. A} & 57°33'\ldots\ldots\ldots & 9.926270 \\ -\text{log. R}\ldots\ldots\ldots\ldots\ldots & & 10. \end{array}\right.$$

$$\text{Log. area. AG}= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2.722150$$

The next less log. 2.722140=527.41 chs. ; the difference between this log. and the log. found is 10, to which adding (**1286 G.**) two ciphers and dividing by 82, we have (very nearly) 22 which is added to the right of the figures 527.41 already found, making as before, 527.4122.

Ex. 2. Required the area of a farm the sides of which are respectively 40½ arp. and 3 arp. 7½ per. and the enclosed angle 57°33′.

$$\begin{array}{l} \textbf{Ans.}\ldots\ldots\ldots\ldots \\ \text{Log. required area.}= \end{array}\left\{\begin{array}{ll} +\text{log. } 40\tfrac{1}{2}\text{ arp. or 405 per.}\ldots\ldots\ldots & 2.607455 \\ +\text{log. 3 arp. } 7\tfrac{1}{2}\text{ per. or 37.5 per.}\ldots & 1.574031 \\ +\text{log. sin. enclosed angle } 57° 33'\ldots & 9.926270 \\ -\text{log. R.}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & 10. \end{array}\right.$$

$$\text{Log. required surface}=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4.107756$$

The next less log. .107549 corresponds to the number 1281 ; the difference between this log. and the log. found is 207 ; adding the 0 and dividing by the dif. (D) 338, we obtain 612426 which are written (**1286 G.**) to the right of the number already found 1281 to get 1281612426 ; but the characteristic of the found log. is 4, which corresponds (**1273 G.**) to 5 integer figures ; then the required area is 12816.12426 perches, or 128 arp. 16.124 (or 16⅛) perches, nearly.

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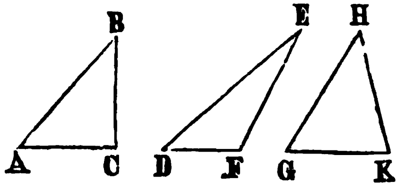
(1) For the tables of logarithms, see the "New treatise of Geometry and Trigonometry, &c.," by the same author.

PROBLEM II.

To find the area of a triangle.[1]



1ST CASE.

When the base and altitude are given.

(**10**) **RULE**. *Multiply the base by the altitude and take half the product. Or, multiply one of these dimensions by half the other,* (**344** *or* **348 G.**)

Ex 1. What is the area of a triangle whose base is 625 and altitude 260 ? **Ans**. 162500.

2. How many square yards of plastering are there in a triangular surface whose base is 40 feet and altitude 30 feet ? **Ans**. 66⅔.

3. How many square metres in a triangular piece of ground whose base measures 30 metres 7 deci-metres, and altitude 17 metres 39 centi-metres ? **Ans**. The required surface=30.7 metres × 17.39 metres=266.9365 sq. m.

4. How many squares of clap-boarding are required to cover a gable whose base is 39 feet 9 inches and height 23 feet 4 inches ?

Ans. 463¾ sq. ft.=4 squares 63¾ sq. ft.

5. To determine the number of squares of roofing with thatch, tile, slate, shingles, zinc, lead, copper or other metal, &c., in a hip-roof whose base is 65.4 feet and height 37.3 feet ?

Ans. 12 squares 19.71 sq. ft.

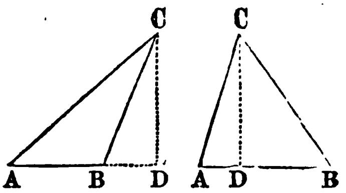
---

(1) See the component or limiting faces of the pyramids and other models of the *tableau*, and the sections or parallel planes as indicated by the line half-way between the parallel bases or faces. The triangle, like the parallelogram, is often met with in the practice of the measurer, &c. The gables of a building, the hips of roofs, the sides of a garret-window, &c., assume that kind of figure; and it is not uncommon either to have to determine the area of a triangular piece of ground.

2ND CASE.

When there are two sides given and the included angle.

(11) **RULE.** *Find the continued product* (**41 G.**) *of the two given sides and of the nat.*[1] *sine of the included angle ; the half of this product will be the required area.*



We have (**1231** 1° **G.**) as in the case (**8 T.**) of the parallelogram, CD=AC × sin. A or BC × sin. B ; or area $ACB=\frac{AB \times CD}{2}$ and since CD=AC × sin. A or BC × sin. B, we obtain for the area of the triangle the expression ½ (AB × AC × sin. A, or ½ (AB × BC × sin. B).

Ex. 1. What is the area of a triangle two sides of which are 30 and 40 and the enclosed angle 30° ? **Ans.** 300 sq. m.

2. Determine the area of a triangle one side of which is 45 yards, another side 37 yards and the enclosed angle 60° ? **Ans.** 720.9661.

3. The other data remaining the same, to determine the area for an enclosed angle=45° ? **Ans.** 588.6664.

(**12**) **By Logarithms.** *Add together the logarithms of the two sides and the logarithmic sine of their enclosed angle ; from this sum take* 10, *log. of the radius, and the remainder will be the log. of double the area of the triangle.*

For, (**G. 1229** 1°) R : sin. A : : AC : AD or R : sin B : : BC : CD ; whence, $CD=\frac{AC \times \text{sin. } A}{R}=\frac{BC \times \text{sin. } B}{R}$, and as area ABC =BA × CD, we have area $ABC=\frac{AB \times AC \times \text{sin. } A}{R}=\frac{AB \times BC \times \text{sin.} B.}{R}$

Ex. 1. Required the area of a triangle whose sides are AB= 125.81, AC=57.65, and the enclosed angle A=57°25′ ?

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(1) For the tables of natural sines, &c., see the " New treatise of rectilineal, and spherical geometry and trigonometry, &c. " by the same author.

Ans. Log. 2ABC=	+ log. AB	125.81	2.099715
	+ log. AC	57.65	1.760799
	+ log. sin. A	57°25′	9.925626
	+ log. R		10.

Log. 2 ABC = ... 3.786140

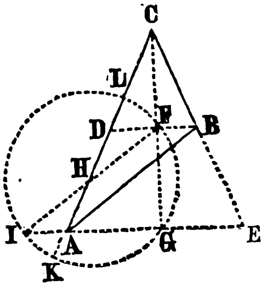
And 2 ABC=6111.4, or ABC=3055.7=area required.

2. How many square yards in a triangle whose sides are 25 feet and 21.25 feet and the enclosed angle 45° ? **Ans.** 20.8694.

3RD CASE.

When the three sides are known.

(**13**) **RULE I.** *Add the three sides together and take half their sum. From the half-sum take the three sides severally. Find the continued product of the half-sum and the three remainders. This product will be the square of the area of the triangle, and the square root* [1] *of this product the area required.*



Let ACB be the triangle. Take CD equal to the side CB and draw DB; draw AE parallel to DB, to meet at E the prolonged side CB : CE will then be equal to to CA. Take CFG perpendicular to DB and consequently also to AE which is parallel to DB ; CFG will bisect DE, AE at F and G. Draw FI parallel to AB, FHI which will meet CA at H and EA prolonged at I. Finally, from the centre H, with radius FH, describe the circumference of a circle ; this circumference will meet at K the prolongation of CA, pass through the point I, on account of AI=FB=DF (whence, HI=HF), and pass also (**444 G**) through the point G, because FGI is a right angle.

Now, since HA=HD=½ AD and CD=CB=½ CD + ½ CB, it is clear that CH is equal to the half sum of the sides AC, BC of the triangle ; that is CH=½ CA + ½ CB ; and since HK=½ IF=½AB, it follows that CK=½AC + ½BC + ½AB=½S, if the sum of the sides is represented by S.

Moreover, HK=HI=½IF=½AB, or KL=AB; whence, CL=CK —KL=½S—AB, AK=CK—AC=½S—AC, and AL=DK=CK—CD

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(1) See the tables at the end of this treatise.

=½S—BC. But, AG × OG = area ACE, and AG × FG = area ABE, whence AG × CF = area ACB; and by similar triangles, AG : CG :: DF : CF, or as AI : CF; therefore AG × CF (area of ACB) = CG × DF = CG × AI; then $\overline{AG \times CF} \times \overline{CG \times AI}$ or, which is the same thing, AG × CF × CG × AI is equal to the square of the area ACB.

But CG × CF = (**576 G.**) CK × CL = ½S × (½S—AB),

and AG × AI = (**572 G**) AK × AL = (½S—AC) × (½S—BC);

whence AG × CF × CG × AI = ½S × (½S—AB) × (½S—AC) × (½S—BC) = area ACB × area ACB = (area ABC)^2.

Ex. 1. Say to find the area of a triangle whose sides are 20, 30 and 40.

20 45 45 45
30 20 30 40
40 — — —
— 25 = 1st remainder. 15 = 2nd rem. 5 = 3rd rem.
2)90
45 = half-sum.

Now 45 × 25 × 15 × 5 = 84375.

The square root of this product is 290.4737, the required area.

2. The three sides of a triangle being 24, 36 and 48; what is its area? **Ans.** 418.282.

3. Required the area of an equilateral triangle whose side is 25? **Ans.** 270.632.

(14) By Logarithms. *After having determined the three remainders, make the addition of the logarithms of the half-sum and three remainders; the half-sum of the four logarithms will answer to the required area.*

Ex. 1. How many square yards of plastering in a triangular surface whose sides are 30, 40 and 50 feet? **Ans.** 66⅔.

2. The three sides of a piece of land measure 505.3, 330.7, and 402.5 metres. What is its area?

505.3 619.25—505.3 = 113.95 = 1st remainder.
330.7 619.25—330.7 = 288.55 = 2nd remainder.
402.5 619.25—402.5 = 216.75 = 3rd remainder.
1238.5
619.25 = half-sum.

+ log. half-sum	619.25	2.7918660
+ log. 1st. remainder	113.95	2.0567143
+ log. 2nd remainder	288.55	2.4602211
+ log. 3rd remainder	216.75	2.4359591
		2)9.6447605
		4.82238025

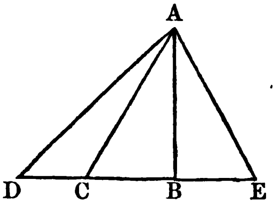
This log. corresponds to 66432.447 which is the required area.

(15) The same example by natural numbers will show the advantage resulting, in the present case, from the use of logarithms to diminish the work ; but, on their side, natural numbers have this advantage over logarithms, that by taking in all the decimals, with even the addition of ciphers to continue if need be the division or the extraction of the fractional part of the required root, precision may be carried to any degree of approximation required, whilst we cannot with exactness give to the answer obtained by logarithms, a greater number of figures than contained in the fractional part of the log. itself, as shown by the inexactness of the last figure (7) of the answer thus obtained.

```
     619.25          70563.53 75          20361108.7456 25
     113.95             2 88.55                     216 75
    -------        ------------         ----------------
    3096 25        352817 68 75         101805543 7281 25
   55732 5        3528176 87 5         1425277612 1937 5
  185775         56450830 00          12216665247 3750
  61925         564508300 0           20361108746 625
 61925          1411270750           407222174912 50
 -----------    --------------       ---------------------
 70563.53 75   20361108.74 56 25    6)4413270320.61 4218 75
                                       36
                                       --

         Proof.              12,6) 813
     66432.4493 +                  756          √≐66432.449304 +
     66432.4493 +                  ---
    --------------           132,4) 5727
     199297 3479                    5296
    5978920 437                     ----
   26572979 72              1328,3 43103
  265729797 2                       39849
 1328648986                         -----
 1892973479                13286,2 325420
 2657297972                         265724
 3985946958                         ------
3985946958                132864,4 5969661
--------------------                 5314576
4413270319.9970 7040 +               -------
                         1328648,4) 65508542
                                    53145936
                                    --------
                         13286488,9)1236260618
                                    1195784001
                                    ----------
                         132864898,3) 4047661775
                                      3985946949
                                      ----------
                        1328648986,0,4)617148260000
```

(16) **RULE II.** *Take for base of any given triangle* ADE, *its greater side* DE ; *find* (**578 G.**) DE : AD + AE :: AD—AE : CD, *difference of the segments* BD, BE *of the base by the perpendicular* AB ; *then* (**367 G.**) $DB = \frac{1}{2}DE + \frac{1}{2}DC$ or $BE = \frac{1}{2}DE - \frac{1}{2}DC$; *now you will have* (**308 G.**) *the perpendicular or altitude* AB *of the triangle* = $\sqrt{AD^2 - BD^2}$ *or, make* (**1229 G.** 1° alt. *or* **1235 G.**) AD : sin. B (= R) :: BD : sin. BAD, *to obtain* (**1231 G.**, 2°) AB = AD × cos BAD, *when* R = 1, *that is, if you work by natural numbers or*, AB = $\frac{AD \times \cos. BAD}{R}$ *if you work by logarithms, where log.* R = 10. *Finally you will obtain area* ADE = $\frac{1}{2}$(DE × AB).



Ex. The data being still the same as in the last example ; we shall have according to the rule

AD = 402.5
+AE = 330.7
= sum 733.2

AD = 402.5
—AE = 330.7
= dif. 71.8

DE = 505.3 = base
÷ 2 = 252.65 = half-base

DC = 104.183178 = dif. of the seg.
÷ 2 = 52.091589 = half-dif.

$\frac{1}{2}$DE = 252.65
+ $\frac{1}{2}$DC = 52.091589
= seg. BD = 304,741589

Nat. sin. found = .7571220 corresponds to 49° 12′ 10.0737″ = BAD.

DE : AD + AE :: AD—AE : BD—BE (or DC)

505.3 : 733.2 :: 71.8 : 104.183178 + = DC

71.8
58656
7332
51324

505.3)52643.76 (104.183178 +[1]
5053

---

[1] It is because this quotient must enter into the calculation to be made to find the sine of the angle BAD that it is necessary to carry the decimals far enough to secure sufficient exactness in the last figures of this sine.

```
21137                         AD   : R ::    BD     :sin.BAD
20212                         402.5:1  :: 304.741589: .7571220—
─────                         28175
 9256                         ─────
 5053                          22991
─────                          20125
  42030                        ─────
  40424                         28665
  ─────                         28175
   16060                        ─────
   15159                          4908
   ─────                          4025
    9010                          ────
    5053                           8839
    ────                           8050
    39570                          ────
    35371                           7890
    ─────
     41990
Nat. sine found=.7571220              Dif. of cos. for 60″=2202
   Next less sine.=.7569951=49°12′   60″ : 2202 : 40.0737″ : 14707
                ───────                               2202
     Difference=    1269                            ──────
    Dif. for 60″=   1900                            801474
       1900 : 60″ :: 1269 : 40.0737″               801474
                     6                            801474
                  ─────                           ──────
              190)7614                         ÷6)882422
                  760                            =147070
                  ─────
                   1400
        AB=AD × nat. cos. BAD
              BAD=49° 12′ 40.0737″
        Nat. cos. 49° 12′ =                   .65342060
        Dif. for   40.07″= —                      14707
                                              ─────────
        Nat. cos. of 49° 12′ 40.0737″         .65327353
                           ×   AD                 402.5
                                              ─────────
                                              326636765
                                             130654706
                                            261309412
                                            ────────────
          AD × nat. cos. BAD=AB=262.942595825
                          × DE                    505.3
                                            ────────────
                                           788827787475
                                         1314712379126
                                        1314712979125
                                        ─────────────
      AB × DE=2 area ADE=               1328648936703
                  ½ AB.DE=              66432.44683 = area ADE.
```

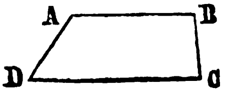
(17) The area found according to this rule is 66432.4468 square metres. The exactness of this result is yet, as it is seen, carried but to the 7th figure, and it cannot be otherwise since the natural sines used and which enter, as elements, into the solution of the problem, extend but to 7 figures, the last of which even is always too great or too small according as it has been, or not, increased by unity when the following figure is greater or less than 5.

(18) Let us remark here that this example, the calculation of which we have just made, in three different manners, allows one to compare the amount of work required by each mode of solution, and enables one to choose when required, whether the most expeditious means (the first) or the one admitting of the greatest precision (the second) or the one requiring no extraction of a root (the third).

(19) It is hardly necessary to remark that this problem, like the preceding one, and the one following, may also be solved by means of a graphic construction allowing one to establish with the help of a sufficiently subdivided scale, the length or value of the perpendicular AB in terms of the base or sides ; and that is often enough the shortest, though not the most precise mode of arriving at the required result.

PROBLEM III.

To find the area of a trapezium.[1]



(20) **RULE** *Find* **(346 G.)** *the sum of the two parallel sides ; multiply this sum by the height or breadth of the trapezium, and half the product will be the required area.*

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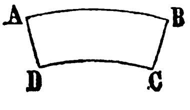
[1] See the component faces (bases and lateral faces) and the sections and parallel planes of the prismoids and other models of the Tableau. The trapezium **(172 G.)** presents itself often enough, in practice, to the calculation of the measurer. Thus, the interior table of a window whose sides are generally splayed, presents the form of a trapezium ; so for the ceiling of a window door or other splayed opening ; and it is plain also that the developed surface ABCD, (part of a concentric ring, see the parallel bases of the hollowed cylinder of the Tableau and the lateral faces of the sections of the hollowed sphere), of the jamb of a curved as well as splayed opening may also be considered as a kind of trapezium with parallel curvilineal bases, but whose area is equally determined by the rule here given, since that figure is nothing else but a



Ex. 1. In a trapezium, the parallel sides are $10\frac{1}{2}$ and $12\frac{1}{4}$ feet, and the perpendicular distance between these sides 3 feet 2 inches. What is the area ? **Ans.** $\frac{1}{2}(10\frac{1}{2}+12\frac{1}{4})\times 3\frac{1}{6}=\frac{1}{2}(10.5+12.25)\times 3.166$ $=11.375\times 3.166=36.01325$ sq. feet.

2. Required the area of a piece of ground whose parallel sides measure respectively 75 and 122 links, and the perpendicular 154 links ? **Ans.** 15169 sq. links.

3. How many square feet area in a board whose length is $12\frac{1}{2}$ feet, breath at one end 15 inches and at the other end 11 inches ? **Ans.** $13=\frac{13}{24}13.541666+$

4. How many square yards in a trapezium whose parallel sides are 240 and 320 feet, and height 66 feet ? **Ans.** $2053\frac{1}{6}$.

5. The parallel sides of a farm are 12.51 and 8.22 chains, and the perpendicular 5.15 chains ; what is the area in square chains ?

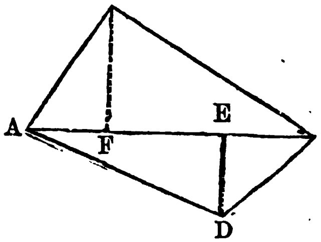
Ans. 53.37975.

PROBLEM IV.

To find the area of a quadrilateral.[1]

(21) RULE. *Multiply* **(351 G.)** *either of the diagonals* **(173 G.)** *of the quadrilateral, by the half-sum of the perpendiculars drawn from the opposite angles to the common base.*

Ex. 1. What is the area of a quadrilateral BD whose diagonal AC is 42 feet, and perpendiculars BF=18 and DF=16 feet ? **Ans.** 714 sq. ft.



2. How many square toises of paving are there in a quadrilateral whose diagonal is 65 feet and the two perpendiculars 28 and $33\frac{1}{2}$ feet ? **Ans.** 55.52083.

---

frustum or part of a circular ring, and that the mode **(1145 G.)** of arriving at the area of that figure is like the one that shows how to determine the area of the trapezium so called. The trapezium is also often met with in the floor or ceiling of a room, two sides of which only are parallel, in the roof of a dormer-window, flight of stairs, roof or ceiling of a garret, and the sides or jambs of a rectangular window assume also this form when the ceiling or table is inclined or splayed. Finally, we are very often called on to determine the area of a lot of ground having the form of a trapezium.

[1] See the bases, lateral faces, sections or parallel planes of certain models of the TABLEAU.

3. How many square metres area in a quadrangular piece of ground one of whose Diagonals is 64 metres and the perpendicular distances from this Diagonal to the two opposite angles, 28 and 32 metres? **Ans.** 1920 sq. m.

4. Determine the number of *squares* of flooring to cover a quadrilateral space, whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches?

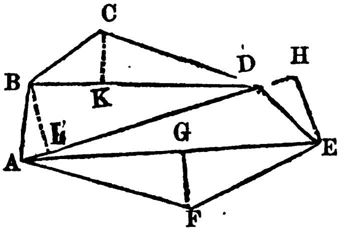
Ans. 63 *squares*, $47\frac{3}{12}$ sq. ft.

5. Required the number of arpents in a piece of land one of whose diagonals measures 70.5 perches, and the perpendiculars 26.5 and 30.2 perches? **Ans.** 19 arp. 98.675 per.

PROBLEM V.

To find the area of an irregular polygon.[1]

(**22**) **RULE.**—*Measure the diagonals which will divide the given polygon into quadrilaterals and triangles. Determine separately the areas of these component figures; their sum will be the area required.*



Ex. 1. Determine the area of the polygon BE, in which BD $=18\frac{1}{2}$, CK $=12\frac{4}{5}$, AD $=27\frac{1}{2}$, BL $=9.5$, EH $=14$, AE $=40$, and FG $=8$.

Ans. $\frac{1}{2}$ (BD × CK) $=\frac{1}{2}$ $(18.5 \times 12.8) = 118.40$ = area BCD, $\frac{1}{2}$ (BL + EH) $=\frac{1}{2}$ $(19.5 + 14) = 11.75$ and quadrilateral area ABDE = AD × $\frac{1}{2}$ (BL + EH) $= 27.5 \times 11.75 = 323.125$, area AEF = AE × $\frac{1}{2}$FG $= 40 \times 4 = 160$. Area ABCDEF $= 118.40 + 323.125 + 160 = 601.525$.

2. Required the number of acres (the acre is 100,000 square links) in a polygonal piece of ground BE whose diagonals BD, AD and AE measure respectively 13 chains (the lineal chain is 100 links)—33 links, 13 chains 99 links, and 14 chains 13 links, and whose perpendiculars CK $=173$ links, BL $=2$ chains, EH $=2\frac{1}{5}$ chains and FG $3\frac{3}{4}$ chains.

Ans. BD × CK $= 1333 \times 173 = 230609 \div 2 = 115304\frac{1}{2}$ = area BCD.
AD × BL $= 1399 \times 200 = 279800 \div 2 = 139200$ = area ABD.
AD × EH $= 1399 \times 220 = 307780 \div 2 = 153990$ = area ADE.
AE × FG $= 1413 \times 375 = 529875 \div 2 = 264937\frac{1}{2}$ = area AEF.

2)13.48064 6.74032 = area ABCDEF.
6.74032 that is 6 acres and 74032 sq. l.

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[1] See the bases, lateral faces, sections or parallel planes of certain models of the TABLEAU.

or 6 acres 2 roods and 24032 links (the rood being the fourth part of the acre, that is 100000 ÷ 4 = 25000 links)
or 6 acres, 2 roods, 38 perches, and 282 links (the lineal perch being the fourth part of a chain, that is 25 links, and the square perch consequently = 25 × 25 = 625 sq. links.)[1]

PROBLEM VI.

To determine the area of a long and irregular figure bounded on one side by a straight line.[2]

(**23**) **RULE**. 1° *Measure, at each end of the straight line, the perpendicular breadth of the figure; measure also this breadth at several intermediate places equidistant from each other.*

2° *To the half-sum of the extreme breadths add the sum of the intermediate breadths; multiply then the sum thus obtained by one of the equal parts of the line of base; the product will be very nearly the area required.*

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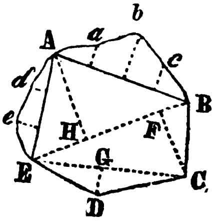
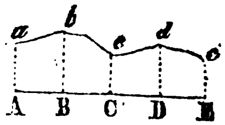
[1] Gunter's chain is 66 english feet, divided into 100 links, each of which is consequently = 66 ÷ 100 = 7.92 english inches. The acre is equivalent to 1 chain × 10 chains = 10 square chains = 4 perches × 40 perches = 160 square perches = 100 links × 1000 links = 100000 square links. The advantage of this division of Gunter's chain into 100 parts consists in this, that all the dimensions which it helps to establish, are immediately applicable and without reduction to decimal calculation. The operation being done, 5 decimals are cut off, the remaining figures to the left being acres since there are 100000 links in the acre and that to cut off 5 figures is equivalent to dividing by 100000. It is plain also that for the roods we have but to multiply first the remainder by 4 and again cut off 5 figures, which is equivalent to dividing at once by 25000 (number of links in a rood) and is by far more expeditious. For perches, the second remainder is then multiplied by 40, cutting off 5 figures as before, since the perch is the 40th part of the rood; or if desirable to neglect the roods, the first remainder may be at once multiplied by 160 (4 × 40) and 5 figures equally cut off. The last remainder .45120 is evidently a fraction of a perch, that is, $\frac{45120}{100000}$ of a perch; and the square perch being 625 links, $\frac{1}{100000}$ of 625 = .00625, this number multiplied by the numerator .45120 gives the 282 links of the answer; that is: for the links the last remainder is simply multiplied by 625 and 5 decimals cut off as before.

[2] The tracts of land which are near, and are bounded on one side by the windings of a road or river, &c., often present to calculation figures of this kind; or, after having determined by the method of the last problem the area of the rectilineal polygon ABCDE which forms part of the irregular polygon A*b*BCDE*ed*A, the method of the present problem will be made use of to obtain the secondary and irregular parts A*abc*B, A*de*E.



Let AE*ea* an irregular figure having for its base the straight line AE. At the points A, B, C, D and E, equidistant from each other, draw the perpendiculars A*a*, B*b*, C*c*, D*d*, E*e* and designate these perpendiculars by the letters *a*, *b*, *c*, *d*, *e*.



Then (**325 G**) the area of the trapezium AB*ba*$=\frac{a+b}{2}\times$ AB,

the area of the trapezium BC*cb*$=\frac{b+c}{2}\times$ BC,

the area of the trapezium CD*dc*$=\frac{c+d}{2}\times$ CD,

and the area of the trapezium DE*ed*$=\frac{e+d}{2}\times$ DE ;

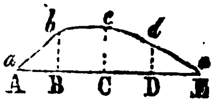
then, their sum, or the area of the whole figure is equal to

$$\left(\frac{a+b}{2}+\frac{b+c}{2}+\frac{c+d}{2}+\frac{d+e}{2}\right)\times \text{AB}.$$

since AB, BC, &c., are equal to each other. But, this sum is equal to

$$\left(\frac{a}{2}+b+c+d\times\frac{e}{2}\right)\times \text{AB}.$$

expression which agrees with the enunciation of the rule.



(**24**) If A*a* becomes very small, we will none the less have area. AB*ba*$=\frac{a+b}{2}\times$ AB and if *a* and A are confounded in one & the same point, or that the trapezium AB*ba* becomes the triangle AB*b*, we will have $\frac{a+b}{2}=\frac{b}{2}$; in this case it is plain that the expression for the area of the figure AE*edb*A becomes $\left(\frac{b}{2}+\frac{b+c}{2}+\frac{c+d}{2}+\frac{d+e}{2}\right)\times$ AB, or, which is the same thing, $(b+c+d+\frac{1}{2}e)\times$ AB. And if E*e* becomes also$=o$, the expression for the area AE*db*A will take the form $(b+c+d)\times$ AB.

Ex. 1 The breadths of an irregular figure at 5 equidistant points, being 8. 2, 7. 4, 9. 2, 10. 2, and 8.6 and the length of the base=40 ; what is its area ?

One of the extreme breadths = 8.2
The other extreme breadth = 8.6

Sum of the extreme breadths = 16.8

Half sum = 8.4
1st intermediate breadth = 7.4
2d intermediate breadth = 9.2
3d intermediate breadth = 10.2

Sum of the breadths = 35.2

the whole base = 40
One of the equal parts = 40 ÷ 4 = 10
Sum of the breadths = 35.2
Multiplied by 10

= area wanted = 352

2. The length of an irregular figure being 84 metres and the breadths, at six equidistant points 17.4, 20.6, 14.2, 16.5, 20.1, and 24.4 metres; required the area. **Ans.** 1550.64 sq. m.

3. The length of a strip of land is 125 perches and its breadth in 15 different and equidistant points, is 5.2, 4.6, 7.2, 8.3, 9.4, 8.1, 7.3, 7.9, 6.6, 7.2, 7.3, 8.4, 7.4, 6.5, and 5.8 perches. What is its content ?

Ans. The sum of the extreme half-breadths and of the intermediate breadths=101.7, the length 125÷14=8.92857 and 8.92857 × 101.7=908.0356 square perches. (1)

(25) REM. Some authors teach how to determine the area of the figure of this problem by finding the product of the whole base AE by the mean of the breadths which is obtained by adding together all these breadths and then dividing their sum by their number. This rule is erroneous, and the more so the less the number of breadths or divisions in the figure to be computed. The error of this method, in case there were but three component parts and con-

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1. *If the lineal perch in question here is* 18 *french feet*, that is, the tenth of an arpent, the area just found will be equivalent to 9 square arpents, 8.0356 square perches, for, as it has already been remarked, the square arpent is 10 × 10 perches=100 square perches, and as the square perch is 18×18=324 square feet (or the square arpent 324×100=32400 square feet) the decimal .0356 of a square perch may be reduced if need be into square feet by multiplying by 324, which gives in this example 11.53 square feet. *If*, on the contrary *the lineal perch were* 16¼ *english feet* which is that of Gunter's chain, we would have after dividing by 160,5 acres, 108.0356 perchs, and if we desired afterwards to reduce into square feet, the decimal of a perch, it is plain that the square perch being 16¼×16¼=272.25 square feet (or the acre=272.25 × 160 or 66×660=43560 square feet) it would be sufficient to multiply .0356 by 272.25 to have 9.69 english square feet.

sequently four heights or breadths, might reach to 25 per cent short of the exact area. It gives for the mean breadth, in this example, 107.2÷15=7.1466 et 7.1466 × 125=893.325 square perches instead of 908.035; or an error of nearly 15 perches of ground.

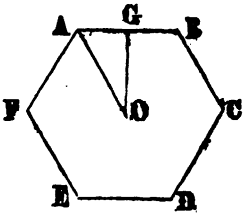
PROBLEM VII.

To find the area of a regular polygon. [1]

(26). RULE 1. *Multiply* **(663 G)** *the perimeter of the polygon by its right half-radius, and the product will be the required area.*

REM. If the polygon be known but by its side, determine first its right radius in the following manner : Divide 360° by the number of sides of the proposed polygon, and the quotient will be **(620 G.)** the angle at the centre ; that is, the angle subtended by one of the equal sides. Now the right and oblique radii of the polygon form with the half-side a rectangular triangle in which the base is known, that is the half-side, and the opposite acute angle, that is, the half-angle at the centre, to find the perpendicular or right radius of the polygon.

Ex. 1. Say to find the area of a regular hexagon whole side is 20 feet ?



Ans. 360°÷6=60 and 60÷2=30° angle AOG, half of AOB. We have also OAG= 90°—AOG=60° and AG=10; then **(1825 G.)** sine AOG : AG :: sine OAG : OG : whence,

	Sin. AOG 30° .... ar. comp. log.	0.301030
is to	sin. OAG 60°	9.937531
as	AG 10	1.000000
is to	OG 17.32052	1.238561

Now as there are 6 sides, each equal to 20, we will have the perimeter 20 × 6=120 and the area=120 × $\frac{1}{2}$ (17.32052) or which is the same=17.32052 × $\frac{1}{2}$(120)=17.32052 × 60=1039.23120 s. f.

Ex. 2. What is the superficial content of an octogon whose side is 20 ? **Ans.** 1931.368.

---

[1] See the parallel bases of the right prisms and prismoids of the *Tableau* and their parallel sections, or planes.

For the angle at the centre=360°÷8=45° the half of which 22° 30' is the angle AOG adjacent to the right radius, and its complement OAG consequently =90°—22°30'=67°30; but we have (**1231, 3° G**) OG=AG × nat. tang. OAG=10 × 2.41421=24.41421 and area =24.41421 × 80 (half-per.)=1931.368.

3. Required the area of a nonagon whose side measures 8 feet and the perpendicular drawn from the centre=10.99 feet ?

Ans. 395.64 sq. f.

4. Find the area of a regular heptagon whose side=19.38 and the right radius=28 ?

Ans. 1899.24.

5. The side of a pentagon=25 metres and the distance from the side to the centre=17.2 metres ; what is its content ?

Ans. 1075 sq. m.

(**27**) With the help of this rule, it is easy to obtain the area of any polygon [1] that is of a polygon of any number of sides. Having calculated and disposed under the form of the following table, the relative areas of the various polygons having for side unity or 1 ; namely :

Name.	*Radius of the circum. circle.*	*Sides.*	*Radius of the inscr. circle*	*Area.*	*The angle* OAB.
Triangle	0.5773503	3	0.2886751	0.4330127	30°
Square	0.7071068	4	0.5000000	1.0000000	45
Pentagon	0.8506508	5	0.6881910	1.7204774	54
Hexagon	1.0000000	6	0.8660254	2.5980761	60
Heptagon	1.1523824	7	1.0382607	3.6339124	$64\frac{2}{7}$
Octagon [2]	1.3065628	8	1.2071068	4.8284271	$67\frac{1}{2}$
Enneagon	1.4619022	9	1.3737387	6.1818242	70
Decagon	1.6180340	10	1.5388148	7.6942088	72
Undecagon	1.7747324	11	1.7028436	9.3656399	$73\frac{7}{11}$
Dodecagon	1.9318517	12	1.8660254	11.1961521	75

And because (**565 G.**) the areas of similar polygons are to each other as the squares of their homologous sides, the area of any given polygon will have to the square of its side the same relation that the area of the polygon of the same name and whose side is 1, has to the square of unity ; whence, we have :

(**28**) **RULE II.** *Square the side of the given polygon ; multiply then this square by the area of the polygon of the same name whose side is 1 : the product will be the required area.*

---

[1] See the bases and parallel sections of the prisms and prismoids, &c., of the *Tableau.*

[2] In the case of the regular or even symmetrical octogon the area is immediately obtained by taking from the square of the double right radius or apothem on one of sides, the square of the other side, as will be seen in the mensuration of solids.

Ex. 1. What is the area of a regular hexagon, whose side is 20?

Ans. $20^2=400$, the area of the hexagon of the table$=2.5980762$, and $2.5980762\times 400=1039.2304800$, as before.

2. Determine the superficial content of a pentagon whose side is 25 yards? **Ans.** 1075.298375 sq. y.

3. The side of a decagon measures 20 metres; what is its area? **Ans.** 3077.68352 sq. m.

4. Find the area of a duo decagon whose side is 6? **Ans.** 403.0614864.

5. The side of a piece of ground having the form of an equilateral triangle measures 3 arpents 7 perches and 6 feet, what is its content?

Ans. $37\frac{1}{3}$ per. $\times$ $37\frac{1}{3}$ per$=1393\frac{7}{9}$ or 1393.77777, $\times\ 0.4330127=603.5234787$ or 6 square arpents, $3\frac{1}{2}$ square perches nearly.

PROBLEME VIII.

To find the circumference of a circle[1] whose diameter is known, or the diameter of a circle of which the circumference is known.

(**29**) **RULE.** *Multiply* (**685 G.**) *the diameter by* 3.1416, *and the product will be the circumference; or divide* (**687 G.**) *the circumference by* 3.1416, *and the quotient will be the diameter.*

Ex. 1. What is the circumference of a circle whose diameter is 25? **Ans.** 78.54.

2. If the diameter of the earth be 7912 miles, what is its circumference? **Ans.** 24884.6136.

3. Determine the diameter, to circumference 11652.1944? **Ans.** 3709.

4. Required the circumference, when the diameter is 17 metres? **Ans.** 53.4072.

5. The circumference of a circle is given$=354$ feet, determine its diameter? **Ans.** 112.681.

---

[1] See the bases and sections or parallel cutting planes of the cylinders, cones and frusta of right cones, frusta and segments of spheres, &c., among the models of the *Tableau*.

REM. The relation 7:22 would give for this diameter 112.636 This last result, is too small by $\frac{45}{1000}$ of a unit or $\frac{4}{10000}$ of the whole, and enables one to judge of the relative exactness of the two ratios.

PROBLEM IX.

To find the area of a circle.

(**30**) **RULE I.** *Multiply* (**431 G.**) *the circumference by half the radius.*

RULE II. *Multiply* (**1024 G.**) *the square of the radius by* 3.1416.

RULE III. *Multiply* (dem. of **684 G**). *the square of the diameter by* .7854.

Ex. 1. What is the area of a circle of which the diameter is 10 !

Ans. 78.54.

If the diameter were 100, the area would be............ 7844

If the diameter were 1000, the area would be............ 785400

2. The diameter is 7, and the circumference 21.9912, what is the area of the circle ? **Ans.** 38.4846.

3. How many square yards are there in a circle whose diameter is 3½ feet ? **Ans.** 1.069016.

4. The diameter being 7, what is the area of the circle ?

Ans. 38.4846.

5. Find the area of a circle whose radius is 30½ perchs ?

Ans. 2922.4734 square perches.

(**RULE IV.**) *Multiply the square of the circumference* by .07958 : *the product will be the area of the circle.* For, let c the given circumference, d the diameter and $\pi=3.14159$; then (**686 G**) $c=\pi d$, and (**687 G.**) $d=\frac{c}{\pi}$; thence the area of the circle$=\frac{\pi d^2}{4}$ since (**1024 G.**) the area of a circle whose radius is $r=\pi r^2$ and that $d^2=4r^2$; but since $d=\frac{c}{\pi}$, we have $d^2=\left(\frac{c}{\pi}\right)^2=\frac{c^2}{\pi^2}$; and as $\frac{\pi d^2}{4}=\frac{1}{4}\pi d^2$, we have $\pi\frac{d^2}{4}=\frac{1}{4}\pi$

$$\frac{c^2}{\pi_2}=\frac{c^2}{4\pi}=\frac{c^2}{4\times 3.14159}=\frac{c^2}{12.56636}=c^2\times\frac{1}{12.56636}=c^2\times 07958.$$

Ex I. Find the area of a circle whose circumference is 10.75.

Ans. 9.196463750.

2. Determine, in acres, the area of a piece of ground whose circumference measures one mile (say 80 chains of Gunter$=66 \times 80=$ 5280 english feet) ? **Ans.** 50.9312.

PROBLEME X.

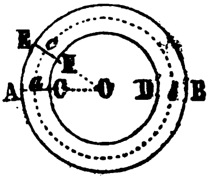
To find the area of a circular ring or the space comprised between two concentric circles. [1]

(32) RULE I. *Find* **(1144 G.)** *by the last problem the areas of the two circles : their difference will be the area of the ring.*

RULE II. *Multiply* **(371 G.)** *the sum of the diameters by their difference : this product multiplied by* .7854 *will be the required area.*

RULE III. *Multiply the half-sum of the circumferences of the two circles by the half-difference of their diameters, that is by the breadth of the ring, and the product will be the required area.*

For each unit of the diameter corresponds to 3.1416 units of the circumference ; then if a C $=$ a A $=$ a unit or any part of the diameter A B or C D, the excess of the circumference a b over the circ. C D will be equal to the excess of A B on a b ; whence a b is an arithmetic mean (**1265 G.**) between circ. A and circ. C. Now, (**428 G.**) A E : a c : C F :: circ. A. circ. : a b : circ. C D ; therefore a c is an arithmetic mean between AE, C F ; and since the arc A E, indefinitely small, may be considered (**430 G.**) as being sensibly a straight line, the part A E F C of the circular ring may be considered as a trapezium ; but, area trapezium A E F C $=$ (**347 G.**) a c $\times$ A C ; then also, area ring A C $=$ circ. a b $\times$ A C.



Ex. 1. How many square inches in the area of a circular ring whose exterior diameter is 30 inches and the breadth $2\frac{1}{2}$ inches ?
Ans. 215.985.

2. The diameters of two concentric circles are 15 and 10 : what is the area of the ring formed by these circles ? **Ans.** 98.175.

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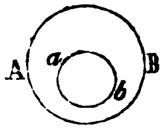
[1] Such would be an alley around a circular garden, the horizontal section of a hollowed column, the ground plan of the wall of a tower, a section perpendicular to the axis of a pipe or tube, &c., &c. See the parallel bases of the hollow cylinder of the *Tableau*.

3. Required the area of the ring whose containing circles have for diameters 9 and 5? **Ans.** 43.9824.

4. The two diameters of a circular ring are 21.25, and 9.75; what is its superficial content? **Ans.** 279.9951.

5. Determine the area of the space comprised between two concentric circles whose diameters are 15 and 16? **Ans.** 24.3474.



(33) If the circles AB, $a\,b$, had not the same centre, as is the case for an eccentric wheel, it is clear that the area of the annular space comprised between the circles would in the same way be obtained by finding (Rule 1) the difference of area of each of them. [1]

PROBLEME XI.

To find the length of an arc of a circle. [2]

(34.) RULE 1. *Multiply the number of degrees in the proposed arc by .0087266 and this product by the diameter of the circle.*

REM. I. Since the circumference is 3.1416 when the diameter is 1, it follows that 3.1416÷360 = 0.0087266 = length [3] of the arc of one degree, to a diameter equal to unity. This quotient multiplied by the number of degrees in an arc, will be the length of this arc in the circle whose diameter = 1; and this product multiplied by any diameter will give the length of the arc in a circle of that diameter.

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[1] Base or central section of the eccentric ring of the *Tableau*; projection on a plane of the opposite bases of a frustum of an oblique cone.

[2] See among the models of the *Tableau* the limiting arcs of the segments and sectors of a circle, bases of the ungula of a cylinder, cones and frusta of right cones, lateral sides, of spherical pyramids and of sections of hollowed spheres, &c.

[3] It has already been observed and besides it is clear that the exactness of a result is limited by that of the elements concerned; it is then hardly necessary to remark that according to the degree of precision wanted, it may become necessary to use a larger or smaller number of the decimals of the unit of such element; thus it is clear that the solution of the problem in question here may require to replace the ratio $\pi = 3.1416$ generally used, by the more exact ratio $\pi = 3.14159$ or by the ratio still more approximative $\pi = 3.141592$, $\pi = 3.1415926$, $\pi = 3.14159265$, &c., with an additional decimal of the term or factor π for each additional decimal of the unit of the result.

REM. 2. Since a minute is the 60th part of [illegible]ee, and a second the 60th of a minute or the (60×60) 3600th [illegible] if the arc proposed contains minutes, they will be reduced, [illegible]ng them by 60, to the decimal of a degree, and if the arc also co[illegible] seconds, the minutes will first be reduced to seconds and the whole afterwards divided by 3600 which will change as before into decimals of a degree the fractional part of the arc.

Ex. 1. The diameter being 18 feet, what is the length of the arc of 30° ? **Ans.** 4.712364.

2. Find the length of an arc of 12°.10′ or 12⅙°, with a diameter of 20 ? **Ans.** 2.123472.

3. In a circle whose diameter is 68, what is the length of the arc of 10°.15 or 10.25° ? **Ans.** 6.082396.

4. Required the length of an arc of 57°17′ 44″ ; the radius of the circle being 25 feet ? **Ans.** 25 feet.

For 57° 17′ 44″ is the 3.1415926th part of 180°, that is the length of the radius in terms of the circumference.

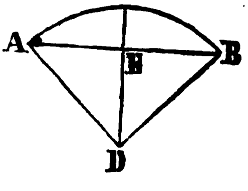
5. Determine in a circle whose radius is 20, the length of an arc of 50° 30′ 3″ ? **Ans.** 15.885.

REM. 3. If the number of degrees in the required arc were not known, it would be easily found by the method of par. (**785 G.**) where the chord and height of the arc are given to find the remainder.

(**35**) **RULE II.** *Determine* (**785 G.**) *the length of the whole circumference of which the given arc forms a part and establish then the following proportion, viz : 360° : the length of the circumference :: the number of degrees in the arc : the length of the arc.*

Ex. 1. Under a radius 14, what is the length of the arc of 60° ? **Ans.** 14 6607720

2. The chord AB of an arc ACB is 30 feet, and the height or versed-sine EC is 8 feet ; find the length of the arc ? **Ans.** 35⅓ feet, nearly.



3. What is the length of the arc whose chord is 48½ and height 18¼ ? **Ans.** 64.767 nearly.

4. If the chord of an arc measures 20.386 perches, and its versed-sine 4 perches ; what is the length of the arc ? **Ans.** 22.402 perches nearly.

5. Required the length of an arc of circle whose chord is 40 and the height 15 ? **Ans.** 53.33 nearly.

(36) **RULE III.** *It is also shown that: the length of an arc is very nearly obtained, by subtracting from eight times the chord of half the arc, the chord of the whole arc, and then taking one third of the difference.*

Ex. 1. The chord of an arc is 36.75 and the chord of half the arc 23.2; what is the length of the arc? **Ans.** 49.616 nearly.

Ex. 2. What is the length of an arc whose chord is 50.8 and the chord of half the arc 30.6? **Ans.** 64.66 nearly.

REM. When the chord and the height only of the whole arc are known, the chord of half the arc, if need be, is obtained equal (365 G.) to the square root of the sum of the squares of the versed-sine and half-chord.

PROBLEME XII.

To find the arc of a sector of a circle. [1]

(37). **RULE I.** *Multiply* (430 2° G.) *the arc of the sector* (that is the length of the arc) *by half the radius.*

RULE II. *Find the area of the whole circle, and then make the proportion: 360 degrees: degrees in the arc of the sector :: the area of the whole circle: the area of the sector.*

Ex. 1. Required the area of a sector, whose arc is 18 degrees and diameter of the circle 3 feet? **Ans.** 0.35343.

2. What is the area of a sector of which the arc is 20 and the radius 10? **Ans.** 100.

3. The arc of a sector is 147°29′ and its radius 25, what is the superficial content? **Ans.** 804.3986.

4. Determine the area of a sector, when the chord of the arc $=28$ and the chord of half the arc$=16$? **Ans.** 275.39.

5. The radius of the circle being 10, what is the area of the sector of which the cord of the arc is 20? **Ans.** 157.08.

6. The chord of the arc is 16 and its height 6; what is the area of the sector? **Ans.** 88.873.

7. To find the content of a sector of which the height of the arc $=4$ and the radius$=8$? **Ans.** 66.858 nearly.

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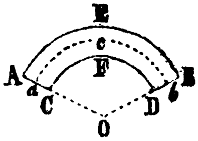
[1] See among the models of the *Tableau*, the lateral faces of the tri-acutangular, tri-rectangular and tri-obtusangular, spherical pyramids.

PROBLEM XIII.

To find the area of a sector of a circular ring or the space comprised between two arcs of concentric circles.[1]

(38). **RULE I.** *Multiyly* (dem. of **32, R. III. T.**) *The half-sum of the interior and exterior arcs of the sector by its breadth; that is by the breadth of the ring of which the sector forms a part, or, which is the same thing, by the difference of the radii of the concentric arcs which contain it.*

RULE II. *Find by the last problem the areas of the two concentric sectors: their difference will be the required area.*



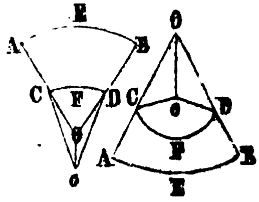
Ex. 1. The arc AEB or CFD of a sector AB of a circular ring is 30° the breadth A C of the ring 2½ and the radius AO of the exterior arc 15 inches? **Ans.** The area=17.99875, say 18 sq. in.

2. The two radii of a sector of a circular ring are 10.625 and 4.875 and the angle at the centre O or A O B that is the arc A E B is 270°; required the area of the sector? **Ans.** 209.996, say 210.

3. The arcs which comprise a section of a circular ring are 11 feet 9 inches and 10 feet 3 inches, and the breadth of the ring 13 inches; what is its area? **Ans.** 11$\frac{11}{12}$ sq. feet.

4. Determine the area of the space comprised between two half circles having a common centre, and whose diameters measure 20 and 30? **Ans.** 39.270 × 5 = 196.35.

(39.) **REM.** If the component sectors ABO, CD° had not the same centre; the area of the space CFDO would first be found by adding to the sector CFD*o*, or taking from it, as the case may be, the sum of the triangles CO*o*, DO*o*, and then taking the difference between AEBO and CFDO; which is plain.



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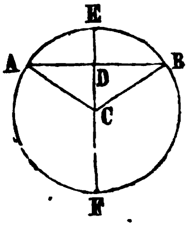
[1] See on the *Tableau* the concentric ring, base of the hollowed cylinder. See also the lateral faces of the sections of the hollowed sphere.

PROBLEM XIV.

To find the area of a segment of circle.[1]

(**40**) **RULE I.** 1° *Find* (**433 G.**) *by problem XII the area of the sector of the same arc.* 2° *Find afterwards the area of the triangle formed by the chord of the segment and the radii of the sector.* 3° *The sum of these areas will be* (**434 G.**) *that of the segment, if the segment be greater than a semi-circle, and if the segment ie less than half a circle, its area will be equal to the difference of these areas.*

Ex. 1. Find the area of the segment AEB whose chord A B is 12 and the radius AC=10.



A D	10	ar. comp. log. 9.000000
: AD=½AB	6	0.778151
:: Sin. D	90°	10.000000
: Sin. ACD	36° 52′=36.87°	9.778151
	× 2	

=73.74° = the degrees in the arc AEB.

Then 73.74 × (**34 REM. 1 T.**) 0.0087266 × 20 = 12.87 = length (nearly) of the arc AEB and AEB × ½AC=12.87 × 5=64.35=area of the sector AEBC.

Now $CD=\sqrt{AC^2—AD_2}=\sqrt{100-36}=\sqrt{64}=8$ et 6 × 8=48 area of the triangle ACB. Thence, sect. AEBC—ABC=64.35—48=16.35 =seg. AEB.

2. Required the area of the segment whose height is 18 and diameter of the circle 50 ? **Ans.** 636. 3138.

3. The chord of a segment=16, the diameter=20 ; what is its surface ? **Ans.** 44.764.

4. The arc of a segment contains 90° with a radius=9 ; what is its area ? **Ans.** 23.1174.

5. Determine the area of a segment of which the chord of the arc is 24 and the chord of half the arc = 13 ? See (**536** or **539 G.**) **Ans.** 82.53332.

(**41**) **RULE II.** 1° *Divide the height or versed-sine by the diameter and find the quotient in the table of versed-sines at the end of this volume.* 2° *Multiply then the number at the right of versed-sine by the square of the diameter, and the result will be the required area.*

See among the models of the *Tableau* the bases and parallel sections of certain ungulæ of cylinders, cones, spindles, &c.

(**42.**) The table in question contains the areas of the segments of a circle whose diameter is 1 and which is supposed to be divided into 1000 equal parts. There will be found the area of a segment whose height is the one thousandth of the diameter, that of a segment whose height is 2 thousandths of the diameter, that of a segment whose height or versed-sine is $\frac{3}{1000}$ of the diameter and so on up to the segment whose height is $\frac{500}{1000}$ of the diameter, that is up to the entire half-circle.

(**43.**) It is plain that this rule is similar to rule II of problem VII and that it does not require a special demonstration, for it is sufficient to remember, to show its exactness, that in two different circles similar segments are (**211 G.**) those that correspond to equal angles at the centre and whose chords (double sines (**1216 G.**) of the halves of those angles) and the versed-sines have consequently to each other the ratio of the diameters of those circles and that (**557 G.**) such similar figures are to each other as the squares of these diameters.

(**44.**) It is hardly necessary to add that if we had to do with a segment greater than a semi-circle it would suffice to operate on the other segment, and then subtract it from the whole circle, and if the quotient of the given versed-sine by the diameter is not found in the table, it will be easy to determine by a simple proportion the difference of area corresponding to the fractional part of such sine.

Ex. 1. The versed-sine of a segment of a circle being 10 and the diameter 50 : find the area of the segment ?

Ans. $10 \div 50 = \frac{10}{50} = \frac{1}{5} = .2 =$ versed-sine of the table : the area which corresponds to this versed-sine is .111823 which multiplied by 2500 the square of the diameter gives for the area of the proposed segment 279.5575.

2. Required the area of the segment whose height is 6 and diameter of the circle 21 ?

Ans. $6 \div 21 = \frac{6}{21} = .285\frac{5}{7} =$ versed-sine of the table to
which corresponds area.................................. .184521
The area which corresponds to the next greater versed-sine is .185425

The difference between these areas is...................... .000904
This difference $\times \frac{5}{7}$ that is $\times 5$ et $\div 7$ gives for area cor. to $\frac{5}{7}$ .000646
To which I add the area which cor. to 285.................. 184521

To obtain the whole area of the segment $285\frac{5}{7}$ of the table.... .185167
Now, multiplying by the square of the diam. $21 \times 21 =$..... 441

We obtain for area of the proposed segment.............. 81.658647

3. Find the area of a segment whose height is 2 and diameter 52? **Ans**. 26.88.

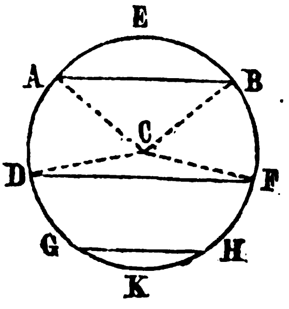
4. The versed-sine is 5 and the diameter 25: what is the area of the segment? **Ans**. 69.889375.

5. The height of a segment is 9 inches and the diameter $3\frac{1}{2}$ feet: find the area? **Ans**. 205.4118 square inches.

PROBLEM XV.

To find the area of a zone of a circle, or the space comprised between any two parallel chords and their intercepted arcs. [1]

(45). RULE I. *First find by the method of par.* **(574 G)** *&c., the diameter or radius of the circle and the other elements of the calculation to be made. Determine then* **(435 G.)** *separately by the problems already given the areas of the component sectors and triangles, and take their sum, if the zone be central; or if the zone be either central or lateral, determine by the last problem the areas of the two segments having for chords the chords of the zone; the difference between these segments, or between the whole circle and the sum of these segments, will be the required area.*



Ex. 1. The two parallel chords of a zone are 12 and 20 and their perpendicular distance is 13; what is the area? **Ans**. 252.87859.

2. Find the area of a zone of a circle whose parallel chords measure 12 and 16 and the distance between them 2? **Ans**. 28.379.

3. Determine the superficial content of a zone whose sides are 96 and 60 and the breadth 26? **Ans**. 2136.82.

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[1] See among the models of the *Tableau* the bases and parallel sections of certain ungulæ of a cylinder, cone, sphere, &c.

4. If two parallel chords of a circular zone are 20 and 15 and their perpendicular distance 17.5 ; What is the area ?

Ans. 395.4369.

5. Required the area of a zone each of whose parallel chords is 40 and the breadth 36 ?

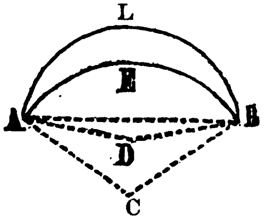
6. One of the parallel chords of a zone of a circle is 30 and passes through the centre of the circle, the other is 16 ; required the area ?

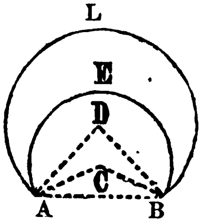
REM. The given segment may also be considered as composed of the trapezium ABFD and of the two equal segments AD, BF and its area determined in this manner.

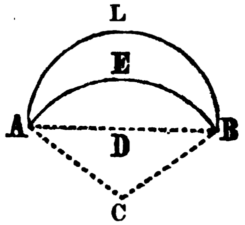
PROBLEM XVI.

To find the area of a lune, or the space comprised between the arcs of two eccentric circles which intersect each other. [1]

(**46**) **RULE.** *Find* (**436 G.**) *by the problem before the last, the areas of the two segments which form the lune : their difference will be the required area.*







Ex. **1**. The chord AB of a lune AEBLA is 20 and heights of the composing segments AEB, ALB, are 5 and 8 ; what is the area of the lune ? **Ans.** 49.392704.

2. The chord=20 and the heights of the segments 10 and 2 ; what is the area of the lune ? **Ans.** 130.204.

3. Determine the area of a lune whose length of the chord is 48, and the heights of the segments 18 and 7 ? **Ans**. 408.608.

4. The base AB of a lune is 10 and the radii AC, AD of the two containing arcs AEB, ALB, are 7 and 6 ; find the area.

5. The chord of a lune being 10 and the heights of the segments 15 and 13 ; what is the area ?

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[1] See among the models of the *Tableau* the opposite bases and the parallel section of the angula of the hollowed cylinder.

PROBLEM XVII.

To find [1] the circumference of an ellipsis.

(47) This figure shown by any section FI,AD (**997 G.**) or FE, RN (**1099 G.**) of a cylinder, or *be* (**1055 G.**), *ac* (**1056 G.**) of a cone by a plane which being inclined to the axis of those solids meets their two sides, is often met with by the measurer [2] It is found in the circus, amphitheatre, garden plot, &c., and on a smaller scale in the oval of a window &c., but it is especially the semi-ellipsis which is met with in the sections of vaults of all kinds, in the arched head of a door [illegible]indow, or of an arched opening between two apartments, [illegible]

(48) One might perhaps think at first that the circumference of the ellipsis should be an arithmetical mean between the circumferences of two circles having for their respective diameters the greater and less diameters of the ellipsis, or which is the same thing that this circumference should be equal to that of a circle whose radius were equal to the half-sum of the great and small radii of the ellipsis, that is whose radius would be of an arithmetical mean between the half-diameters of the ellipsis ; and it is very nearly so for ellipses whose diameters differ from each other but from 25 to 20 per cent, but to be convinced that it is not always so, it is sufficient to resort to an extreme case, (as we have already done at par. (**928 G**). For instance, let us suppose that whilst the small axis of the ellipsis is 1, the great axis be 1,000,000 ; it is evident that the circumference of such an ellipsis will be sensibly equal to the double of its great diameter, that is 2,000,000 while the half-sum 500000 + .5 or 500000 (for the 1.5 may be neglected) of the axes × 3.1416=1,570809 ; and if the small diameter were infinitely small with regard to the greater axis supposed to be equal to 2, the exact circumference would be 4 (double the great axis) while the arithmetical mean circumference would be

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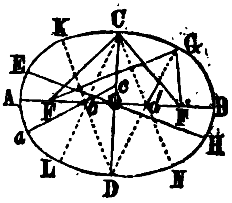
[1] Although it is not possible with the principles hitherto mentioned to give a demonstration of this rule and the four following, we have however thought proper to insert them here in order to complete the rules necessary to the measurement of plane surfaces, or of those (**1140 G.**) which having simple curvature may be developed into plane surfaces.

[2] See among the models of the *Tableau*, the bases and sections of the oblique cylinders, cones and conoïds. &c., and frusta of such bodies. These ellipses are of various degrees of eccentricity or have their diameters in varied ratios.

3.14159, &c., the error being in that case 4—3.1416=8584 or nearly one fourth. But if the circumference of an ellipsis cannot be correctly obtained in this manner, it is demonstrable that it can be arrived at by the following method :

(**49**) **RULE 1.** *Multiply the square root of the half-sum of the squares of the two diameters of the ellipsis by* 3.1416, *and the product will be the required circumference.*

Ex. 1. The greater diameter AB of an ellipsis is 15 and the smaller diameter 12 ; what is the circumference :



ANS. $\left(\frac{AB^2 + CD^2}{2}\right)^{\frac{1}{2}}$ (**39 G.**) $=\sqrt{184.5}=13.583$ et $3.416 \times 13.583=42.6723528$.

2. The greater and smaller axes being respectively 25 and 20 ; determine the periphery of the ellipsis ? **Ans.** 69.3979.

3. The semi-diameters of an ellipsis are 12½ and 7½ ; what is the perimeter ? **Ans.** 64.7667.

50. It is plain that the *semi-ellipsis* CBD is equal in perimeter and area to the semi-ellipsis ACB, and that each of them has for measure the semi-circumference and the half-area of the whole ellipsis. This rule and the following which show how to find the circumference and area of the whole ellipsis give then also the means *of arriving at the peremeter* ACB or CBD *or area of the semi-ellipsis* of the same name.

It is moreover evident that any other diameter E H divides the ellipsis into two parts of the same area and perimeter.

(**51**). *There is an important property of the ellipsis which permits one to trace it with facility or to discover whether a curvilineal figure resembling an ellipsis is really one or not ;* it is that the sum FC + F′C, FG + F′G, of the radii drawn from two points, F′F′ on the greater diameter and which are called foci or centres of the ellipsis, to any third point C or G, &c., on its circumference, is constant and equal to the greater diameter AB ; then it is clear that this very property permits us to establish the foci. Indeed, the two diameters of any ellipsis being given, from the point C or D end of the smaller axis, as centre and with radius CF=CF′=OA or OB=½AB, AB will be intersected in the required foci F and F′, from the points F and F′ as centres, with radii FG, F′G of which the sum=AB, that is, with any radius FG less than FB and another radius F′G equal to the difference between the first radius FG and the diameter AB, arcs may be traced the intersection of which in G will give a point, and by repeating the operation a series of points through which may be drawn a curve which will be the required ellipse.

(**52**) Or, there may be fixed at F and F′ needles to which will be fastened the ends of a thread of a length such as to give $FC + F'C$ or $FG + F'G = AB$; it will then be sufficient to hold the thread tight by means of a pencil or point which can be moved round the two foci to complete the outline of the ellipsis.

(**53**) *To perform the same operation on a large scale;* after having taken FG or F′G at pleasure, less than AF′ or BF′ but greater than AF or BF′, knowing the other radius $= AB - FG$ or $AB - F'G$, as the case may be, and FF′ being also known $= 2OF = 2\sqrt{GF^2 - CO^2} = 2\sqrt{OA^2 - OC^2}$ one will have but to compute either FF′G or F′FG of the two angles at the base of the triangle GFF′ and draw either of the two radii of the required length and with the required angle to give a point G of the proposed circumference; this operation repeated will give a series of points through which may be traced a line which will be the required circumference. Let us also observe that the measuring of the radius G F or GF′ may be avoided, by computing each of the angles at F and F′ and afterwards making an intersection G of the directrices FG, F′G.

(**54**) Let us add that a geometrical or graphic construction on a small scale would have the advantage of giving in a more expeditious manner and often accurate enough all the angles GFF′, GF′F, &c. necessary to determine the intersections or points G of the required perimeter.

(**55**) *The ellipsis is also traced as follows:* Let $ac = AO$ or BO the semi-greater axis, $ab = CO$ or DO half-smaller axis. In moving the right line *ac* so as to keep the point *e* on the diameter DC and the point *b* on the diameter AB, the point *c* will describe the required ellipsis. In practice the right line *ac* is any rod with projecting points at *a*, *b* and *c*, and along the diameters A B, C D are disposed rods, grooves or slides to guide the points *b* and *c*.

(**56**) **RULE II.** *When the diameters are not very unequal, the circumference of the ellipsis is prettly correctly obtained by multiplying the half-sum of these diamaters by* 3.1416.

Thus the three last examples computed in this manner will respectively give for answers 42.41 instead of 42.67, 69.11 instead of 69.40. and 62.83 instead of 64.76; so that when the difference between the diameters does not exceed $\frac{1}{5}$ or $\frac{1}{4}$ or when the ratios between the diameters are 5 : 6 or 4 : 5, the error in the result does not exceed $\frac{1}{150}$ or $\frac{1}{160}$, and when the difference between the diameters is $\frac{2}{3}$ or when these diameters are to each other as 15 : 25 the error becomes nearly $\frac{1}{30}$ of the whole result. When the diameters are to each other as 1 : 2, the circumferences obtained by the two rules are to each other as 47.12 : 49.66, the error being in that case $\frac{1}{20}$ nearly. The diameters being as 1 : 3 the circumferences are nearly :: 63 : 70, the error being in that case $\frac{1}{10}$ nearly. When the diameters are :: 1 : 5, the circumferences are :: 94 : 113 and the error $\frac{1}{5}$ nearly. Finally if the diameters to each other :: 1 : 10 the perimeters would be :: 173 : 223, and the error $\frac{5}{22}$ or $\frac{1}{4}$ nearly. Which will enable one to choose either of the rules according to the degree of accuracy required in the result.

REM. Besides it is plain that we might also, after having found the circumference, according to this second rule, correct it by the addition of the error or deficiency proportioned to the ratio between the diameters, and as established above.

PROBLEM XVIII.

To determine the area of an ellipsis. [1]

(57) RULE. *Multiply the product of the two diameters by .7854 ; the result will be the required area.*

Ex. 1. What is the area of an ellipsis whose diameters are 24 and 28 ? **Ans.** 24 × 28=432=AB × CD, and 432 × .7854= 339.2928 = area ABCD.

2. If the axes of an ellipsis are 35 and 25, what is its area ?

Ans. 687.225.

3. Required the area of an oval whose length is 70 and breadth 50 ? **Ans.** 2748.9.

4. The greater axis of an ellipsis measures 840 links, the smaller axis 612 links ; required the number of acres within this enclosure ?

Ans. 4 acres 6 perches.

(58) REM. Since the rule gives for area of the ellipsis the expression AB.CD × .7854 or which is (**87G**) the same thing $\left(\overline{AB.CD}\right)^2$

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[1] The component faces of several of the models of the *Tableau* present ellipses of various degrees of eccentricity, or whose diameters to each other in various ratios.

× .7854, it evidently follows *that the ellipsis is equal in area to a circle whose diameter would be a mean proportional between the two diameters of the ellipsis.* Let d this mean diameter, we have $AB : d :: d : CD$ and since (**104 G**) $AB^2 : d^2 :: d^2 : CD^2$ it is plain also that *the area of the ellipsis is a mean proportional between those of the inscribed and circumscribed circles, that is, between those of two circles having for respective diameters the two diameters of the ellipsis.*

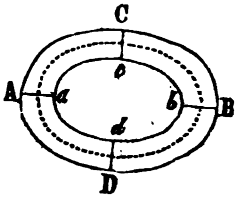
(**59**) **REM.** The two rules which show how to determine the circumference and area of an ellipsis may be with advantage substituted to the less precise and longer method of par. (**437 G.**) in the computation of the perimeters and areas of the curvilineal, that is (**47 T.**) elliptical bases of the oblique cylinder and of the frustum of a cylinder (**997** and **1099 G.**) as well those of the oblique cone and frustum of a cone (**1055, 1065, 1067, 1140,** &c. **G.**)

PROBLEM XIX.

To find the area of an elliptic ring.

(**60**) **RULE I.** *Determine separately the areas of the two concentric ellipses, and take their difference which will be the required area.*

RULE II. *Multiply the half-sum of the parallel circumferences of the two limiting ellipses by the breadth of the ring.*



Ex. 1. What is the area of an elliptic ring whose interior diameters are 10 and 20 and the exterior diameters 12 and 22 ?

Ans. 10 × 20 × .7854 = 157.08, 12 × 22 × .7854 = 207.3456; the difference 50.2656 of these two results is the required area of the ring.

2. The exterior circumference of an ellipsis is 100, the interior circumference 90, the breadth of the intermediate space being 3.5; required the area of the ring ? **Ans.** 332.5.

3. Determine the area of an elliptic half-ring, whose parallel perimeters measure 93 and 77 inches and breadth 10 inches ?

Ans. 850 square inches or 5.9028 sq. feet.

4. Compute the area of any part A*a* *c*C of an elliptic ring, whose exterior arc AC is 15, parallel arc *a c* 12, and breadth 3 ? **Ans.** 40.5.

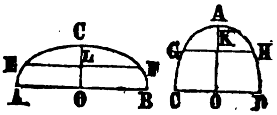
REM. It is hardly necessary to remark that if the breadth of the annular space were not everywhere equal, or even if the interior ellipsis had any other position relatively to its exterior envelope, or any ratio whatever between its diameters, the required area would none the less be obtained by the first of the two rules of this problem.

PROBLEME XX.

To find the area of a segment of an ellipsis whose base is parallel to either of the axes of the ellipsis. [1]

(61) *Divide the height of the segment by that of the two diameters of which this height forms a part, and find in the table annexed to this treatise the segment of a circle whose versed-sine is equal to the quotient. Next find the continued product of the segment thus found and of the two axes of the ellipsis; this product will be the required area.*

Ex. 1. Compute the area of the elliptic segment AGH whose height AK= 10, and the two axes AB, CD, 34 and 25 ? **Ans.** 162.02.



2. What is the area of the segment of an ellipsis, whose base GH is at 36 from the centre O, the axes being 120 and 40 ? **Ans.** 536.75.

3. Determine the area of an elliptic segment whose height CL is 8 inches; the two axes being 4 and 3 feet ? **Ans.**

(62) REM. If the segments of ellipses ACD, *acd*, *ace*, of the figure of par. (**1140 G.**) answer to the definition of the ennunciation of this prob. we may if need be apply the rule here given to express their areas. The area of the elliptic segment which forms the upper surface of the ungula fig. 2 of par. (**1143 G.**) could be computed in the same way if required. And if the segment to be computed were the *zone* or part AEFB, CGHD, the required area would be equal to the difference between the semi-ellipses ACB, CAD and their respective segments ECF, GAH.

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[1] Several of the ungulas of a cylinder, cone and spheroid of the *Tableau* present in their sections segments of ellipses, some greater, others smaller than the semi-ellipsis, others, semi-ellipses, and others zones of ellipses.

PROBLEM XXI.

To find the area of a parabola. [1]

(**63**). This figure is that of the section of a cone by a plane parallel to its inclined side. (ADC, fig. of par (**1140 G.**) gives an idea of it). It has this peculiarity that any point E, H, &c., of the curve is equidistant from a point F called the focus and from a straight line MN perpendicular to the axis CD) called the directrix and whose distance SC from the apex C of the parabola is equal to the distance FC from the focus to the apex ; so that one has always EF=EM, HF=HN, &c. Now it is proved that the position F of the focus is found by bisecting B D in T, joining CT and drawing TR perpendicular to CT to obtain DR=CF=CS. The focus F found and the position of the directrix MN determined, the curve is traced by drawing a series of indefinite straight lines GH (called ordinates) parallel to A B or perpendicular to the axis CD ; then, from the focus F as a centre and with a radius US equal to the distance between the parallels GH, MN intersect GH in G and H, which determines two points in the perimeter of the parabola. This operation sufficiently repeated will give a series of points, through which may be traced a curve which will be the figure required.

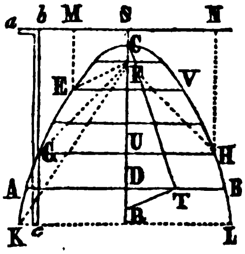
(**64**) The parabola is also traced with a square *abc*, whose branch *bc* is equal to the distance between the directrix MN & the base KL of the proposed parabola. At the extremity *c* of the square and to the focus F, tied a thread *c* GF equal in length to *cb*. The branch *ab* of the square is then made to slide along the directrix MN holding at the same time the thread tight along the branch *bc*, by means of a point or pencil whose motion describes the required parabola.

(**65**). **RULE**. *Multiply the base by the height and take two thirds of the product for the required area.*

Ex. 1. Find the area of the parabola ACB whose base AB is 20 and height CD 18 ? **Ans.** 240.

2. The base of a parabola is 13.5, and the height 11.25 ; what is the area ? **Ans.** 101.25.

3. CD=10,AD=8 ; what is its area ? **Ans.** $106\frac{2}{3}$.



This figure, like all the other figures, treated of in the " mensuration of areas," is to be found amongst the component faces or sides of the models of the *Tableau*. See the conical and conoidal ungulas of the *Tableau*.

(66). REM. It follows from the definition of the parabola that any part GCH, ECV of the parabola ACB terminated by a base GH, EV, parallel to A B, is still a parabola, and not a mere segment, as in the case of the ellipsis; for the cone may be considered cut by a plane parallel to its base and this as well on the one as on the other side of that base at KL without ceasing to be a cone and consequently, without the definition of the section KCL or ECV, &c, being in any way thereby altered.

Whence it results that to arrive at the area of a zone, or segment AEVB of a parabola by any line EV parallel to its base, one will have but to take the difference between the entire and partial parabolas ACB, ECV.

(67). There is still the **hyperbola** [1] (section of a cone by a

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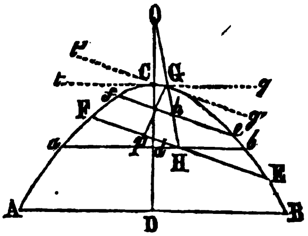
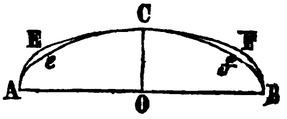
[1] The *hyperbola* ACB is shown by sections of certain ungulas of cones and conoids which will be found among the models of the *Tableau*.



This curve is, but in a contrary sense, analogous to the ellipsis. Thus, whilst in the ellipsis (**51 T.**), it is the sum of the radii drawn from the two foci which is constant or invariable: in the hyperbola, on the contrary, it is the difference of these same radii which remains constant; which causes the two halves, parts or branches of the curve (conjugated hyperbolas as they are called) to present to one another, not their concave sides as in the ellipsis, but their convex ends, apices or sides. To trace this curve without any condition of dimensions, that is, without any condition as to the dimensions of the cone or the position of the plane of section: having taken at will any two points, F, F, at any distance from each other, from one of these points or foci with any radius, described an arc on each side of the axis (that is of the line which connects the foci), and describe, from the other focus as a centre and with a radius greater than the first by a given difference, two other arcs which at the point of their intersection with the two first arcs will determine 2 points of the required curve. This operation repeated with two new radii, taking care however that the second radius be always greater than the first by the given difference (which, as has been seen, must remain constant) will give two other points in the curve to be described; and other points in the curve may also be determined till their sequel and direction render plain the course of the hyperbola. If now, the radii be transposed, it is plain that we will have a new series of points. that of the conjugated hyperbola. The point O which is half-way between the two foci is called the centre of the hyperbola as of the ellipsis.

The hyperbola may also as the parabola and ellipsis, be traced by a mechanical operation. Taking a ruler fastened at one end to one of the foci of the curve to be described, and so that the ruler may be movable round the said focus, the other end ot the ruler will be fastened to the second focus by a string or line which must be shorter than the ruler, by the required difference between the radii; then a plane which meets its base at an angle greater than that made by the side of the cone with that base) the **cycloid** (which is described by a point placed on the circumference of a circle maintained in the same plane, during an entire revolution of the said circle along a straight line called the base of the curve, which is very much like a semi-ellipsis, fig. of paragraph (**68**) **and** several **other curvilineal figures**, whose areas and perimeters one may have to compute, and for which there are special rules which allow of establishing with all the required precision their relative or absolute areas and circumferences;but it is to be remarked here as has already been done (**1134 G.**) that generally it will be necessary to enquire first as to the nature of the proposed figure ; and the mere labour entailed by this preliminary operation will often be sufficient to cause one to decide on resorting immediately to the method of the following problem.

(**68**). A practized eye will often find it difficult to understand the nature of the figure to be computed, and may sometimes make pretty grave mistakes thereby. There is for instance the curve AECFB, called " flat arch " (*anse-de-panier*) and others of the same kind often met with in the arched heads of openings, and which one may be sometimes disposed to consider as an ellipsis, so as to compute its superficial content by the rule applicable to that figure ; now, it is seen thatin this case the difference AECe + BFCf (or 2 AECe) between the two figures, may be too great to allow it to be neglected.



PROBLEM XXII.

To determine the area of any curvilineal figure.

(**69**). **RULE** *Divide the whole figure, if it be irregular,* (*that is, if the corresponding parts are not symmetrical*) *the half or fourth part, if regular, into trapeziums of the same breadth or height, and proceed then in the manner of problem VI., doubling or quadrupling if need be the area thus found to obtain the whole area of the figure.*

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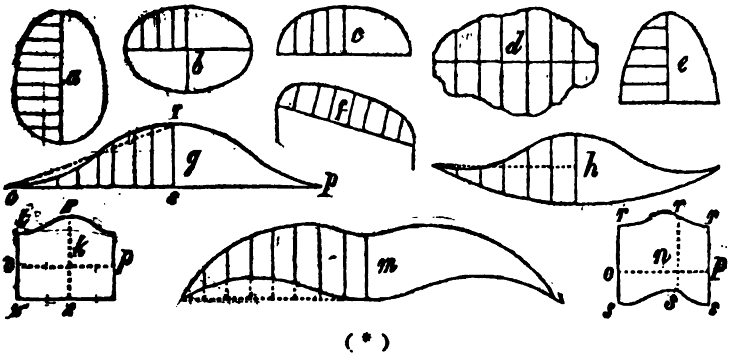
pencil or point which will hold the string tight and at the same time in contact with the ruler, will describe, the ruler going round the first focus, the required hyperbola and the transposition of the ruler and string will allow of describing at will the other branch of the curve.



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(70) The method of computation by trapeziums, will be the more accurate as there will be in the figure to be computed concavities and convexities *adb*, *bec*, compensatory of each other such as are

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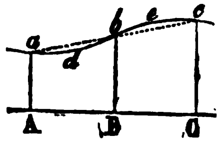
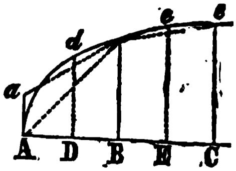
*. Among these figures, *a* is the ovum or oval; (such is the vertical section of the egg, &c.,) *b* is the ellipsis, (such is the section of the melon, &c.,) or any other analogous figure, the bulls-eye window, the section of the spheroid, the amphitheater, &c.; *c* is the half-ellipsis, or flat-arch, cycloid, the flat arched head of an opening, the section of a vault, &c.; *d*, any irregular curvilineal figure; *e* a parabola or any other similar figure, hyperbola, the raised arched head of an opening; the section of a vault, the vertical section of a conoid, dome, &c.; *f* is the raking arch or the section of an inclined vault, *g* is the developed concave or convex surface of an ungula of a right cylinder; *h*, the developed lateral surface of an ungula of a right cone; *m* the development of the surface of the ungula of an oblique cylinder or cone. The lunettes or intersections of vaults, already mentioned at article **(1143 G.)** present also surfaces the development of which offers to the consideration of the measurer the three last figures which have just been defined. The developed lateral surface of the frustum of a right cylinder presents the form *k*, and it follows from par. **(997 G.)**. and from the dem. of par. **(1099 G.)** that it suffices to multiply the half-sum of its less and greater height *tv*, *rs*, by the length *op* perpendicular to *rs* or *vt*, this breadth being evidently equal to the developed circumference of the section of a cylinder by a plane perpendicular to its axis or side. The development of the lateral surface of an oblique cylinder **(997)** presents the figure *n*, the height of which *rs* which is that of the inclined side of the cylinder, is everywhere uniform, the area of the envelope being consequently equal to the product of *rs* by the breadth *op*, the perimeter of a section perpendicular to the axis or side of the solid.

It is useful to state also that if the ungula of the right cylinder of which the figure *g* is the envelope, instead of being partial as KLNE or KLRF page 409, G. is entire or complete as AD*d*, page 388 G, the area of *g* will be obtained by finding the product of *op* by the half of *rs*, for in this case *g* will be but the envelope *k* of the frustum of a cylinder whose less height *vt* would be equal to zero. By adjusting to the lateral face of an ungula or frustum of a cylinder or cone, etc. of the *Tableau*, a sheet of paper, to trace or cut it out afterwards at will, the pupil will get an excellent idea of the nature of the developed surfaces here mentioned.

seen in the figures *g*, *h*, *m*, *k*, since the segment *bec* which is neglected by considering as a trapezium the part BC*ceb* of the area to be computed, will be compensated by the segment *adb* which is in excess in the trapezium AB*ba*.



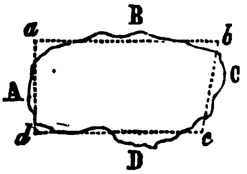
(**71**) But when the figure is entirely convex one will add to the precision by taking in the sum of the segments *abd bce*, &c., determining by mere inspection or otherwise the mean breadth which will be multiplied by the corresponding perimeter *adbec* to obtain its area.



(**72**) Let us observe also that instead of considering as null the initial height of the figure, at the point A of the springing of the curve, which would give for the area of the part AB*bda*A of the figure the triangle AB*b*, one will obtain more accuracy by considering as a straight line the almost vertical part A*a* of the curve, which will then give for a more approximate area of this component part of the figure, the trapezium A*ab*B instead of the triangle A*b*B.

It is also plain that a continued subdivision D*d*, E*e*, sufficient to allow of considering the parts *ad*, *bd*, *be*, &c., of the convex or concave circumference of the fig. as being sensibly straight lines will also have for result to add considerably to the accuracy of the operation.

(**73**) There is also a pretty correct and expeditious mode of arriving at the area of an irregular figure ABCD : that of reducing it to any equivalent rectilineal or regular figure by compensatory lines *ab*, *bc*, that is, such that the sum of the parts cut off by these lines be equal in area to the sum of the parts comprised in their inclosure, a graphic or mechanical operation for the accuracy of which one must often trust to an ocular estimation.



(**74**) Finally, as to the evaluation of the developed lengths of the perimeters of the figures in question here, let us remark again as was done at page 596 G, that often the most expeditious manner and not the least exact, of arriving at them, will consist in the use of a thread or ribbon, or wooden or metallic rods thin enough to allow of adjusting them to the perimeter to be computed, in order to immediately deduce from them the required dimensions.

MENSURATION

OF

BODIES OR SOLIDS.

(See the models of the *Stereometrical Tableau*)

(75) The mensuration of solids, comprises that of their surfaces[1] as well as that of their volumes or solidities.

It has already been seen (**5, T.**) that the unit of measure for plane surfaces is a square the side of which is the unit of length.

Any curve line is also referred to a unit of length, and its numerical value is the number of times the line contains this unit. It is also to be observed here that the rule already given (page 177, 2° **G.**) to find the numerical relation between two straight lines or to determine their common measure or the greatest common divisor, applies equally to any two curved lines of the same radius, since that equality of curvature will allow of the superposition and entire and perfect coincidence of these lines in the same manner as if they were straight. Now if it is supposed that the lineal unit be reduced to a straight line and that a square be constructed on this line, this square will also be the unit of measure for curved surfaces.

(76) The unit of volume is (**1014 G.**) a cube the component face of which is equal to the superficial unit which serves to compute the area of the solid, and the side equal to the lineal unit used to express its lineal dimensions.

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[1] The opposite bases, the lateral faces and the sections of the various models of the *stereometrical tableau* present, among others, all the plane figures already treated of in the "Mensuration of surfaces," comprising the *square, rectangle, parallellogram, triangle, polygon, circle, sector, segment, zone, lune, ellipsis, parabola, hyperbola, &c., &c.*

PROBLEM I.

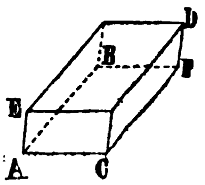
To find the area of a right prism [1] (946 G.)

(See the *stereometrical tableau.*)

(77) **RULE.** *Multiply* (**992 G.**) *the perimeter* [2] *ABFCA or ABCDEA* (*fig. of the following page*) *of the base by the height AE or AF as the case may be, and the product will be the lateral area. To this area add those of the two bases when the whole area is required.*

Ex. 1. What is the area of a cube whose side is 20 ?

Ans. 2400.



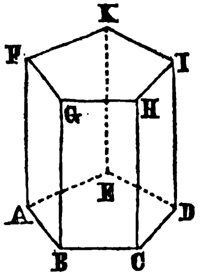
2. Determine the whole area of a triangular prism, the base of which is an equilateral triangle having 18 inches side and the height 20 feet ? **Ans.** 91.949 sq. feet.

3. Required the weight of copper necessary to cover the inside of a cistern the length of which measures 10 feet, breadth 5 feet and height or depth 4 feet, the copper to be used being 5 pounds to the square foot ? **Ans.** 850 pounds.

---

[1] The prism, which comprises also the cube and parallelopipedon, presents itself every day to the measurer's consideration. It is seen in the body and wings of buildings of all kinds as well as in the figure of the various apartments forming part of them. It is found again in the walls, pillars and piers of all kinds of buildings and on a smaller scale in each of the composing stones or bricks of these bodies. Gabled roofs most often present the figure of the right trianglar prism and the gables of the walls forming their parallel bases are also prisms of the same name. The body or square of a garret-window is generally nothing but a triangular prism or right half-parallelopipedon and the roof of a garret window, if it be hip'd, is an oblique triangular prism, provided the inclination of the hip be equal to that of the roof, and if the plane of the hip be not parallel to that of the roof, it is then a frustum of a prism the solid and superficial content of which is to be valued. There are also in the arts and trades a thousand and one objects affecting the form of a cube, right, oblique or truncated parallellopipedon, right. oblique or truncated polygonal prism or which may be decomposed into solids of this kind. The cuttings and embankments of railroads and other roads, often enough present to the consideration of the measurer quadrangular prims having for parallel bases trapeziums.

[2] Each of the edges or sides (AB,CF, 1st fig. or AF, BG, CH, &c., 2nd fig.) of the prism being of the same length, it is evidently the same thing to multiply successively each side or edge (base of the parallelogram which goes to make up the lateral surface of the prism) by the length of the corresponding parallelogram or to add according to the rule all these breadths so as to multiply them at once by the length of the side of the prism.



4. How many square metres are there in the lateral area of a building the length of which is 100 metres, breadth 23.3 metres and height 17 metres ? **Ans.** 4192.3

5. A room measures 40 feet by 25, and its height is 15 feet ; how many square yards of plastering will be required to cover the four walls and ceiling ? **Ans.** 3279$\frac{7}{9}$.

6. What would be the cost to line with lead of 7 pounds to the foot and at 4 pence a pound, the inside of a rectangular vessel the length of which is 3 feet 2 inches, breadth 2 feet 8 inches, and height 2½ feet.

Ans. Area to be covered $=37.\frac{7.333\times}{12}$ square feet, $=263\frac{5}{18}$ pounds $=£4.7.9\frac{1}{9}=\$17.55.185$.

7. What is the lateral area of a deal of 10 feet, by 12 inches, and 3 inches ? **Ans.** 25 sq. feet.

8. How many superficial feet of cut stone in the lateral area of an octagonal pillar whose side is 15 inches and height 10 feet ?

Ans. 100.

9. How many squares of wainscot to cover the lateral area of a hexagonal building whose oblique radius is 20 feet and height 33 feet?

Ans. 39.60.

10. What is the lateral area of a polygonal post 3 feet perimeter and 10 feet high ? **Ans.** 30 square feet.

11. The perimeter of an iron bar is 3¾ inches, its length 7 feet ; what is its lateral area ?

Ans. $3.75\times 74=315$ square inches.

PROBLEM II.

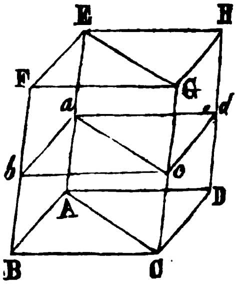
Find the volume of a right prism.

(See the *tableau.*)

GENERAL FORMULA.

78. *To the sum of the areas of the parallel ends or bases (AF, ED or AD, FI) of the solid (see the figures of the last problem and the following) add four times the area of the section half way between them ; multiply the whole by the sixth part of the height (AF or AE) or length of the* *body (or, which is the same, multiply this sum by the whole height and then take the sixth part of the product) the result will be the required volume.*

(79.) REM. The prism (**940 G.**) being a body whose size (breadth, thickness or diameter) is everywhere the same, it is plain that any section of this solid by a plane *abcd* parallel to the base BD or FH, is equal to the base, which therefore in the case of the prism reduces the rule or general formula at the top of the *stereometrical tableau* to the more simple following expression : (see : Introduction, page 8, last paragraph.)



RULE. *Determine first the area of the base ; multiply this area by the height, the product will be* (**1020**) *the volume of the prism.*

Ex. 1. What is the solid content of a cube whose side is 24 inches ? **Ans.** 13.824.

2. How many cubic feet in a block of marble whose length is 3 feet 2 inches, breadth 2 feet eight inches and height or thichness $2\frac{1}{2}$ feet ? **Ans.** $21\frac{1}{9}$.

3. How many gallons of water contained in a cistern having the dimensions of the preceding example, the gallon being 282 cubic inches ? [1] **Ans.** $129\frac{17}{47}$.

4. What is the volume of a triangular prism whose height is 10 feet, and three sides of its triangular basis 3, 4 and 5 feet ?

Ans. 60.

5. Required the number of cubic feet of stone in a pillar 15 feet high and whose base is a regular hexagon the side of which is 1 foot 4 inches ? **Ans.** 69.282.

6. Determine the number of toises of masonry (the toise being 6 x 6 x 2=72 french cubic feet) in an octagonal prism of 12 feet high and 3 feet side ? **Ans.** 7 toises 17.47 cubic feet.

7. The pier separating two splayed windows, and whose base is consequently a trapezium, measures 13 feet high, 2 feet thick, 9 feet broad outside and 7 broad inside ; required the number of bricks that have been required to build it, at 20 bricks to each cubic foot ?

Ans. 4.160.

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[1] N.B. The english imperial gallon is 277.274 english cubic inches, the old beer gallon=282 cubic inches and the wine gallon actually used in Canada is 231 english cubic inches.

8. A stone gable 3 feet thick, measures 40 feet at its base and 20 feet high; how many cubic yards of masonry does it contain?

Ans. $44\frac{12}{27}$.

9. The facade of a building is 33 metres, its height 17 metres and the thickness of the wall 73 centimetres; what is the volume in cubic metres? **Ans.** $33 \times 17 \times .73 = 409.53$.

10. Required the number of cubic metres in an embankment whose length is 100 metres and each of the parallel planes constituting its ends is a trapezium having for parallel bases 3 metres and 13 metres for its height 3.3 metres. **Ans.** 2640.

11. A well must be 27 feet deep, and its plane must be a regular hexagon whose radius of the circumscribed circle be 5 feet; how many cubic yards of rock are to be mined to give it its required dimensions? **Ans.** The side of the hexagon is (**643 G.**) 5 feet; $5^2 = 5 \times 5 = 25$, and 25×2.5980762 (area (**27 T.**), of the hexagon whose side is 1) $=64,9519$ square feet = area of the given hexagon, and, $\frac{64.9519 \times 27}{27} = 64.9519$ cubic yards.

Ex. 12. What is the solidity of a rectangular iron bar $4\frac{1}{2} \times 1$ inches and 14 feet long? **Ans.** 756 cubic inches.

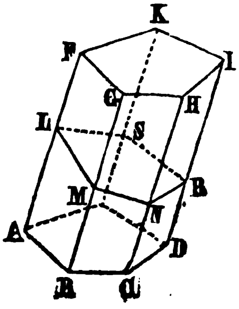
13. Required the volume of an eight sided post whose height is 10 feet and breadth of each side 7 inches?

Ans. $\overline{7 \times 7} \times 4.8284271 =$ (**27 T.**) area of the base $=236.5929279$, and $\times 120 = 28391.15$ cubic inches, and $\div 1728$(or $12 \times 12 \times 12$) $= 16.43$ cubic feet.

PROBLEM III.

To find the area of an oblique prism.

(See the *tableau*)



(**80**) **RULE.** *Multiply* (**996 G.**) *the length AF, BG, CH &c., of the side by the perimeter of a section LMNRSL perpendicular to the side.*

Ex. 1. What is the area of the face and two sides of an inclined beam or rafter with parallel bases the length of which is 12 feet, the breadth of the face 9 inches, and that of the sides $13\frac{1}{2}$ inches?

Ans. 36 square feet.

2. The length of a cornice under a

flight of stairs between parallel walls is 20 feet and the circumference or perimeter of a section of the cornice perpendicular to its direction is 27 inches; what is its developed area?

Ans. 45 square feet.

PROBLEM IV.

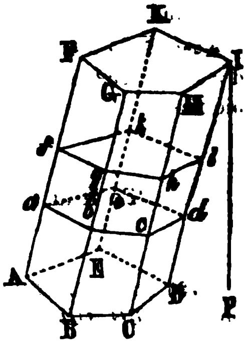
To find the volume of an oblique prism.

See the *tableau.*

REM. The oblique prism, being, as the right prism, of invariable diameter throughout its whole length, every section of this solid by a plane parallel to the base would form a surface equal to that of the base; whence it is plain that for the oblique prism, as for the right prism, the general formula is reduced to the following simplified expression.

(81) **RULE**. *Multiply* (**1020 G**) *the area of the base ABCDE or FGHIK by the height IPperpendicular to this base; the product will be the required volume. Or, which is the same thing,*

Multiply (**1025 G**.) *the side AF, BG, CH, &c., of the solid by the area of a section abcdea perpendicular to this side.*



Ex. 1. How many cubic feet of oak will be required for the carriage of a flight of stairs 17 feet long and 15 × 4 inches square.

Ans. 6¼

2. The horizontal base of the breast of a chimney built obliquely, measures 7 feet by 18 inches, the perpendicular height being 7 feet 3 inches; how many bricks does the parallelopipedon contain at 18 bricks to the cubic foot?

Ans. 76⅛ cubic feet × 18 = 1370¼ bricks.

3. The triangular side of a garret or dormer window has for its horizontal length 7 feet, for vertical height 5 feet, the breadth of the dormer being 4 feet; the roof of the dormer window is hip'd, parallel to the roof of the building; the height of the triangle which constitutes its vertical section is 2 feet; what is the total volume?

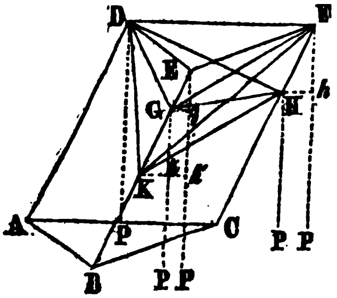
Ans. The body or square of the dormer (right triangular prism) (1)$=\frac{1}{2}(7\times 5\times 4)=70$ cubic feet, the roof (oblique triangular prism)$=\frac{1}{2}(7\times 4\times 2)=23$ cubic feet; the required volume is consequently 98 cubic feet.

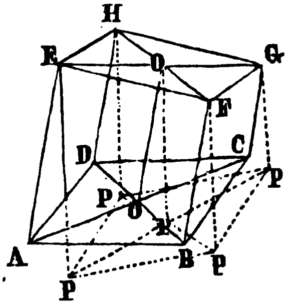
PROBLEM V.

To determine the area of the frustum of a prism.

(See the *tableau.*)

(**82**) **RULE**. *Find separately* (**1039 G.**) *the area of each of its component faces; their sum will be the area required.*





Ex. What is the number of superficial feet of cut stone in the circumference of the top of a chimney situated obliquely on an inclined roof, that is the component faces of which are not parallel to those of the building; the plane of the chimney being a rectangle 3 feet by 4 feet and the respective heights of its four sides or edges 7, 8, 9½ and 8½ feet?

Ans. $\frac{1}{2}(7+8)\times 3=22+\frac{1}{2}(8+9\frac{1}{2})\times 4=35+\frac{1}{2}(9\frac{1}{2}+8\frac{1}{2})\times 3=27+\frac{1}{2}(8\frac{1}{2}+7)\times 4=31=115\frac{1}{2}$.

PROBLEM VI.

To find the solidity of the frustum of a triangular prism.

(See the *tableau.*)

GENERAL FORMULA.

(**83**) *To the sum of the areas of either of its three pairs of parallel bases or faces AOFD—BE, ABED—OF, BOFE—AD) add four times the area of a parallel section or plane half-way between them.*

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[1] Here the prism in question does not rest on one of its parallel bases; but this circumstance cannot prevent one from deciding immediately on the nature of the solid to be measured; for, it is evidently indifferent, respecting the volume required, whether the position of the polyhedron be vertical, horizontal or inclined.

This sum multiplied by the sixth part of the corresponding height of the solid, will be the required volume.

(80) In fact, though the frustum of a triangular prism, when considered with respect to its non parallel bases or ends ABC, GHK, or ABC, DEF, cannot be measured at once by the general formula and must be decomposed by a plane of section parallel to one of these bases and passing through the nearest point of the other base, that is, through the end of its edge or of its shortest side, into a prism and a pyramid, which may be measured separately by the general formula, to take afterwards the sum of the results; however, if attention be paid to the nature of the solid, to wit, that the faces and edges or opposite sides are parallel, and that the sides or edges being but simple lines, the area of each of them is equal to zero (0) it will immediately be seen how to apply the rule to measure at once the proposed prism.

(85) Example. Let ABC—DEF (fig. of the following page) a frustum of a prism where AD=8, FC=7, BE=9, CK=4, height,=5, (the base GHK being considered perpendicular to the sides or edges AD, FC, BE). We will have by the formula : volume=area AGFD + 4 times the area *acfd* + the area BE, which is null, the whole multiplied by $\frac{1}{6}$ of the height.

The upper base BE is but a line and is equal to.................. 0

The lower base$=\frac{AD+CF}{2}\times GK=\frac{8+7}{2}\times 4=7\frac{1}{2}\times 4=$.......... 30

The section *acfd* (Imagine such a section *acfd* parallel to the base ACFD and half-way between this base and the apex BE) gives $ad=\frac{AD+BE}{2}=\frac{8+9}{2}8\frac{1}{2}$, $cf=\frac{FC+BE}{2}=\frac{7+9}{2}=8$, and the breadth of the trapezium *acfd* is $gk=KG\div 2=2$, whence area $afcd=8\frac{1}{2}+8$, that is $16\frac{1}{2}\div 2$ or $8\frac{1}{4}\times 2=16\frac{1}{2}$ and four times this area$=16\frac{1}{2}\times 4=$.............................. 66

The sum of the areas is...................... 96

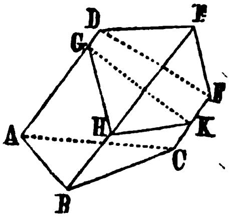
which multiplied by the 6th part of the height or by $\frac{5}{6}$, gives 80 units for the volume of the proposed frustum, which agrees with the result given here below of the calculation of the same frustum by another method and proves evidently the accuracy of the formula.

Here again, as for the prism, the general formula is reduced to the following more simple expression :

(86) RULE 1. *Multiply* **(1093 G.)** *the base of the frustum by one third of the sum of the heights of its three parallel sides or edges.*

RULE II. *Multiply the third of the sum of its three parallel sides by the area of a section perpendicular to these sides.*

REM. This second rule, it has been said (**1096 G.**) derives evidently from that of paragraph (**1025 G.**) but should this conclusion, perhaps too immediate for the pupil to perceive its truth, not be found rigorous and satisfactory enough, it is however easy to show its exactness, in different ways, the following of which though the most expeditious is not the least conclusive. Let then ABC—DEF the frustum of an oblique triangular prism, divided into two frusta of right prisms GHK—ABC, GHK—DEF by a plane GHK perpendicular to the parallel sides AD, BE, CF of the solid. The volume of each composing frustum is equal (**1693 G.**) to the product of the common base GHK by one third of the sum of the perpendiculars GD, HE, KF........GA, HB, KC; but $GHK \times \frac{1}{3}(GD + HE + KF) + GHK \times \frac{1}{3}(GA + HB + KC) = GHK \times \frac{1}{3}(GD + GA + HE + \overline{HB + KE + KC)}$ $= GHK \times \frac{1}{3}(AD + BE + CF)$; therefore, &c.



Ex. I. The base of the frustum of a right triangular prism is 10 square feet, the sides are 7, 8 and 9 feet; what is its solidity ?
Ans. 80 cubic feet.

2. The three sides of the frustum of an oblique prism are $7\frac{1}{2}$, $8\frac{2}{3}$ and $9\frac{1}{3}$ feet; the base and height of a section perpendicular to the side are respectively 5 and 3 feet; what is the solidity of the solid ?
Ans. $8\frac{1}{2} \times 7\frac{1}{2} = 63\frac{3}{4}$ cubic feet.

3. The three sides of the base of an inclined prism measure respectively 3, 4 and 5 metres and the heights of its three apices are 6, 7 and 8 metres; what is its solid content ? **Ans.** 42 cubic metres.

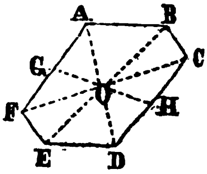
PROBLEM VII.

To find the solidity of the frustum of a prism whose base AD or section perpendicular to the side is a regular polygon or having symmetrical halves (1097 G)

(**87**) **REM.** Here again the general formula is reduced to and may be replaced by either of the following simplified expressions, or one may at will compute separately the volume of each of the component frusta of triangular prisms, as in the following problem, and then take their sum.

(**88**) **RULE I.** *Multiply* (**1097 G.**) *the base by half the sum of the heights of two opposite sides ; the product will be the volume required.*

RULE II. *Multiply the half sum of two of the opposite sides or edges of the frustum by the area of a section perpendicular to those parallel sides.*



REM. This second rule derives again from par. (**1093 G.**) since one may suppose the frustum of the polygonal prism divided into frusta of triangular prisms, and for each of these component frusta make the same proof as for the frustum of the triangular prism of the last problem.

Ex. I. How many cubic feet of stone in the top of a chimney having for horizontal section a regular hexagon whose side is 2 feet, the heights or lengths of two opposite edges of the frustum being 13 and 17 feet ?

Ans. $2.5980762 \times 2^2 \times \left(\frac{17+13}{2}\right) = 155.884572.$

2. Find the number of cubic inches of birch in a staircase baluster having for horizontal section a regular octagon 3 inches diameter and whose least and greatest length or height measure respectively 27 and 29 inches.

Ans. The required side of the octagon is pretty accurately obtained in this case by describing a circle 3 inches diameter thence to find the chord of one eighth of its circumference. This operation gives for the breadth of one of the sides of the baluster $1\frac{6}{40}$ inches nearly, say 115 ; or $(1.15)^2 = 1.3225$, and 1.3225 + (**28 T.**) 4.8284.271, or to abridge $4.83 \times 1.32 = 6.375$ square inches = area of the section of the baluster ; finally, $6.375 \times \frac{1}{2}(27+29) = 6.375 \times 28 = 178\frac{1}{2}$ cubic inches.

PROBLEM VIII.

To determine the solidity of any frustum of a prism.

(See the *tableau.*)

(**89**) **RULE.** *Frst find separately* (**1098 G.**) *by the preceding rules the volume of each of the frusta of the component triangular prisms, and then take their sum.*

Ex. An embankment of earth presents the form of the frustum of a right prism having for its base the polygon ABCDEA ; the area of the component base ABC is 50 square yards, that of the base ADC=73 yards and that of the base ADE=65 yards ; the heights or lengths of the parallel sides A, B, C, &c., are respectively 7, 8, 9, 13 and 11 feet ; what is the number of cubic yards in the proposed solid ? **Ans.** (1103.20° G.) $\overline{450 \times \frac{1}{3}\ (7+8+9)} + \overline{657 \times \frac{1}{3}\ (7+9+13}$ $\overline{+585 \times \frac{1}{3}(7+13+11}=3600+635\ 1+6045=16.996$ cubic feet ; dividing by 27 we have $592\frac{12}{27}$ cubic yards.

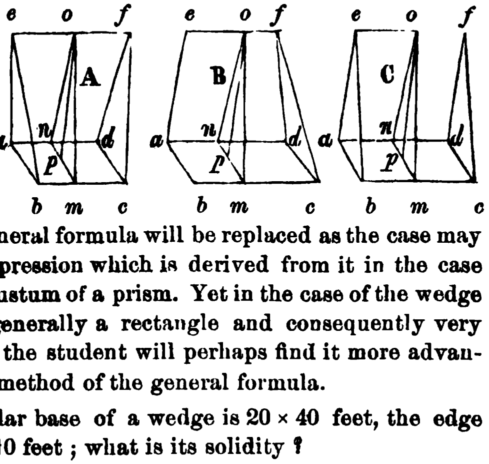
REM. Here we have reduced into square feet the areas of the bases given in square yards, and divided by 27, but it is plain that since 3 times 9=27, it would be the same thing to multiply immediately by the yards and then divide them by 3.

PROBLEM IX.

To find the solidity of a wedge.

(See the *tableau.*)

(90). **RULE.** *To the sum of the areas of either of its three pairs of parallel bases, add 4 times the central or half-way section and multiply the sum by the 6th part of the height corresponding to such bases. The result will be the required volume.*



REM. The wedge, as already stated (1090 G.) is but a triangular prism or the frustum of a prism, as the edge *ef* is equal or unequal to the two other sides ; thus the general formula will be replaced as the case may be, by the simplified expression which is derived from it in the case of the prism, as of the frustum of a prism. Yet in the case of the wedge of which the base is generally a rectangle and consequently very simple to be measured, the student will perhaps find it more advantageous to keep to the method of the general formula.

Ex. The rectangular base of a wedge is 20 × 40 feet, the edge 35 feet and the height 10 feet ; what is its solidity ?

Ans. $\frac{(40+40+35)\times 20\times 10)}{6}$ or (**1094 G. REM.**) $\frac{1}{3}$

$40+40+35)\times\frac{1}{2}20)\times 10)=3833.33.$

2. What is the solid content of a wedge the base of which measures 5 feet 4 inches by 9 inches, the length of the edge 3½ feet and the perpendicular height 2½ feet ? **Ans.** 4.1319 cubic feet.

3. An inclined plane meets a horizontal plane and forms with this last a wedge the edge of which measures 100 feet; the rectangular base 80 feet by 20 feet and the perpendicular distance between the edge and the base 300 feet ; what is the volume of the solid ?
Ans. 260,000 cubic feet.

PROBLEM X.

To find the solidity of a prismoid. (1)

REM. A simple examination of the models of the *tableau* shows at once the nature of the intermediate section or plane parallel to the bases and half-way between them. This question however is treated of in detail under the heading of problem LIX to which one may refer for any thing concerning the prismoid.

(91) RULE. *To the sum of the areas of the two parallel bases, AC ac, add four times the area of a parallel section EG equidistant from*

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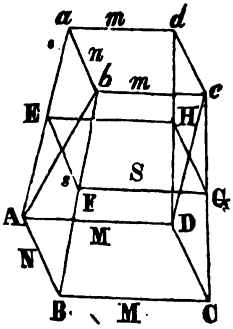
1. This solid, like the prism, is very often met with by the measurer. Rectangular vats with inclined sides are of this form ; a flat hip-roof presents the same figure ; large reservoirs are nothing but reversed prismoids; it is found again, in the basins, wharves, pillars, abutments and constructions of such kind ; excavations and earth works, mounds and embankments, &c., generally assume this form ; the continued embankment of a railway is subdivided by vertical sections into prismoids each resting on one of its lateral faces and whose parallel bases are consequently perpendicular to the horizon ; the prismoid is met again in every piece of squared timber the ends of which are unequal rectangles, it is again seen in the pilings of balls, and shells, and it is also often repeated on various scales in the arts and trades, &c. It has been remarked (note page 412) that one must take care not to confound the prismoid with the frustum of a pyramid, or rather, should it have been said, the frustum of a pyramid with the prismoid, for it evidently follows from the definition of the prismoid that any frustum of a pyramid with parallel bases is at the same time a prismoid and can be measured by the rule applicable to this latter ; but the so called prismoid is not the frustum of a pyramid, and one cannot consequently determine its solidity by the rule applicable to the frustum of a pyramid, though however in certain cases this last rule may give an approximation very near the truth. Let us add also that since when it is required first to determine the nature of the solid to bemeasured, one must in the case of the frustum of a pyramid ascertain the proportionateness of the sides as well as their parallelism, and that their parallelism alone is sufficient to constitute the prismoid ; one will also often save a useless work by considering as a prismoid any solid the lateral faces of which were inclined to one another and the sides of the opposite bases parallel to each other.

these bases : then multiply the sum thus obtained by one sixth of the height or perpendicular distance between the parallel planes (**1101 G.**) *and the result will be the required volume.*



Ex. One of the bases of a rectangular prismoid is 20 × 25 feet, the other is 10 × 15 the height is 12 feet ; what is its solidity ?

Ans. $\overline{(25 \times 20) + (15 = 10) + 4(20 \times 15) \times} \frac{12}{6} = 1850 \times 2 = 3700.$

2. A wharf or pillar has for its parallel bases, rectangles measuring respectively 100 × 50 feet and 80 × 40 feet, the height is 30 feet ; what is its content in cubic yards ? **Ans.** $4518\frac{14}{27}$.

3. A pile of broken stone measures 100 × 20 feet at the bottom, 97 × 16 feet on the top, and 3 feet high or thick, what is its content in cubic toises ?

Ans. $\overline{(100 + 20) + (96 \times 16) + 4 \times 98 \times 18)} \times \frac{3}{6}$(or by $\frac{1}{2}$) $\div 216 = 24\frac{112}{216}$ cubic toises.

4. A cutting or excavation presents the form of a reversed prismoid ; the inferior area of the excavation is 10,000 metres, the upper area 14,400 metres, the half-way area between the parallel bases is 12,100 metres and the height or depth of the excavation is 9 metres ; how many cubic metres have been dug out ? **Ans.** 109200.

5. How many cubic feet of water can be contained in a reservoir the base of which is a rectangle 100 × 50 feet, its upper base a rectangle 180 × 130 feet and depth 20 feet ? **Ans.** $262,666\frac{2}{3}$.

6. What is the cubic space contained within a roof the base of which is a rectangle 40 × 60 feet, the upper part a rectangular flat 20 × 40 and height 12 feet ? **Ans.** 18,400 cubic feet.

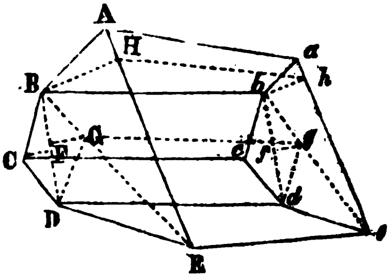
7. What is the solidity of a piece of square timber whose length is 24 feet and the ends parallel and rectangular planes 30 × 27 inches and 24 × 18 inches ?

Ans. 102 cubic inches.

8. A trough the depth of which is 20 inches, has for its parallel bases rectangles of 36 × 30 inches and 30 × 24 inches ?

9. Ans. 10.3475 cubic feet.

9. An embankment for a railway measures 300 yards in length, its ends are trapeziums, the parallel sides of one are 4 and 34 yards and the height 10 yards, the sides of the other are 4 and 19 yards and the height 5 yards; how many cubic yards does it contain?



Ans. Area of one end $=\frac{1}{2}(4+34)\times 10=190$ yards, area of the other end $=\frac{1}{2}(4+19)\times 5=57\frac{1}{2}$ yards, intermediate area $=\frac{\frac{1}{2}(4+4)+\frac{1}{2}(34+19)}{2}\times\frac{1}{2}(10+5)=15.25\times 7.5=114.375$ square yards.

$114.375\times 4=457.500$, $190+57.5+457.5=705$, and $705\times\frac{1}{6}(300)=705\times 50=35{,}250$ cubic yards.

10. A causeway on a sloping or inclined ground measures 100 metres in length; the areas of the quadrilaterals with parallel sides forming the vertical ends or bases of the prismoid perpendicular to its length, are 120 and 80 square metres, and the area of a half-way section between these latter is 100 metres; how many cubic metres have been required to form it? **Ans.** 10,000.

11. What is the cubic space occupied by a pile of cannon balls whose rectangular base is 30 feet by 10, the upper plane 25 feet by 5 and the height 4 feet? **Ans.** 833⅓ nearly.

12. The pedestal of an equestrian statue the height of which is 10 feet, has for its parallel bases rectangles of 15 × 7 feet and 12=4 feet; what is the solidity of the mass of stone which it is formed of? **Ans.** 750 cubic feet.

PROBLEM XI.

To find the area of a regular pyramid.

(See the pyramids of the *stereometrical tableau.*)

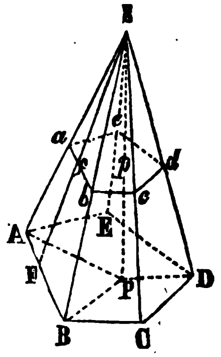
(92) RULE. *Multiply* **(1039 G)** *the perimeter* (*ABCDEA*) *of the base by the inclined half-height* (SF); *the product will be the lateral or convex area. To the lateral area add that of the base, when the whole area is required.*

Ex. 1. What is the lateral area of a regular triangular pyramid, the inclined height of which is 20and each side of the base 3. **Ans.** 90.

2. Required the whole area of a regular pyramid the inclined height of which is 15 metres, and the base a pentagon the side of which is 25 metres ?

Ans. 2012.778 square metres.

3. How many squares of shingle, zinc or other metal, &c. would be required to cover a roof having the form of a regular pyramid the base of which is 200 feet perimeter, and the inclined height 33 feet ? **Ans.** 33.



PROBLEM XII.

To find the lateral area of the frustum of a regular pyramid with parallel bases. (Fig. to Prob. XI.)

(See on the *tableau* the models of this solid.)

(**93**) **RULE.** *Find* (**1040 G.**) *the product of the half sum of the perimeters* (*ABCDEA, abcdea*) *of the two bases by the inclined height* (*fF*) *of the frustum ; you will have the required area.*

Ex. 1. What is the lateral area of the frustum of a heptagonal pyramid, the inclined height of which is 55, each side of the inferior base 8, and each side of the superior base 4 ? **Ans.** 2.310.

2. An eight sided roof, terminated by a platform has for measure of its inclined height 17 feet ; the length of the side of the regular octagon constituting its base is 20 feet, and the side of the superior polygon is 10 feet ; required the weight of the lead that covers it, lead being 6 pounds per square foot ? **Ans.** 12240 pounds.

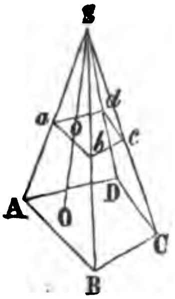
3. How many superficial feet of cut stone are there in the lateral area of of a polygonal tower the inferior and superior perimeters of which measure respectively 100 feet and 80 feet and the inclined height of which is 40 feet ? **Ans.** 3600.

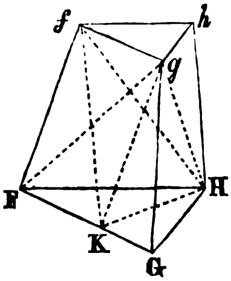
PROBLEM XIII.

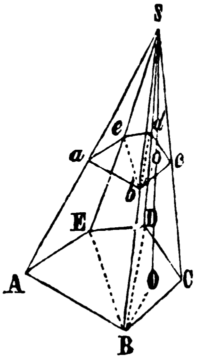
To determine the area of a pyramid, or of any frustum of an oblique or irregular pyramid.

(See the models of the *tableau.*)

RULE. *Get* (**1059 G.**) *separately the area of each of the component faces and take their sum for the required area.*







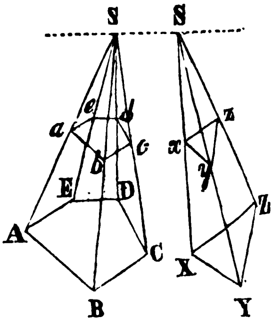
PROBLEM XIV.

To find the solidity of any pyramid.

(See the models of the *tableau*)

GENERAL FORMULA.

To the sum of the areas of the two bases or ends of the pyramid add four times the area of a section half-way between them ; the sixth part of the product of this sum by the height of the solid, will be the required volume.



95. REM. Here, the superior base S, (apex of the pyramid) is but a point and its area is therefore null, or=0. The area of the half-way section has exactly and in all cases, the value of the fourth of that of the base, since its linear dimensions are [1] the halves of those of the base, and the figures AC, *ac*,—XYZ, *xyz* are (**1033 G.**) similar polygons of which the areas are as the squares of the homologous sides ; which gives, as already stated, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Now the superior area being, as just seen, equal to 0, the formula is, for the pyramid: *the area of the base plus 4 times the area of the half-way section multiplied by the sixth part of the height.* But the half height area being the fourth part of that of the base, and as 4 times ¼ are 1, the formula is further simplified and becomes: twice the area *of the base by the 6th part of the height,* expression which reduces at last to the following. (Introduction page 9.)

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[1] In the similar triangles ABS, *ab* S, the homologous sides are (**520 G.**) proportional. Therefore, since *ab* is half-way between AB and S, we have A $a = a$ S $= \frac{1}{2}$ AS and consequently AB : AS :: $a\,b$: a S, that is $a\,b$ = AB ÷ 2.

(**96**) **RULE**. *Multiply* (**1049 G.**) *the area of the base by one third of the height* (**957 G.**) *and the product will be the required solidity.*

Ex. 1. What is the solidity of a pyramid, of which the base is a square 30 feet each side, and the height 25 feet? **Ans.** 7500.

2. The side of the equilateral triangle which forms the base of a pyramid is 3 feet, its height is 30 feet; what is its solidity?

Ans. 38.9711.

3. What is the solid content of a hexagonal pyramid of which the height is 6.4 feet and each side of its base 6 inches?

Ans. 1.38564.

4. The height of a pyramid is 12, and each side of its pentagonal base is 2; its cubic content is required? **Ans.** 27.5276.

5. What is the volume of the space occupied by the roof of an octagonal tower of which the side is 5 metres, the height of the roof being 10 metres?

Ans. $5^2 \times 4.8284271$ (**28T.**) $=120.7106775$ metres is the area of the octagonal base of the roof and $120.71 \times 10 \div 3 = 402.366$ cubic metres.

PROBLEM XV.

To find the solidity of the frustum of a pyramid with parallel bases.

(See the frusta of a pyramid of the *tableau*)

(**97**) **RULE 1.** *To the sum of the areas of the two bases add* (**1102 G.**) *four times the area of a section half-way between them, that is, of a section of which the lineal factors are arithmetical means* (**1265 G.**) *between those of the two bases; multiply then the sum thus obtained by one sixth of the height of the frustum; the product will be the required solidity.*

(**98**) **RULE II.** *Find* (**1061**) *first a mean proportional between the two bases; then add together this mean proportional and the* *two bases of the frustum ; and multiply this sum by one third of the height of the frustum ; the product will be the required solidity.*

REM. In the case of the frustum of a pyramid the stereometrical formula cannot be simplified ; it cannot be reduced as in the case of the prism or of the whole pyramid, to a more simple expression. On the contrary, every other rule to obtain the volume of the frustum of a pyramid is more complicated and requires more work than the formula of the *tableau*, whether we would cube the frustum by taking the difference of the whole and the partial pyramids (AS, *as*, fig. to page 73, 74) or arrive at the result by Legendre's formula which will be found here below (**RULE 11**). In fact, we must in the first case compute by rules of proportion, the lineal dimensions of the partial pyramid which is wanting in the frustum under consideration to form the whole pyramid of which the frustum is a part, afterwards calculate each of these pyramids ; and in the second case there is to be found a mean proportional between the two bases, that is, between the areas of these bases, operation which requires the extraction of the square root of the product of these two areas the one by the other ; whilst by the formula of the author, we arrive immediately and without any difficulty at the area of the intermediate section the factors of which are each an arithmetical mean between those of the opposite bases of the solid.

Ex. I. What is the number of cubic feet in a piece of timber the length of which is 24 feet and the ends are squares of 15 and 6 inches the side ?

Ans. $\sqrt{15^2 + 6^2} = 90.225 + 36 + 90 = 351 = (\div 144)$ 2 feet 5¼ square inches, which multiplied by one third of 24 gives 19.5 cubic feet.

2. Required the volume of a pentagonal pedestal the height of which is 5 feet, each side of the inferior base 18 inches and each side of the superior base 6 inches ? **Ans.** 9.31925.

3. A fort, the height of which is 15 metres, has for base a regular octagon the side of which is 10 metres, the side of the superior polygon is 9 metres ; what is the solidity of the tower ?

Ans. Area inf. oct.$=$(**28 T.**) $4.8284271 \times 10^2 = 482.84271$ square metres, area sup. oct. $= 4.8284271. \times 9^2 = 391.1025951$, proport. mean area$=\sqrt{482.84 \times 391.10} = 434.56$, the sum of the 3 areas$=482.84 + 391.1 + 434.56 = 1308.50$, and $1308.5 \times \frac{1}{3}\,(15) = 6542.5$ cubic metres.

Ans. By the rule (**1101 G.**) of the prismoid, we have for area half-way of the parallel bases $(10 + 9) \div 2 = 9.5$, and $(9.5)^2 \times 4.8284271 =$

$1435.76, \times 4 = 743.04$, $1743.04 \div 482.84 + 391.1 = 2616.98$, and $2616.98 \times \frac{1}{6}(15) = 6542.45$ as before, for the difference .05 between the two results comes only from our not having taken into the two calculations a greater number of decimals.

REM. In this last example, the area of the smaller base $= 4.8284271 \times 9^2$ and that of the greater $= 4.8284271 \times 10^2$; the product of these two areas, the one by the other is $4.8284271 \times 9^2 \times 4.8284271 \times 10^2 = 4.8284271^2 \times 9^2 \times 10^2$ the square root of which is $4.8284271 \times 9 \times 10 =$ the proportional mean area required. It is then plain that in the calculation of the solidity of the frustum of a pyramid by the first of the two rules here given, one will save a long and useless work by using the method just indicated to determine the proportional mean area required, instead of multiplying the one by the other the areas 482.84271, 391.1025951, afterwards to extract its square root. This remark applies also to the frustum of a cone, prob. XXVIII.

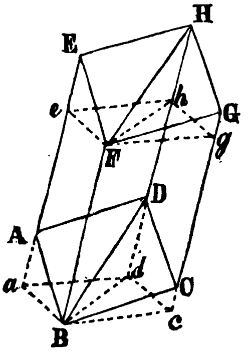
PROBLEM XVI.

To find the solidity of the frustum of any pyramid, that is with non parallel bases.[1]

(See among the models of the *tableau*, the frustum of a triangular pyramid with non parallel bases.)

(99) RULE I. *Divide the frustum to be cubed by a plane of section* e F g h *parallel to one of the bases* a B c d, *and passing through the nearest point* F *of the other base, into the frustum of a pyramid with parallel bases and a solid which will be decomposed into as many pyramids as the given frustum has sides less two. Find separately the solidity of each of these elements by the preceding rules : their sum will be the required solidity.*

(100) RULE II. *Determine* **(1067 G.)** *separately the respective solidities of the whole and partial pyramids ; the difference of these volumes will he the required solidity.*



Ex. The inferior and superior or opposite areas of the frustum of a pyramid with non parallel bases, are 30 and 20 metres, the respective heights of the whole and partial pyramids are 33 and 17 metres ; what is the solidity of the frustum ?

Ans. $\overline{30 \times \frac{1}{3}(33)} - \overline{20 \times \frac{1}{3}(17)} = 320$ cubic metres — $113\frac{1}{3}$ cubic metres $= 216\frac{2}{3}$ cubic metres.

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[1] The figure is not that of the frustum of a pyramid, but imagine it to be one.

PROBLEM XVII.

To find the area of a right cylinder.

(See the cylinders of the *tableau.*)

(101): *Multiply* **(§93 G.)** *the circumference of the base by the height to obtain the lateral area. To this area add those of the two bases if the whole area is required.*

Ex. I. What is the lateral area of a cylinder of which the diameter of the base is 20, and height 50? **Ans.** 3141.6.

2. What is the number of superficial feet of cut stone in the convex surface of a circular pillar the height of which is 7 feet and circumference 8 feet 4 inches? **Ans.** $58\frac{1}{3}$.

3. How many yards of coating are there in the circumference and ceiling of a circular room which is 20 feet in diameter and 10 feet high?

Ans. Cir. $= 3.1416 \times 20 = 62.832$, convex area $= 62.832 \times 10 = 628.32$ area of the ceiling $= \overline{20 \times 20} \times .7854 = 314.16$, required area $= \frac{628.32 + 314.16}{9} = 104.72$ square yards.

4. What is the cost of polishing the convex area of a marble column of which the diameter is 12 inches and length 10 feet, at one dollar a superficial foot? **Ans.** $31.42.

5. A cylindrical tower the height of which is 10 metres and diameter also 10 metres, has for lateral area?

Ans. 314.16 square metres.

6. Required how many feet in area there are in a running foot of the interior surface of a cylindrical drain the diameter of which is 3 feet? **Ans.** $3.14159 \times 3 = 9.42477$.

7. A cut stone vault is semi-cylindrical, its diameter is 10 feet and length 50 feet; what is its concave area?

Ans. 785.4 square feet.

8. What is the number of square inches of gilding in the surface of an iron bar the length of which is 14 feet and the diameter $1\frac{1}{4}$ inches? **Ans.** circ. $3.927 \times 168 = 659.37$.

9. How many superficial inches of silvering would be required to cover the interior, that is the concave surface and the bottom of a cylindrical vessel 7 inches in diameter and 9 inches high?

Ans. The bottom = 7 × 7 × .7854 = 38.4846 square inches, the concave area = $\overline{3.1416 \times 7} \times 9 = 197.9208$ square inches, in all 236.4 square inches?

PROBLEM XVIII.

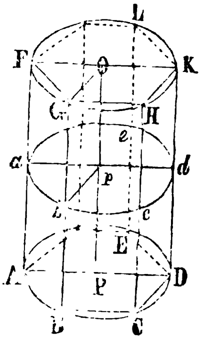
To find the solidity of a right cylinder.

(See the *tableau.*)

REM. The general formula is reduced here, as in the case of the right prism, since the cylinder is nothing else but a prism having an infinite number of sides, to the following expression.

(102) RULE. *Multiply* **(1023 G.)** *the area of the base by the height ; the product will be the solidity.*

Ex. I. Required the solidity of a cylinder of which the height is 20 and circumference of the base 5½?



Ans. $(5.5)^2 \times$ **(31, T.)** $.07958 = 2.4073$ = area of the base, and 2.4073 × 20 = 48.146.

2. A bucket or other cylindrical vessel is 15 inches diameter and 12 inches high ; how many gallons of wine will it contain, the gallon being 231 cubic inches ?

Ans. 15 × 15 × .7854 × 12 = 2120.58 cubic inches, ÷ 231 = 9.18 gallons or 9 gallons, 1 pint and half a pint, nearly.

3. A bar of wrought iron is 14 feet long and 1¼ inches diameter, what is its solidity in cubic inches ?

Ans. 1.25 × 1.25 × .7854 × 168 (or 14 × 12)=206.1675.

4. A stone column is one foot diameter and 10 feet high ; what is its solidity ? **Ans.** 7.854 cubic feet.

5. What is, per lineal foot, the capacity of a pipe or drain of 3 feet diameter ? **Ans.** 7.0686 cubic feet.

6. The foundation of a chimney is a cylindrical mass of which the diameter is 10 feet and height 10 feet; how many cubic yards of masonry does it contain ?

Ans. 785.4 cubic feet÷27=29 cubic yards, 2 cubic feet.

7. The axle-tree or iron spindle of a mill-wheel is 10 foot long and 9 inches diameter ; what is its solidity in cubic feet ?

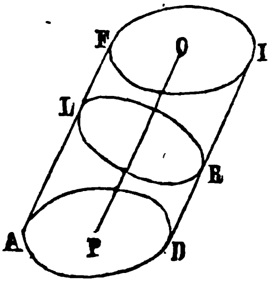
Ans. 9 × 9 × .7854 ÷ 1728=4.418 cubic feet.

PROBLEM XIX.

To find the area of an oblique cylinder.

(See the tableau.)

(103) RULE. *Multiply* **(997 G.)** *the length* AF *or* ID *of the side by the circumference of a section* LR *perpendicular to the side or axis* PO *of the cylinder ; the product will be the lateral area.*



Ex. 1. The semi-cylindrical vault of the opening or arch of a bridge crossing a river obliquely, is 30 feet diameter and 20 feet long ; what is its concave area ?

Ans. 942½ square feet, nearly.

2. A stair railing terminated at each end by the vertical faces of the newels, measures 10½ inches circumference and 15 feet long ; what is the number of superficial yards of varnish laid over ?

Ans. 10½ inches=.875 feet, and 15 × .875=13.125 square feet= 1 yard 4⅛ feet.

3. What is the area of the zinc in a pipe the diameter of which is 9 inches, and of which the length, 5 feet, is terminated by the parallel planes of two alternate elbows (**153 G.**) or facing in opposite directions.

Ans. cir.=3.1416 × 9=28.2744, cir. × 60 and÷144=11$\frac{1}{12}$ square feet nearly.

PROBLEM XX.

To find the solidity of an oblique cylinder.

(See the *tableau.*)

REM. The general formula is reduced here, as for the oblique prism to the following simplified expression.

RULE I. *Multiply the area of the base by the height of the solid; the product will be the required solidity.*

(104) RULE II. *Multiply* **(1026)** *the length of the side by the area of a section perpendicular to the side or axis ; the product will be the required solidity.*

Ex. What is the solid content of the stair railing of the last problem ?

Ans. Area sect. perp. = (**31, T.**) 10.5 × 10.5 × .07958 = 8.7737 square inches, and 8.7737 × 180 (the length in inches) = 1579.26 cubic inches, or .914 cubic foot, or $\frac{9}{10}$ nearly.

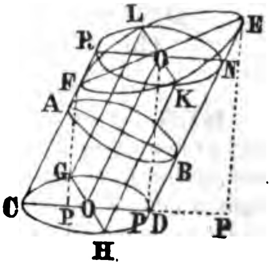
Ex. The area of the base of a cylinder is 3.33 square metres and the perpendicular distance which separates its two bases, is 10 metres ; what is its solidity ? **Ans.** 33.3 cubic metres.

PROBLEM XXI.

To find the area of the frustum of a right cylinder or of the frustum of an oblique cylinder whose large and small axes CD, FE or GH, LK of the opposite bases, are (1099 G.) in the same plane CDEF or GHKL.

(**105**) **RULE I.** *If the frustum be right, multiply* (*dem. of* **1099** *and* **1097 G.**) *the half-sum of the greatest and least heights EP, FP of the frustum by the perimeter of the base of which PP is the axis ; the product will be the lateral area.*

RULE II. *If the frustum is oblique, multiply the half-sum of the lengths of the least and greatest sides DE, CF of the frustum by the perimeter of a section AB perpendicular to the axis of the cylinder.*



Ex. 1. The diameter of a cylinder is 10, its least height is 9.4 and its greatest height 10.6 ; what is its convex area ?

2. A half-elbow of a stove pipe EP, FP or of any conduit (the rectilineal elbow is nothing but a double frustum of a right cylinder, that is two frusta of right cylinders meeting at any angle whatever) of which the diameter is 7 inches, has for its least and greatest lengths, 4 and 11 inches respectively ; what is its lateral area ?

Ans. 3.1416 × 7 × ½ (4 + 11) = 164.9 or say 165 square inches, or (÷144) = 1 square foot and 21 square inches, or 1⅐ square feet nearly.

3. Between the two component frusta of the rectilineal elbow of a cylindrical hand railing, is a third frustum of which the greatest length is 3 inches and least length null. What is its area, the diameter of the rail being 9 inches ?

Ans. It is plain that the proposed frustum is nothing but a double ungula of a right cylinder, that is two ungula united by their perpendicular bases; the required area of which $=3.1416 \times 9 \times \frac{1}{2}(3)=28.2744 \times 1.5=42.4$ square inches.

4. In a cylindrical vessel inclined to the horizon is a liquor of which the least distance from the surface to the bottom is 67 decimetres and the greatest is 1.33 metres, the diameter of the vessel being 1 metre; required the area of the surface exposed to the action of the liquor ?

Ans. $1 \times 3.1416 \times \frac{1}{2}(67+1.33)=3.1416$ square metres, = lateral area, the bottom $= 1^2 \times .7854 = .7854$ square metres, the whole area $=3.1416+.7854=3.9270$ sq. m.

5. A semi-cylindrical vault is terminated by two walls, unequally oblique to the axis or direction of the vault; the diameter is 20 feet and the least and greatest lengths 36 and 30 feet, what is the area ? **Ans.** 1036.73 square feet.

6. The drum of a circular stair of which the diameter is 10 feet, is terminated by the inclined roof of the building; its least height from the level of the floor of the last story is 7 feet, its greatest height 13 feet; what is its lateral area in square yards ?

Ans. $314.16 \div 9 = 34\frac{8}{9}$ nearly.

PROBLEM XXII.

To find the solidity of a frustum of a right cylinder, or of a frustum of an oblique cylinder of which the great or small axes CD, FE or GH, LK of the opposite bases, are (1099 G.) in the same plan CDEF or GHKL.

REM. To apply here the general formula, it would be necessary to decompose the solid by a plane parallel to one of its bases, and passing through the point F (¹) the nearest of the other base, into a right or oblique cylinder as the case may be and the ungula of a cylinder with a circular or elliptical base and of which the general rule gives the exact volume.

(106) RULE I. *Multiply* **(1099 G.)** *the base CHDG, that is the area of the base, by the half-sum of the least and greatest heights EF, FP of the frustum; the product will be the required solidity.*

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(1) See the figure of the last problem and imagine a plane of section parallel to CD or RN and passing through the point F; then the section KNL of the ungula parallel to the plane passing through F will be the section of the ungula half-way between its base and its apex E.

RULE II. *Multiply* (**1099 G.**) *the area of a section AB perpendicular to the axis* 00 *of the cylinder, by the half-sum of the lengths of the least and greatest sides DE CF of the frustum.*

Ex. I. In a cylindrical vessel the bottom of which sunk and disturbed its vertical position, the least inclined height of the liquid contained is 13 feet and the greatest height 15 feet, the diameter of the vat being 20 feet; required the number of gallons of liquor (say 7½ gallons to the cubic foot) in the vat?

Ans. 4398.24 cubic feet=32986.80 gallons.

2. The semi-cylindrical coping of a wall meeting two others at unequal oblique angles, measures 3 feet diameter and its mean length is 100 feet; what is its solidity?

Ans. area perp. sec. $=\frac{3 \times 3 \times .7854}{2} \times 100 = 353.43$ cubic feet.

PROBLEM XXIII.

To find the area and solidity of any frustum of a cylinder.

(See the *tableau.*)

(**107**) **RULE I.** *Imagine the frustum cut (at AB, fig. to problem* **XXI**) *by a plane perpendicular to the axis of the cylinder. Refer to that common base, the two component frusta; get by the two last problems the area or solidity of each of them, and take their sum; or, which is the same thing, multiply the common base or the circumference, as the case may be, by half the sum of the two greatest and two least sides AC, BE-AF BD of the two frusta: the result will be the solidity or area, as the case may be, of the proposed frustum.*

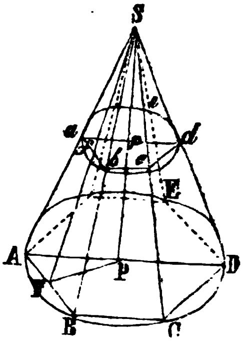
RULE II. *The area of the base CD multiplied by the half-sum of the least and greatest heights FP, EP of the frustum, will give its solidity.*

PROBLEM XXIV.

To find the area of a right or regular cone.

(See the *tableau.*)

(**108**) **RULE.** *Multiply* (**1041 G.**) *the circumference of the base BE by half the side, or inclined height AS, or BS, or &c.; the product will be the convex area; to this area add that of the base, if the whole area is required.*



Ex. 1. What is the lateral area of a cone the side of which is 50 and diameter of the base 8½? **Ans.** 667.59.

2. What is the convex area of a cone the side of which is 36 and diameter of the base 18? **Ans.** 1272.348.

3. The bottom of a boiler is a reversed cone the diameter of which is 10 feet and side 6 feet; what is its lateral area? **Ans.** 94.248 square feet.

4. A vessel the diameter of which is 10 inches has a conical lid the side of which is 5¾ inches, what is the area of the latter? **Ans.** 10 × 3.1416 × 2.875=90.321 square inches.

5. A reservoir the plan of which is circular and the vertical section of which drawn through the centre is an isosceles triangle, is 60 metres wide and the length of its inclined side is 33 metres; how many bricks would be required to line its area, at 75 bricks per square metre?

Ans. Diam. 60 × 3.1416 × 16½ × 75=233,264.

6. A tower is 150 feet circumference and the inclined side of its conical roof measures 30 feet; how many squares of shingle roofing would be required to cover it? **Ans.** 22½.

7. What will be the weight of the conical cover of a gazometer the circumference of which is 180 feet and the inclined side 30 feet, the iron being 5 pounds to the square foot? **Ans.** 13,500 pounds.

PROBLEM XXV.

To find the area of the frustum of a right or regular cone with parallel bases.—(See the *tableau.*)

(109) *Multiply* **(1042 G.)** *the half-sum of the circumferences of the two bases by the inclined height of the frustum; you will have the convex area; to which add the areas of the two bases, to obtain the whole area.*

Ex. I. The side of a frustum of a cone is 12½, and the circumerences of its bases 8.4 and 6; what is its lateral area? **Ans.** 90.

2. What is the whole area of the frustum of a cone the side of which is 16 feet and radii of the bases 3 and 2 feet?

Ans. lat. area=251.328, area inf. base=28.2744, area sup. base =12.5664, whole area=292.1688.

3. The conical part of a funnel has for greater diameter 10 inches, the smaller diameter 1 inch, and the inclined side 15 inches; what is its lateral area?

Ans. 259. square inches=1.8 square feet.

4. The inclined roof of a circular tower the diameter of which is 30 feet and the side 20 feet is terminated at the top by a platform the circumference of which is 33 feet ; required the number of *squares* of zinc to cover it, including the platform.

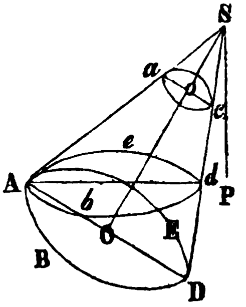
Ans. lat. area=1272.48, area sup. base=$(33)^2$ × .07958=86.66, required area=1359.14 square feet=13½ *squares* 9 square feet.

5. How many square inches of gilding will be required to cover the interior of a goblet the inferior circumference of which is 6 inches, the sup. circ. 7 inches and the side 3½ inches ?

Ans. The lateral surface = 3½ × ½ (6 + 7)=22.75 square inches, the bottom=6 × 6 × .07958=2.865, the whole=25.615 square inches.

PROBLEM XXVI.

To determine the area of a cone or of any frustum of an oblique or irregular cone.—(See the *tableau*.)

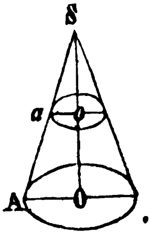


(110) RULE. *Divide the lateral area of the cone, by lines drawn from the apex to the base, into triangles or sectors, and the lateral area of the frustum of the cone by lines drawn between the two bases, into trapeziums, &c. ; calculate separately the area of each of the component parts and take their sum for the required area.*

PROBLEM XXVII.

To determine the solidity of a right or oblique cone. [1]

(See the models.)



REM. The general formula is reduced for the right cone, as with the regular pyramid, to the following simplified expression, for $ao = \frac{1}{2}AO$ and consequently half-way area $o = \frac{1}{4}$ area O and as $4o = O$, it follows that $\overline{4o + O} \times \frac{1}{6}SO = O \times \frac{1}{3}SO$.

(111) RULE. *Multiply* (**1050 G.**) *the area of the base by one third of the height, and the product will be the required solidity.*

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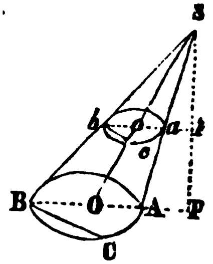
[1] Read the general formula and the remark relating to the solidity of the pyramid, problem XIV.



Ex. 1. What is the solidity of a cone the height of which is 27 feet and base a circle 10 feet diameter ? **Ans.** 706.86.

2. The circumference of the base of a right cone is 9 feet, its height being 10½ feet, what is its solidity ? **Ans.** 22.56.

3. The area of the base of an oblique cone is 1000 metres and its height 30 metres ; what is its solid content ?

Ans. 10,000 cubic metres.

4. A rock or hillock having the form of an irregular cone has for its base a figure the area of which is 5300 square yards, the height of the body being 105 yards ; how many cubic yards of stuff would it be necessary to take away to remove it ? **Ans.** 185,500.

5. What is the volume of the space included under a conical roof the height of which is 30 feet and diameter 30 feet ?

Ans. 7068.6 cubic feet.

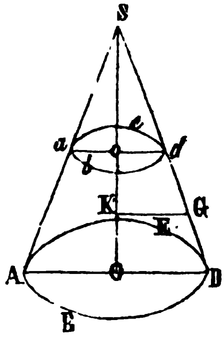
6. How many cubic inches of sugar plums can a cornet 3 inches diameter and 9 inches long contain ? **Ans.** 21⅕.

7. The circumference of the conical bottom of a boiler is 10 feet and the height of the cone 1 foot ; how many gallons will that part of the vessel contain ?

Ans. $10 \times 10 \times .07958 \times \frac{1}{3} = 2.652666$ cubic feet, × 1728 and ÷ 231 = 19.843 gallons.

PROBLEM XXVIII.

To determine the volume of a frustum of a right or oblique cone, that is, of a frustum of any cone, with parallel bases.—(See the models.) [1]



(112) RULE I. *To the sum of the areas of the two bases, add four times the area of a section half-way between them, that is of a section whose lineal factors are arithmetical means* (**1265 G.**) *between those of the two bases ; then multiply the sum thus obtained by one sixth of the height of the frustum ; the product will be the required solidity.*

RULE II. *Find* (**1063 G.**) *first a proportional mean between the two bases ; get afterwards the continued sum of the*

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[1] See the remark which relates to a frustum of a pyramid, problem XV.

mean and the two bases of the frustum; then multiply this sum by one third of the height of the frustum and the product will be the required solidity.

Ex. 1. Required the solidity of a frustum of a right cone, the height of which is 18, the diameter of the inf. base 8, and that of the sup. base 4 ?

Ans. Inf. base$=\overline{8 \times 8} \times .7854=50.2656$, sup. base.$=\overline{4 \times 4} \times .7854$ 12.5664, the arith. mean factor between 8 and $4=\frac{1}{2}(8+4)=6$, $6 \times 6 \times .7754 \times 4=113.0976$, the sum of these areas$=175.9296$, multiplying by 3 (the sixth part of the height 18) we obtain 527.7888.

2. How many cubic feet of water may be contained in a reservoir having the form of the frustum of an inverted cone the greater diameter of which is 200 feet, smaller 100 feet, and depth 25 feet ?

Ans. 458.153 cubic feet.

3. A conical pipe unites two drains of 10 and 20 inches circumference, its length or the perpendicular distance between its two bases is 25 inches; what is the capacity of that part of the drain ?

Ans. Area small end$=$(**31 T.**) $(10)^2 \times .07958=7.958$, area large end$=(20)^2 \times .07958=31.832$, the arithmetical mean circ.$=\frac{1}{2}(10+20)=15$, $(15)^2 \times .07958 \times 4=71.622$, the sum$=111.412$, this sum $\times \frac{1}{6}(25)=$ 464.51666 cubic inches.

4. What is the capacity of a firkin the height of which is 20 inches, the inf. diam. 10 inches, and the sup. diam. 16 inches ?

Ans. 2701.876 cubic inches $\div 1728 = 1.55$ cubic feet.

5. A vessel presenting the form of two frusta of cones, united by their greatest bases, measures 40 inches long, 28 inches at the bung or centre and 20 inches at the head or ends; how many gallons will it contain ?

Ans. $20 \times 20 \times .7854 = 314.16$, $28 \times 28 \times .7854=615.7536$, $24 \times 24 \times .7854 \times 4=1809.5616$, the sum of the areas$=2739.4752$; $\times \frac{1}{6}(20)=9131.584$ cubic inches$=$the content of one of the component frusta, $\times 2=18263.1680$ cubic inches, $\div 231=79.06133$ gallons.

Ans. By the 2nd rule we obtain :

area lesser base$=$ $.7854 \times 20^2 = 314.16$

area greater base$=$ $.7854 \times 28^2 = 616.7536$

area mean proport.$=$(**98 REM. T.**) $.7854 \times 20 \times 28 = 439.824$

1369.7376

multiplying by one third of the height of the frustum $6\frac{2}{3}$

we obtain for sol. of the frustum 9131.5840

2

doubling, we have for total sol. as before 18263.1680

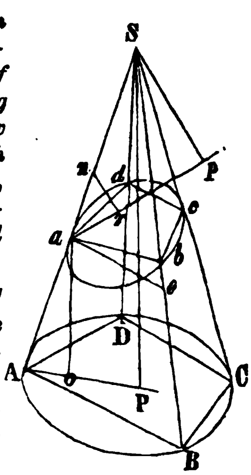
REM. It is hardly necessary to say that instead of multiplying separately by .7854 or by .07958, as the case may be, the squares of the diam. or circ. of the opposite bases, and 4 times the *square* of the diam. or circ. of the intermediate base then to take their sum; the sum of these squares may be multiplied at once by the factors .7854 or .07958.

PROBLEM XXIX.

To find the volume of any frustum of a cone with non parallel bases.—(See the *tableau.*)

(113) RULE I. *Decompose the given frustum into a frustum of a cone with parallel bases and an angula of a cone, by a plane of section parallel to one of the bases and passing through the nearest point of the other base. Now get the volume of the frustum of the cone with parallel bases by the rule of problem XXVIII, then that of the ungula of the cone by the following ploblem ; the sum of these volumes will be that of the proposed frustum.*

(114) RULE II. *Determine separately* (**1067 G.**) *the respective volumes of the entire and partial cones ; the difference of these volumes will be the required solidity.*



Ex. The inf. and sup. areas of the frustum of a cone with non parallel bases are 30 and 20 metres, the respective heights of the entire and partial cones are 33 and 17 metres; what is the volume of the frustum ?

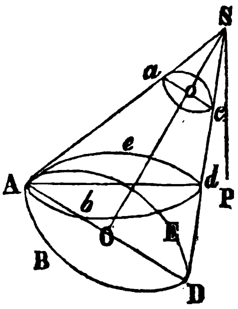
.$(30 \times \frac{1}{3}\,33) - (20 \times \frac{1}{3}\,17) = 330 - 113\frac{1}{3} = 216\frac{2}{3}$ cubic metres.

PROBLEM XXX.

To find the volume of the ungula of a cone.

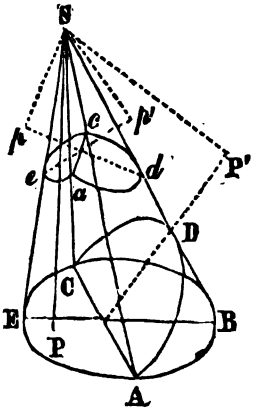
(See the *tableau.*)

(115) REM. Say the ungula DAd with entire bases ABDEA or AbdeA, that is, each of the bases of which is an ellipsis, or the one a circle and the other an ellipsis, or the ungula ABC–D with partial bases, that is, each base of which is the segment of an ellipsis or the one the segment of a circle and the other the segment of an ellipsis, a parabola or a hyperbola, according as the ungula in each of the two cases be that of a right or of an oblique cone. Imagine a plane of section parallel to either of the two bases and drawn half-way between such base and the apex D or *d*, B or D, as the case may be, of the ungula, that is, passing through the middle of Dd or DB ; the section will be in the first case a semi-circle if the base to which it is parallel is a circle, a semi-ellipsis if the base to which it is parallel is an ellipsis, and such semi-ellipsis will also be similar to the one to which it is parallel ; in the second case the section of the ungula ABC—D will be the segment of a circle, if the base parallel to it is a circle, or the segment of an ellipsis if the plane of section is parallel to an elliptical base, or finally a parabola or a hyperbola. The apex of the ungula is evidently but a point d or D, D or B and its area is consequently null or $=0$. Therefore the general formula " sum of the areas of the two opposite bases or ends, plus 4 times the area of the half-way section, multiplied by the sixth part of the height of the ungula " is reduced as for the cone or pyramid to the following expression :



(**116**) **RULE I.** *To the area of the base add four times that of the half-way section parallel to such base and multiply the sum of these areas by the sixth part of the height of the solid ; the result will be the solidity nearly of the proposed ungula.* (See the problems relating to the areas of the segments of circles and ellipses, to that of the parabola, &c., problems XX and XXI, &c.

Ex. I. Cube the frustum of a right cone the heightof which 20, the inferior diameter = 15, superior diameter = 9.6, intermediate diam. $(15 + 9.6) \div 2 = 12.3$. Imagine a plane of section passing between the opposite ends of the inferior and superior diameters ; this section will divide the given frustum into two ungulæ, the one having for its base the inferior base of the frustum of the cone, the other, the superior base of the frustum. The intermediate section of the ungula, the diameter of whose base is 15, will be the segment of a circle whose versed-sine $= 15 \div 2 = 7.5$ and the intermediate section of the other ungula will also be the segment of a circle having for its versed-sine $12.3 - 7.5 = 4.8$. The area of



this latter segment=(**41, T.**) 42.94 and that of the other segment is equal to that of the circle the diam. of which is 12.3 minus 42.94=118.82 —42.94=75.88. Now, volume inferior ungula=75.88× 4 + 176.71 (area of the circle the diam. of which is 15) × 20÷6=1600.75, the real solidity=1596.98, the difference=3.77=less than the fourth part of one per cent in excess.

(**117**) The other ungula has for its solidity 42.94 × 4÷72.38 (area of the circle whose diameter is 9.6)=171.766 + 72.38=244.1486 × 20÷6 - 813.829. This volume added to 1600.75 gives for the sum of the solidity of the two ungulæ 2414.579. The solidity of the frustum =area circle, diam. 9.6 + area, circle diam. 15 + 4 area circle diam. 12.3, the whole multiplied by 20÷6=2414.63376 which differs from the last result only because we did not take in all the decimals.

Ex. 2. With the same frustum of a cone as in the last example, suppose the plane of section passes through the end of the superior base and cuts the inferior base at 5 from the opposite end of that base; then (**41, T.**) for the area of the segment of the base the versed-sine of which is 5 we will have 51.5637, and as the area of the circle to diam. 15 is 15 × 15 × .7854=176.715, we will obtain, deducting from it 51.5637, the area of the other segment of the base the versed-sine of which is 10=125.1513. Now for the respective intermediate bases of the two ungulæ, we obtain for the one whose versed-sine is 5 (half of 10) 45.3492 and for the other segment whose versed-sine is 12.3—5=7.3 we obtain area circle to diameter 12.3—45.3492=118.8232—45.3492= 73.474. We consequently obtain for the solidity of one of the two ungulæ (the one having for its base the inf. base of the frustum) 125. 1513 + 4 times 45.3492=306.548 and 306.548 × 20÷6=1021.8267; the real solidity=1015.701, the difference is 6 +, say the 6 tenths nearly of one per cent in excess.

Ex. 3. Another frustum of which the height=40 and of which the inf. and sup. diam. are 60 and 38.4 and the intermediate diam. consequently=49.2=$\overline{60 + 38.4}\div 2$, the frustum divided into two ungulæ with bases not truncated as in the example n. 1, gives for respective areas of the segments constituting the intermediate bases of the two ungulæ whose versed-sines are 19.2 and 30, 687.050251 and 1214.120405 whose sum=1901.170656 (area of the circle forming the intermediate section of the frustum); area sup. base.=1158.119424, area inferior base=2827.44, solidity inf. ungula=51.226, the solidity of the superior ungula=26.042, the sum of the two=77.268=solidity of the frustum. The first solidity 51.226 is .123 more than the real solidity which is 51,103, the error being 0.24 or ¼ per cent nearly, the second, volume 26.042 is less than the real solidity which which is 26.165, by .123 or nearly the half of one per cent in defect.

. The solidity of the other ungula=51.5637 + 4 times73474 · 72.3825,the whole × 20 ÷ 6 = 1392.80706, which added to 1021.8267 volume of the other ungula gives for the solidity of the frustum 2414.63376. But the frustum calculated separately=as in the last example 2414.63376.

(118) **REM**. Since the formula applied to the frustum of the cone gives an accurate result, and the solidity of the ungula having for inferior base the base of the frustum, is greater than the real solidity, as ith as just been seen, by about ½ per cent (more or less) it follows that the solidity of the other ungula is less than the real solidity precisely by the quantity which is in excess in the first ungula, which is evident.

What is certain, is that in practice one would prefer disregarding as the case may be, the half per cent of error in question to taking the trouble, of arriving at a more accurate result, through the long and difficult calculation required by the accurate formula and lose a time which generally would be more valuable than the stake of half a unit in a hundred, or of one unit in 200.

Ex. 4. The segments of ellipses being the bases of the ungula of a cone, are (**61 T.**) respectively of 20 and 15 feet area, and the heights of the whole and partial cones perpendicular to those bases are (**1067 G.**) 30 and 33 feet; what is the volume of the ungula?

Ans. $(20 \times \frac{1}{3}\overline{30})-(15 \times \frac{1}{3}\overline{23})=300-115-185$ cubic feet.

PROBLEM XXXI.

To determine the solidity of the ungula of a cylinder.

(See the several ungulæ of a cylinder of the *Stereometrical Tableau*,)

(199) **NOTE**. The ungula of a cylinder, like the ungula of the cone is sometimes met with in the practice of the measurer at the intersections of vaults, &c.; and in the practice of the gauger who has sometimes to calculate the quantity of liquor in a cylindrical or conical vessel inclined to the horizon. The component parts of certain bodies may also offer to calculation solids of this kind, as when we decompose with respect to its measurement, the frustum of a cylinder or a cone with non parallel bases into a frustum with parallel bases and an ungula, to determine separately their solidity and get afterwards the um of these solidities.

(120) REM. If the base of the ungula is entire or not truncated, that is, if this base is, either a circle or an ellipsis, according as the ungula or the cylinder of which the ungula forms a part is right or inclined, then the ungula may be considered as the frustum of a cylinder with nonparallel bases and whose least height=0, and whose solidity, as it has been seen (**107, T.**) is equal to the product of the area of its base by half the height of the solid; for the area of the half-way section parallel to the base is in that case equal to half the base, being evidently half a circle or half an ellipsis as the case may be; now 4 times the half-circle or half-ellipsis in question=twice the base, and twice the base plus the base, (that is three times the base) multiplied by the 6th part of the height of the solid is equal to multiplying the base by half the height. The general expression is therefore simplified in the present case and gives for

RULE. *Multiply the base of the ungula by half its height, the product will be the required solidity.*

(121) REM. II. If the ungula is truncated by a plane parallel to its base we must resort to the general formula.

RULE. *To the sum of the areas of the opposite bases of the frustum of the ungula add* 4 *times the area of a section half-way between the two ends of the solid and the product of this sum by the* 6*th part of the height of the solid will be the required solidity.*

NOTE. The bases will be according to data, circle and the segment of a circle, or an ellipsis and the segment of an ellipsis and the half-way section will be either the segment of a circle or the segment of an ellipsis.

(122) REM. III. When the ungula of a cylinder to be computed is an ungula properly so called, that is, with partial base, it is measured exactly like the ungula of the cone (prob. XXX) that is, by adding to the area of its base 4 times the area of the half-way section to multiply afterwards the whole by the 6th part of the height of the solid. If the ungula is that of a right cone, the base is the segment of a circle and the parallel section also a segment of a circle. The same if it is the ungula of an oblique cylinder, the half-way section parallel to the base will be, like the base, the segment of an ellipsis. In each of the two cases, the height or versed-sine of the segment the area of which is to be valued is half that of the base. As to the chord of the segment if this segment is of a circle we will have (**539, G.**) the half-chord=the square root of the product of the versed-sine by the rest of the diameter and if the segment is of an ellipsis, we will obtain the half chord by making :[1] the greater axis or

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[1] See, in relation to this, the articles on the ellipsoid and elliptical spindle.

diameter of the ellipsis is to (:) its minor axis or diameter as (::) the square root of the versed-sine of the segment multiplied by the rest of the greater axis is to : the half-chord. Besides we may also in practice obtain directly the chord of the segment by tracing and measuring that chord on the solid itself.

(**123**). **REM. IV.** If the ungula of the last paragraph is truncated by a plane parallel to its base, forming thus what may be called the frustum of an ungula of a cylinder with parallel bases, we may still easily obtain either by direct measurement of the model or solid to be cubed, or by calculation, the chord and versed-sine of the intermediate section, the versed-sine being the half-sum of those of the circular or elliptic segments of the opposite bases.

(**124**) **REM V.** If the ungula is central, its apex, diameter of the cylinder, will be a mere line and the area=0. In that case, the central or half-way section will be the central zone of a circle or of an ellipsis, according as the ungula be right or inclined. If the ungula is eccentric, its apex, chord of a section, will still be a line and its area=0. In such case its half-way section will be the eccentric zone of a circle or an ellipsis as the case may be.

(**125**) **REM. VI.** If the ungula, central or eccentric, is truncated by a plane parallel to the base, it will then be the frustum of a central or eccentric ungula of a right or oblique cylinder one of whose bases, if entire, will be a circle or an ellipsis, the other base a central or excentric zone of a circle or ellipsis and the half-way section also the zone of a circle or ellipsis either central or eccentric as the case may be. If the base of the ungula is not entire, it will be either a central or eccentric zone of a circle or ellipsis, this segment being as the case may be, more or less than half the whole base of the cylinder of which the ungula forms a part.

Ex. I. Let it be required to cube a lateral ungula of a right cylinder, its height=24, the versed-sine of the segment of a circle constituting its base=2 and the diameter of the cylinder of which the ungula forms a part=10.

The area of the base=(**41, T.**) 2÷10=.2=versed-sine of the table at the end of this volume, and the tabular area corresponding to this sine = .111823 which being multiplied by 10^2 or by 100 gives for required area 11.1823. Thus also we obtain the area of the segment at half-height of the ungula=1÷10=.1=.040875=tabular area which × 100=4.0875 and this × 4=16.35, adding 11.1823, we have 27.5323=sum of the areas of the base and 4 times the half-way section, this sum × 4(=24÷6) gives for the volume of the proposed ungula 110.1292. The real solidity is 109.4334, the difference .6958=

.636, that is : the rate of error is less than the $\frac{2}{3}$ of 1 per cent in excess.

Ex. 2. The height of the lateral ungula of a cylinder is 24 and the diam. of the cylinder 10 as in the last example. The base of the ungula is a semi-circle. Required the solidity. Area base=10 × 10 × 7854÷2=39.57, the versed-sine of the half-way segment=5÷2=2.5, 2.5÷10=.25 corresponding in the tables to .153546 which × 100=15.3546. Now 15.3546 × 4=61.4184, adding 39.27 we obtain 100.6884 multiplying by 4, one sixth of the height, we have for the prismoidal volume 402.7536 ; the accurate solidity is 400, the difference is 2.7536 equal to a rate of error of .688 or a little more than the two-thirds of one per cent in excess.

Ex. 3. With the same diam. (10) of a cylinder and the same height of an ungula (24) as in the two last examples, the base of the ungula which in the 1st example is less than a semi-circle, in the 2nd example equal to a semi-circle, is in this 3rd example greater than a semi-circle, its versed-sine being = 8 = 10—2 ; we have seen that the area of the segment having 2 for its versed-sine or height= 11.1823 and as the whole base of the cylinder=10^2× .7854=78.54 we will obtain the area of the required segment = 78.54—11,1823 = 67 3577. The half-way section at half-height has for height or versed-sine 8÷2=4, 4÷10=.4=tab. v. s.=.293369=tab. area × 100=29.3369 and × 4 = 117.3476,117.3476 + 67.3577 = 184.7053 = sum of the areas, which × 4, 6th part of the height ; we obtain 738.8212 for the volume by the general formula, the real volume=734.218, the difference is 4.6032 equal to a rate of error of .627 per cent or less than the two thirds of one per cent.

Ex. 4. A lateral ungula of a cylinder whose diameter=25, has for its height 60 and for base a segment of a circle whose versed-sine=5. What is the volume of the ungula ?

Since 5 : 25 and 2½ : 25 in this example as 2 : 10 and as 1 : 10 in example N° 1, we have for tab. area as in the first example .111823 and .040875 which multiplied each by 25^2 or by 625 give for area of the base 69.89 and for area of the half-way section 25.546875. This last × 4 times=102.1875, adding 69.89 we have 172.077 which × 10 (the height÷6)=1720.77. The real solidity=1709.90, the difference=10.87=.635=less than two thirds of 1 per cent.

Ex. 5. Let us compute now the complemental ungula to that of example 4, that is, the ungula which, with that of the last example, make up the cylinder of which each of them forms a part.

The area of the base of the cylinder=25 × 25 × .7854=490.875 and this base × the height 60=29452.5= solidity of the cylinder.

The ungula to be computed has for its inferior base the base of the cylinder less the base of the ungula already computed, that is, 490.875—69.89=420.985, the superior base being entire=490.875 and the half-way section=490.875—25.546875 (area of the segment whose v. s.=2.5)=465.328125. This last area taken 4 times=1861.3125, adding the areas of the opposite bases, we obtain 2773.1725 for the sum of the areas, multiplying by 10 (6th part of the height) we obtain for the vol. of the ungula 27731.725, if to this vol. we add 1720.77 vol. of the 1st ungula we have 29,452.495=volume of the cylinder as herein above determined, the difference between .5 and 4.95 being caused by our taking 69.89 for 69.889375 for the area of the inferior segment of the 1st ungula.

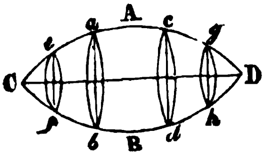
If 1709.9 is the real vol. of the 1st ungula; then, as 29452.5 is the real vol. of the cylinder, it follows that the accurate solidity of the complemental angula just computed in this example to 27731.725 is 29452.5—1709.9=27742.6. Therefore the vol. of the complemental ungula is in this example too small by 11 nearly=.04=$\frac{4}{100}$ per cent or $\frac{1}{25}$ of 1 per cent.

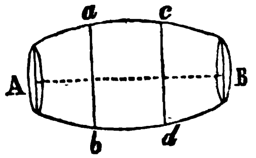
THEOREM.

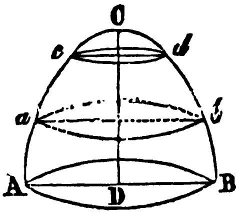
(126) General expression for the lateral area (convex or concave) of any solid of revolution, or of a segment or frustum of such a solid with a single base or two parallel bases, and whose plane of section is perpendicular to the axis of the generating curve.

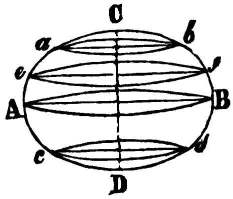
Divide the generating curve into equal parts so small that each of them be sensibly a right line; draw through each point of division a circumference parallel to the base or perpendicular to the axis of the solid. These parallel circumferences will divide the area to be computed into zones of equal breadth; each of these zones will be a continued trapezium the area of which will be obtained by multiplying the half sum of its parallel bases or circumferences by the height or breadth of the zone, and the whole area of the proposed solid will be equal to the sum of the areas of its component zones.

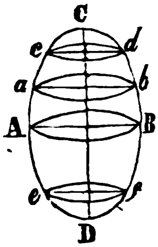
The required area will therefore be obtained by adding to the half-sum of the circumferences of the opposite bases or ends of the solid, the sum of the intermediate circumferences of all the component zones, and afterwards multiplying the whole by the breadth of one of these zones: expression analogorus to that of par. (**23 and 69 T.**) for the area of any plane figure.







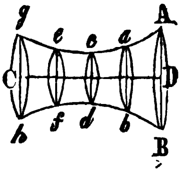


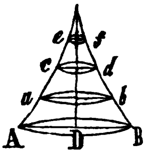


Thus AB–C being any conoid, a half-spindle, a hemisphere, a half-spheroid or any segment of a sphere, spheroid or spindle, with a single base AB, we will have the lateral area=(½ circ. AB + circ. *ab* + circ. *cd*) × A*a*=*ac*=*e*C.

If the given segment or frustum AB *dc* has two bases AB, *cd*, the area will be=(½ circ. AB + circ. *ab* + circ. etc. + ½ circ. *cd*) × A *a* or *ac*. If the opposite halves of the solid are symmetrical as in the cask AB or any other central frustum or segment of a spindle or speroid, it is hardly necessary to observe that it will suffice to operate on one of the symmetrical halves and afterwards double the result.

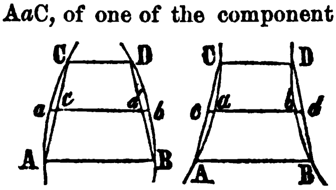
If the solid AB–C in question is with a concave surface, that is, generated by the revolution of a curve AC or A*g* which presents its convexity to the axis CD of the solid, it is plain that we will in the same manner obtain the area=(½ circ. AB + circ. *ab* + circ. *cd* + circ. *ef*) × A*a* or *ac*, &c., in the case of the conoid or segment with a single base, or=(½ circ. AB + circ. *ab* + circ. &c., + ½ circ. *gh*.) × A *a* or *ac*, &c., in the case of the frustum or segment with two parallel bases AB, *gh*.





It follows evidently from what precedes that if the generating line of the surface to be computed is mixed, that is, partly convex and partly concave, or if this line is partly right and partly curved, the same process will quite as simply lead to the determination of its area or superficies.

REM. It is to be remarked that the general formula just established will generally give, for any convex surface, a result which will be in defect of the required area of the solid, and in the same way, the result obtained in the case of a concave surface will be in excess of the real area of the proposed body.



In fact, in practice, the breadth A*a*C, of one of the component zones of the surface to be measured, will differ more or less from the straight line AcC, according as AC is a greater or smaller portion of the generating curve. Therefore instead of considering AC as a straight line with a length = AcC we will add to the accuracy of the result by taking for the breadth of the zone the developed breadth A*a*C of that zone, which will be obtained pretty accurately with the help of a scale of equal parts sufficiently subdivided and thin enough to be adjusted to the convex or concave direction of the length of the arc to be computed.

However, though we shall have added to the precision of the operation by substituting for the rectilineal breadth AcC of the zone, its real breadth A*a*C; we will still be in defect or in excess of the required area, although by a very small quantity, with regard to the whole area. This quantity will be, very nearly, equal to $(ac + bd)$ (or $2\,ac$) $\times 3.1416 \times \frac{1}{2}$ A*a*C or to $3\frac{1}{7}$ times the double of the area of the space Ac C*a*A, or to $12\frac{4}{7}$ times the area of the triangular space having *ac* for its base and for its height the developed length of the arc *a*C; for $2ac \times 3.1416$ is evidently the difference between the circumference *ab* and the mean *cd*, of the circumferences AB, CD, and it is precisely by the product of that difference by the length of the arc *a*C or *a*A, or which is the same thing, by the product of the half difference *a*C by the whole arc A*a*C, that the required convex area is slight or wantting, or that the concave area to be determined is great or in excess; but on account of AC being very small, the difference, either in excess or in defect, between the accurate area and the area obtained by the formula, will always be, as we have just said, a relatively small and insignificant quantity, which will soon be seen with the problems and solutions shortly to be submitted in order to compare their accuracy with that of the results which the ordinary rules furnish, and to judge at the same time of the amount of labour incident thereto.

THEOREM.

General expression for the volume of any solid.

(See the models of the *stereometrical tableau.*)

(127). *Of every right or oblique prism or cylinder—of every regular or irregular pyramid or of every right or oblique cone—of every frustum of a pyramid or cone comprised between parallel bases—of the sphere—of every spherical sector or pyramid—of every spheroid—of every segment of a sphere or spheroid with a single base or of every frustum of these bodies with two parallel bases inclined in any way to the axes of the solid—of every right or inclined paraboloid or parabolic conoid—of every right or inclined hyperboloid or hyperbolic conoid—of every segment of a paraboloid or hyperboloid with a single base or of every frustum of these bodies with two parallel bases inclined in any way to the axes of the solid—of every wedge or other frustum of a triangular prism—of every part of such wedge or of such a truncated prism separated from the whole solid by a plane parallel to either of its lateral faces—of every other prismoid or cylindroid—of every ungula of a sphere or spheroid comprised between planes of section passing in any direction through the centre of the solid—of every ungula of a prism or prismoid, cylinder or cylindroid, pyramid, cone or conoid comprised between planes of section having their common intersection in the axis of the solid and of every segment of such ungula with a single base, or of any frustum of such ungula between parallel bases* (**and approximatively,**) *of the semi-spindle or semi-central frustum of a spindle (cask) comprised between parallel planes perpendicular to the fixed axis of the solid,—of any ungula whatever of a prism or prismoid, cylinder or cylindroid, pyramid, cone or conoid, sphere, spheroid or spindle, and of every frustum of an ungula comprised between parallel bases) : the volume is equal to the sum of the area of its base, if there is but one, or of its parallel bases, if there are two, and of four times the area of a section half-way between the bases, between the base and apex, or between the opposite apices, as the case may be, multiplied by one sixth of the height of the solid.*

Let A and B the opposite bases, base and apex, or opposite apices of any one of the bodies just enumerated, let S a parallel section half-way between A and B, and H the height of the solid; we will obtain, as the case may be, volume=(area A +area B+4 area S) × $\frac{1}{6}$ H, or (area A + 4 area S) × $\frac{1}{6}$ H, or (4 area S) × $\frac{1}{6}$ H, according as area apex B=0 or area apex A + area apex B=0.

(**128**) Now, of the five regular polyhedrons, the tetrahedron is a pyramid, the hexhaedron is a cubė that is a prism, and each of the three others is a compound of pyramids equal to each other; every frustum of a polygonal prism is a compound of frusta of triangular prisms each having for its base one of the lateral faces of the given frustum and the edges or arrises of which all unite and become confounded in one of the parallel edges of the solid or in any right line parallel to the sides of the frustum, situated in its interior and which may be considered as an axis of the prism of which the frustum forms a part; every frustum of a cylinder may also be considered as a compound of frusta of triangular prisms, since the cylinder itself is but an infinitary prism [1]; every circular, elliptical, parabolic, &c., spindle, elongated or flattened, as the case may be, will be decomposed, as has already been shown, by sections perpendicular to the fixed axis of the solid (**1138 G.**) into cones and frusta of cones, or, if possible, into frusta of a sphere or spheroid, or of a parabolic or hyperbolic conoid, subdivisions to which may be added if required, the cylinder and spherical segment. The conoid or spheroid whose generating curve is not that of a section of a cone, can be decomposed (**1139 G.**) like the spindle, into frusta of cones or conoids, segments of a sphere, segments of spheroids, of paraboloïds or of hyperboloids, &c.; the ungula of a cylinder, cone or conoid, &c., will be considered as a compound of rectilineal or spherical pyramids, &c., and every other body will be subdivided, as the case may be, into elements (**1143 G.**) of the kind just enumerated.

The expression is therefore general, as has been said at the head of this article, and will serve at will to determine the solidity of any body.

(**129**) Used till now (**1103 G.**) to the consideration of so varied a number of expressions for the volume of the several solids in

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[1] We have seen elsewhere (**83** to **88 T.**) that as regards the frustum of a regular prism, that is, the bases of which are regular or symmetrical polygons, and (**105** to **107 T.**) as to the frustum of a cylinder, this subdivision or imaginary decomposition by planes of section is not at all necessary, since such bodies are osily measured without that.

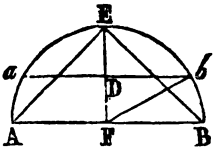
question, and that, without even including the spheroid, paraboloid and the segments of those bodies, which still give rise to additional formulae; the pupil will at first perhaps be astonished and will even doubt the existence of a formula which may be applied at once, to bodies as dissimilar to each other as are the prism or cylinder, the sphere, the segment of a sphere, the pyramid or cone and the wedge, &c., and whose limiting surfaces are indifferently plane or curved or both; but the following reflections will be sufficient to prove the accuracy of the enunciation of the proposition.

(**130**) **In the first place**, the *prism* or *cylinder* has for its solidity (**1103** 1° and 6° **G.**) the area of its base multiplied by its height; but the opposite bases of a prism or cylinder are equal and every section of such solids parallel to the base (**943 G.**) is equal to the base; the sum of the 2 bases plus 4 times the half-way section between them, is then equal to six times the base, and it is the same thing to multiply 6 times the base by one sixth of the height or to simply multiply the base by the whole height.

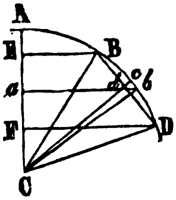
(**131**) **In the second place**, the solidity of the *pyramid* or *cone* (infinitary pyramid) is (**1103** 2° and 7° **G.**) one third of the product of its base by its height; but the parallel section half-way between the base and the apex is equal to the fourth part of the base, since the sides or other homologous lines of that section are halves of those of the base and the areas are as the squares of the homologous sides, that is, :: 1 : 4 when the sides are :: 1 : 2. Therefore in this case the base plus 4 times the section between the base and the apex is equal to twice the base, and it is the same thing to multiply twice the base by one sixth of the height or to simplify the formula by multiplying the base by one third of the height.

(**132**) **Besides**, as it is shown (**1102 G.**) by the def. of the prismoid, *the frustum of a pyramid* is at the same time a prismoid and the *frustum of a cone* (frustum of an infinitary pyramid) is still a prismoid, and these frusta, supposing that their height be indefinitely increased, will at last become the very solids of which they at first formed but a part; and the formula (area A + area B + 4 area S) will always hold good, whatever may be the area of the apex or of the superior base B, and when B is but a point and consequently its area become equal to 0, the formula will become: (area A + 4 times area S) × $\frac{1}{6}$ of the height.

(**133**) **In the third place**, the solidity of the *sphere* is (**1075 G.**) equal to its area multiplied by one third of its radius: now this area is precisely equal to four of its great circles, that is to four times the area of a section of the sphere equidistant from any two opposite apices or points of its convex area; thence therefore the accuracy of the formula, since the sixth part of the height of the sphere, that is of its diameter, is one third of the radius or semi-diameter.



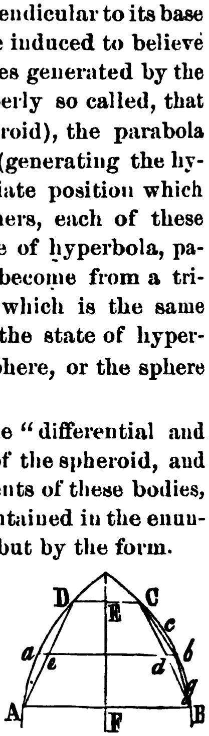
(**134**) With respect to the *hemisphere*, its solidity is equal (**1077 G.**) to the convex area by one third of the radius; but its convex area is equal to two great circles, since the area of the entire sphere is equal to 4 great circles, and we have (4 great circles × $\frac{1}{6}$ E F)=(2 great circles + $\frac{1}{3}$ EF); but area section *a*D*b* (or ED=FD)= $\frac{3}{4}$ area base AB, since $Db^2 = bF^2 - DF^2 = FB^2 - (\frac{1}{2}FB)^2 = 1 - \frac{1}{4} = \frac{3}{4}$ and as four times $\frac{3}{4} = 3$, we obtain 4 area *ab* + area AB = 4 area AB; therefore 4 area AB × $\frac{1}{6}$ EF or 2 area AB × $\frac{1}{3}$ EF = (area AB + 4 area *ab*) × $\frac{1}{6}$ EF; therefore, &c.



(**135**) **And in general**, had we to do with any *any segment* ED of the *sphere*, its solidity is equal (**1088 G.**) to the sum of the solidities of the truncated cone ED and segment BD; but the solidity of BD, that is of the solid generated by the revolution of the segment BD is (**1089 G.**) the difference between the spherical sector generated by the revolution of the sector BCD and the solid generated by the revolution of the isosceles triangle BCD; this difference is equal to (**1089 G.**) $\frac{2}{3}\pi(CB^2 - Cd^2)EF = \frac{2}{3}\pi(Cc^2 - Cd^2)EF$; but $Cc^2 - Cd^2 = Cb^2 - Cd^2 = ab^2 - ad^2$ because of *a*C being common to the rectangular triangles *ab*C, *ad*C; therefore the solidity of the solid generated by BD (and which with the truncated cone generated by the revolution of the trapezium EBDF forms the spherical segment we have to do with) $= \frac{2}{3}\pi(ab^2 - ad^2)$ EF. Now, $\pi ab^2 =$ (**1024 G.**) area circle *ab*, $\pi\, ad^2 =$ area circle *ad* and consequenly $\pi\,(ab^2 - ad^2) =$ area of the circular ring *db*. It is plain also that we may write $\pi(ab^2 - ad^2)\,\frac{2}{3}$ EF or $4\pi\,(ab^2 - ad^2)\frac{1}{6}$ EF, since $\frac{2}{3} \div 4 = \frac{1}{6}$; therefore the solidity of BD = (4 area *db*) × $\frac{1}{6}$ EF or 4 times the area of the ring generated by the revolution of *bd*, multiplied by one sixth of the height EF of the segment. But the solidity of the component truncated cone= (**112 T.**) (area base FD + area base EB + 4 times area parallel section *ad*) × $\frac{1}{6}$ EF; therefore the entire solidity of the segment of a sphere = (area base FD + area base EB + 4 area section *ab* equidistant from EB and FD) × $\frac{1}{6}$ EF; therefore, &c.

(136) In the fourth place. After having shown the accuracy of the "*general expression*" in the case of the sphere and cone, solids generated by the revolution of the circle and triangle, the two extreme sections of the cone (and the most dissimilar) the one by a plane parallel to its base, the other by a plane perpendicular to its base and passing through the apex of the cone, we are induced to believe that it will be the same by analogy, with the bodies generated by the revolution of the three other conic sections properly so called, that is: the ellipsis (generating the ellipsoid or spheroid), the parabola (generating the paraboloid) and the hyperbola, (generating the hyperboloid), and this on account of the intermediate position which these three sections occupy between the two others, each of these latter having to pass successively to the state of hyperbola, parabola and ellipsis, or vice-versa, in order to become from a triangle, a circle or from a circle a triangle ; or which is the same thing, the cone having to pass successively to the state of hyperboloid, paraboloid and ellipsoid, to become a sphere, or the sphere by the reverse process to become a cone.

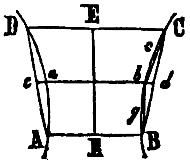
And in fact, the expressions furnished by the "differential and integral calculus" for the respective solidities of the spheroid, and parabolic and hyperbolic conoids, or of the segments of these bodies, are easily reduced to and translated into those contained in the enunciation of this article and from which they differ but by the form.



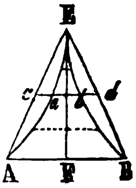
(137) Finally. It remains to demonstrate that when the *segment* AC of a spindle, for instance, *or of any other solid of revolution*, &c., is not that of a sphere, spheroid, regular conoid or cone, we none the less obtain its solidity, at least very nearly, by the formula $(E + F + 4ab) \times \frac{1}{6}$ EF. In fact we always have sol. truncated cone AC=(area E + area F + 4 area *ed*) $\times \frac{1}{6}$ EF, which generally offers a very near approximation to the required solidity.

We have again (by the formula) for the volume of the solid generated by the revolution of the segment B*b*C about the axis EF, 4 times the area of the ring, the breadth of which is *db*, multiplied by one sixth of the height EF; and, drawing the right lines *b*C, *b*B, the solids generated by the revolution of the triangles *bd*B, *bd*C, considering them as continued triangular prisms, will have for solidity the area of the ring *bd*, their common base, by half the height EF, or which is the same thing, three times the area of the annular base *db* —*ae* by one sixth of the height EF, or sol. B*b*C=3 area *bd* $\times \frac{1}{6}$ EF, which added to that of the component truncated cone AC of the solid to be computed, furnishes a new approximation still nearer than the first to the required solidity. There still remains to complete the solidity given by the formula $(E + F + 4\,ab) \times \frac{1}{6}\,EF$, the product of $\frac{1}{6}\,EF$ by once the area of the ring described by *bd*, and to cover or meet this last product we have the solids generated by the revolution of the segments *bc*C, *bg*B. Now, it is plain that the sum of these latter is to the solid generated by the segment B*b*C, in the ratio, nearly of the respective areas of the sum of the segments *b*B, *b*C to the segment BC; but these areas are to each other, very nearly, as 1 is to 4; whence it follows that the remainder (area $bd \times \frac{1}{6}\,EF$) just spoken of will sensibly correspond to the solidity of the sum of the solids *b*B, *b*C which go to make up the given segment ABCD; therefore, &c.

(**138**) **Let us remark** that the difference between the accurate solidity of the proposed segment and its approximate solidity by the formula $(E + F + 4\,ab)\ \frac{1}{6}\,EF$, is always in excess, which is partly owing to the fact that considering the solid generated by the revolution of the segment B*b*C about the axis EF as a continuous prism, (or as a solid ring having for section the segment B*b*C) with a mean length equal to the half-sum of the circumferences *ab*, *ed*, we take this length a little too great, since the continued prism in question loses more of its length in C than it gains in B; which induces us to observe also that since the solid ring generated by the revolution of the segment B*b*C is rather the continued frustum of a prism or a series of frusta of prisms, we might obtain its solidity with sufficient accuracy by making (**1095 G.**) the product of the generating area B*b*C (section of the prism by a plane perpendicular to its sides or edges) by one third of the sum of the circumferences at B, *b* and C (respective lengths of the edges of the ring or frustum) and we might still add to the accuracy of the solidity to be obtained by multiplying the generating area B*b*C of the ring by one fifth of the sum of the five circumferences at B, *g*, *b*, *c* and C or by the sum of any number of circumferences (taken at equidistant points from each other) divided by the number of these circumferences.

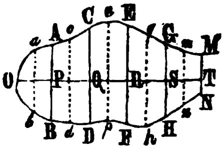


(**139**) The rule which has just been given to obtain the solidity of the segment of a solid with a convex surface, is also applicable to the *segment of a solid with a convex surface*, the same demonstration being admissable in both cases, as indicated by the letters in the figure; with this exception only that the difference between the accurate and the approximate solidities will evidently be in defect instead of being in excess, for in this case the mean length of the continued frustum of a prism or of the solid ring generated by the revolution of the segment B *b* C is less than the mean to be obtained by taking into consideration the circumferences at B and at C. We will therefore obtain very nearly the solidity of the segment AC, equal to the difference of the solidities of the frustum of a cone AC and of the solid ring produced by the revolution of the segment B*b*C, that is by making the product of the sixth part of the height EF by the sum of the areas of the bases AB, CD and four times the section *a b* half-way between those bases.

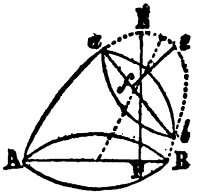


(140) The same rule will also give with sufficient accuracy in practice, the solidity of the *conoid* AEB *with a concave surface*, and often we will indefinitely add to the accuracy of the volume of the solid to be obtained by a continued subdivision of the body to be computed, into parallel segments, smaller and smaller and of equal height or thickness. However in most cases, it will not be necessary to carry the number of the subdivions beyond 3 or 5 to obtain a sufficient precision in the result.

(141) In general we will obtain very nearly the solidity of *any regular or irregular body* OT by dividing it into slices or segments, by parallel and equidistant planes. We will separately obtain by the prismoidal formula $(O + AB + 4\,ab) \times \frac{1}{6}$ OP, the solidity of each of these slices the sum of which will be the solid content of the proposed body. We will thus obtain for the solidity of the segment OAB, (area O + area AB + 4 area *ab*) $\frac{1}{6}$ OP, for the solidity of the next slice BC we will obtain $(AB + CD + 4cd)\ \frac{1}{6}$ PQ, and so on; whence it is plain that the entire volume of the solid$=O + 4\,ab + 2\ AB + 4\,cd + 2\ CD + 4\,ef + 2\ EF + \&c. + MN) \times \frac{1}{6}$ OP, &c., that is:



to the sum of the areas of the ends O, T, of the given solid, or of the exterior bases of the first and the last slices, we will add twice the sum of the other bases AB, CD,&c., of those two slices and of the other component slices, plus four times the sum of the sections *ab*, *cd*, *ef*, &c., of those slices, and then multiply the whole by the sixth part of the height OP or PQ, &c., of one of them; the result will be the solidity of the proposed body, (formula analogous to that of par. **(23, T.)** to obtain the area of any plane figure.



(**142**) It is plain also that to arrive at the solidity of *any* frustum or segment A B *ab*, of a sphere, spheroid or conoid with non parallel bases AB, *ab*, it will suffice to compute separately the volume of the whole solid AB—E and that of the partial solid *ab—e* and then take the difference of those solidities. We will thus obtain vol. AB *ba*=(area AB + 4 area intermediate section between AB and E × $\frac{1}{6}$ EF) less (area *ab* + 4 area intermediate section between *ab* and *e* × $\frac{1}{6}$ *ef*.)

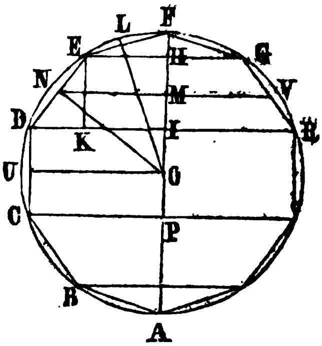
(**143**) Let us now apply this general formula to the solution of the several problems relating to it, (excepting however the prism or cylinder, the pyramid or cone, the frustum of a pyramid or cone, and the prismoid, which have already been treated of), and let us also take the opportunity of comparing, in certain cases, the results thus obtained and those furnished by the ordinary rules, so as to judge of their comparative accuracy and of the amount of labour necessary to work out the result.

PROBLEM XXXII.

To find the area of a sphere.

(**144**) **RULE I.** *Multiply* (**1071 G.**) *the circumference of one of its great circles by its diameter* AF.

RULE II. *Multiply* (**1072 G.**) *the square of its diameter or four times the square of its radius by* .7854 *and by* 4, *or at once by* 3.1416.



Ex. 1. What is the area of a sphere the diameter of which is 7 ?

Ans. 153.9384.

2. The diam. of a sphere is 24 inches ; what is its area ?

Ans. 1809.5616.

How many square inches of gilding would be required to cover a spherical ball the circumference of which is 78.54 inches ?

Ans. 78.54÷3.1416=25=diam. and 78.54 × 25=1963.4 sq. inch.

4. What is the area of the earth if the diam. is 7912 miles ?

Ans. 196,663,355.7504.

5. How many superficial feet of lead or other metal would be required to cover a hemispherical dome the diameter of which is 33 feet 4 inches ?

Ans. 33⅓ × 33⅓ × 7854 × 2=1755 square feet ; for, if the area of the whole sphere is equal to 4 great circles, it is plain that that of the hemisphere is equal to 2 great circles.

6. The vault of the apsis of a church is in the form of a quarter of a sphere the radius of which is 15 feet: required the number of yards of plastering necessary to cover its surface ?

Ans. 30 × 30 × .7854÷9=78.54 or 78½ yards ; for, since the entire sphere is equal to 4 great circles, the quarter of a sphere is equal to one.

7. What will be, at 5 pounds per square foot, the weight of a hemispherical copper boiler the circumference of which is 188½ inches ?

Ans. 188.5÷3.1416=diam. = 60 and 188.5 × 60=11310 square inches of which the half 5655÷144 = 39.27 square feet, this area multiplied by 5 (the weight per sq. foot) gives 196.35 pounds.

(**145**) **RULE III.** *Consider the surface of the sphere as a compound of continued trapeziums or zones of equal breadth AB=BC=CD=DE=EF and proceed in the manner of paragraph.* (**126 T.**)

Ex. 1. What is the area of a hemisphere the diameter of which is 263 ?

Ans. The circumference=263 × 3.1416=826,2408, the fourth part of the circumference 206.5602 divided into 5 equal parts, gives for the developed width of one of the component zones 41.31204. The intermediate diameters of these zones obtained by means of a scale of 40 units to the inch., measure respectively, computing from the base to the apex, 250, 213, 145, and 82 : the sum of these intermediate diameters plus the half (131.5) of the diameter 263 at the base, is 830.5; this sum × 3.1416 gives the sum 2609.0988 of the circumferences to enter into the computation ; this latter × 41.31204, width of one of the zones, finally gives for answer 107,787 units of area.

REM. The two first rules give each of them for area of the proposed hemisphere 108,650.66 units. The difference between these results is 863.5, 863.5÷108.650=008 nearly, whereby the rate of error is ⅘ of 1 per cent nearly. We therefore conclude that in every analogous case, it will suffice to increase by .008 or. 01, nearly the result obtained by this rule, to come very near the required area.

Ex. 2. Let it be required now to operate with 10 sections or zones instead of 5, the diameter of the hemisphere remaining the same?

Ans. The 9 intermediate diameters being as follows: 260,250, 234, 213, 186, 154, 119, 82 and 42; their sum + 131.5 (half the diameter 263 at the base) is 1671.5, this sum × 3.1416 = 5251.1844, sum of the circumferences to be used as an element of the proposed computation; the breadth of one of the component zones will in this case be $\frac{1}{10}$ of the quarter of the circumference, that is 862.2408 ÷ 4 ÷ 10 or at once by 40 = 20.65602; but, 5251.1844 × 20.65602 = 108,468.57. It has already been seen that the accurate result is 108,650.66; the difference of these results is now but 182, equal to .0017, that is: the deficiency is no more than $\frac{1}{6}$ of 1 per cent.

This rate of error added to the result of any other analogous operation would therefore give a pretty near approximation to the truth.

Ex. 3. Let us see now in how far the precision of the result will be added to, by working out the solution of the same problem, by means of an additional number of subdivisions, say 20 for instance.

Ans. The intermediate diameters are 262, 260, 256, 250, 243, 234, 224, 213, 200, 186, 171, 154, 138, 119, 101, 82, 62, 42, 21; the sum of the intermediate diameters + the half-diameter at the base = 3349.5; multiplying by 3.1416 and by 10.32801 (breadth of one of the sections) we will obtain 108,679.5 against 108,650.66 the true area. The difference is in this case in excess instead of less than the required area, as it should be (**126**) and as it would be in fact if we had computed the intermediate diameters of the component zones instead of obtaining them graphically or mechanically, as has been done with the aid of a small diagram on paper and a scale of equal parts. This difference or excess is but of 29 units in 108, 650, say .0027 or less than $\frac{1}{40}$ of one per cent; it is due to our neglecting, in measuring the intermediate diameters, the fractions of units which if needed could be taken into consideration; but it may be admitted that in practice a result which, like this, deviated from the truth but by $\frac{1}{3000}$ in excess or deficiency, would be equivalent to perfect accuracy.

(146) REM. If we put this third rule among those that may be used to determine the area of a sphere or part of a sphere; it is not that we should think proper to apply it to arrive at the area of a sphere properly so called or at the solution of any anologous problem that can be solved by more simple and direct rules; but it is because in practice, it is rare enough that we have to deal with a perfect sphere, part of a perfect sphere, a spheroid or part of a spheroid properly so called, a true paraboloid or hyperboloid, or in general with a solid of revolution, the generating curve of which is an exact section of a cone, such as the circle, the ellipsis, the parabola and hyperbola. It is therefore evident that in all cases in which we might not have to operate on a perfect spheroid or conoid, or the kind of which could only be established but by much preliminary labour, it would be better to proceed immediately by Rule III than to have recourse to another rule which did not accurately apply to the proposed problem.

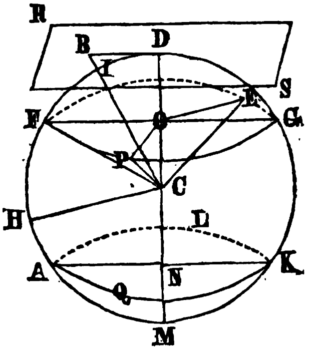
(147) Let us add that *if the surface to be measured, instead of being every where of equal curvature, as that of the sphere, were, as that of the paraboloid, &c., of unequal curvature,* we might, before proceeding to the subdivision into zones of equal breadth, first divide the area to be computed in two or several parts which would afterwards be subdivided into a less or greater number of zones according to the lesser or greater curvature in the corresponding part of the generating arc. One might then calculate separately the parts of unequal curvature and afterwards take the sum of those parts.

(148) *Usually also, the measurer or geometrician, in considering the degree of precision to be brought to bear in the practice of the details of his art, will not lose sight of the importance of not devoting to the solution of a problem, a labour and time which would not be justified by circumstances.* It would for instance be idle, we may even say unjust, that to establish to within a millionth, thousandth, hundredth or any other unit near of the accurate result, a proposed area or volume, one should devote to it a time which would cost those interested more than a fraction of the value of such unit. We say "usually;" for it is plain that there may be circumstances, either in a question or cause in litigation where the cost of doing justice to the parties may be more and in fact is often more, in an unlimited proportion, than the value at stake.

PROBLEM XXXIII.

To find the solidity of a sphere. (See the *tableau.*)

REM. A plane or plane surface RS touches the sphere but in one single point D; therefore the areas of the opposite and parallel ends or bases D, M are each of them null or=0, which reduces the formula in the case of the sphere to multiplying 4 times the area of a great circle, that is, of a section passing through the centre C, by the sixth part of the height DM perpendicular to that section.



(**149**) **RULE I.** *Multiply* (**1075 G.**) *the area by one third of the radius.*

RULE II. *Cube* (**1103**, 10°) *the diameter and multiply the number thus found by* $\frac{1}{6}\pi$ *; that is, by* 0.5236 *or the solidity of a sphere the diameter of which is* 1 *;* for (**1084 G.**) the solidities or volumes of any two spheres are as the cubes of their diam.

RULE III. *Multiply* 4 *times the area of a section of the sphere equidistant from its opposite ends or apices by the sixth of the height perpendicular to that section.* This rule, in the case of the sphere, is evidently analogous to the first, for the area of the sphere is equal to 4 great circles, the great circle is the section of the sphere by a plane passing through the centre C, that is, equidistant from two opposite points D, M, of its surface, and the 6th of the height DM is but the sixth of the diameter or the third of the radius.

Ex. 1. What is the solidity of a sphere the diameter of which is 12 ? **Ans.** $\overline{12 \times 12 \times 12} \times .5236 = 904.7808$.

2. If the mean diameter of the earth is 7918.7 miles, what is its solidity in cubic miles ?

Ans. $(7918.7)^3 \times .5236 = 259{,}992{,}792{,}082.6374908$ cub. m.

3. The top of a steeple is terminated by a spherical ball the diameter of which is $2\frac{2}{3}$ feet, ; what is its solidity ?

Ans. $2\frac{2}{3} \times 2\frac{2}{3} = 7\frac{1}{9} = 7.1111111$, $7 \times 2\frac{2}{3} = 18.6666666$, $2\frac{2}{3} \times \frac{1}{9}$ or $2.6666666 \div 9 = 2962962$, $18.6666666 \times .2962962 = 18.9629629 = (2\frac{2}{3})^3$, and $18.9629629 \times .5236 = 9.9290074$ cubic feet.

4. What is the solid content of a cannon ball having a diameter of 10 inches ?

Ans. $10^3 = 1000$, and $1000 \times .5236 = 523.6$ cubic inches.

5. How many cubic inches of gunpowder to fill a shell the interior diameter of which is 12 inches ?

Ans. $12 \times 12 \times .7854 \times 4$ or $12 \times 3.1416 = 452.3904$ = area of the sphere and that area $\times \frac{1}{3}$ radius or $\frac{1}{6}$ diam., that is, by $2 = 904.6808$ cubic inches.

6. How many cubic feet of air may be contained in a buoy of a spherical form with an int. diameter of 10 feet ?

Ans. 523.6 cubic feet.

7. A stone ball is 3 feet diameter ; what is its weight at 150 pounds per cubic foot ?

Ans. $3 \times 3 \times 3 \times .5236 \times 150 = 2120.58$ pounds.

8. How many gallons of liquor (231 cubic inches per gallon) may be contained in a hemispherical boiler 10 feet diameter ?

Ans. The content of the vessel in cubic feet $= 10^3 \times .5236 \div 2 = 261.8$, the number of gallons per cubic foot $= 1728$ cubic inches $\div 231 = 7.4805195$, say $7\frac{1}{2}$, and $261.8 \times 7\frac{1}{2} = 1963\frac{1}{2}$ gallons, or more correctly $261.8 \times 7.48 = 1958.26$ gallons.

9. A hemispherical vault of the uniform thickness of one foot, measures 10 feet interior diameter ; how many bricks have been required to built it, at 20 bricks per cubic foot ?

Ans. It is plain that the required solidity is equal to the difference of the solidities of the exterior and interior hemispheres ; but, the exterior hemisphere $= 12^3 \times .5236 \div 2 = 452.39$ cubic feet, the interior hemisphere $= 10^3 \times .5236 \div 2 = 261.8$ cubic feet, the difference of these solidities is 190.59 cubic feet and $190.6 \times 20 = 3812$ bricks.

10. The thickness of a bomb is 5 inches and its exterior circumference 62.83 inches ; what is its weight, at 480 pounds per cubic foot ?

Ans. We have for the exterior diameter of the bomb $62.83 \div 3.1416 = 20$ inches ; therefore the diam. of the hollowed part is 10 inches ; now the solidity of the bomb is the difference of the solidities of the exterior and interior spheres. The sol: of the exterior sphere $= 20^3 \times .5236 = 4188.8$, the int. sol. $= 10^3 \times .5236 = 523.6$, the difference of these solidities is 3665.2 cubic inches ; now, 1 cubic foot or 1728 cubic inches : 480 pounds weight :: 3655.2 cubic inches : 1018 pounds weight.

PROBLEM XXXIV.

To determine the convex area of a spherical segment or of any spherical zone [1]

(See the *tableau.*)

(150) RULE I. *Multiply* **(1073 G.)** *the height oC, OO of the*

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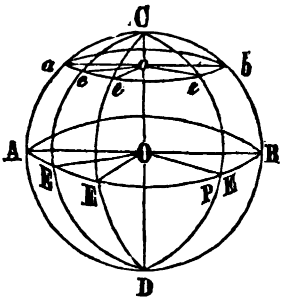
[1] The spherical segment is any part *aebC*, of the sphere, cut off from the whole sphere by a plane of section *aeb*, a small circle of the sphere.

The spherical zone is any part *aeb*–AEB of the sphere comprised between two parallel planes *ab*—AB. It is, as the case may be, lateral, central, excentric·

segment, or the height Oo of the zone by the circumference of a great circle of the sphere ; the product will be the required area.

REM. If the diameter of the sphere is not given it may easily be found by the method of par. (**540 G.**) by dividing the square of radius of the base of the segment by the height, to obtain the remainder of the diameter ; the remainder thus found + the given height will be the required diameter of the sphere.



Ex. The diameter of a sphere being 42 decimetres, what is the convex area of a segment the height of which is 9 decimetres ?

Ans· 42 × 3.1416 = circ. 131.9472 which × 9= 1187.5248 square decimetres..

2. The radius of the base of the roof of a lantern in the form of a spherical segment, is 10 feet, the height of the roof is 4 feet. How many superficial feet of lead or other metal would be required to cover it ?

Ans. $10^2 \div 4=25$, $25 + 4 =$ diam. of the sphere$=29$, $29 \times 3.1416=$ circ. 91.1064, now 91.1064=4364.4256 square feet.

3. Required the area of the lid of a boiler in the form of a spherical segment the circumference of which is 91.1 inches and height 10 inches ?

Ans. $91.1 \div 3.1416 = 29=$diam. of the lid the radius of which is consequently 14.5 inches ; to obtain the diam. of the sphere of which the segment forms a part, we have $(14.5)^2 \div 10=21.025=$the remainder of the diam. of which the height of the lid forms a part ; therefore the required diam. $= 21.025 + 10 = 31.025$, this diam. × $3.1416 = 97.46814$ $=$circ. of a great circle, this latter × 10 gives 974.6814 for the required convex area in square inches.

4. A hemispherical dome, a segment of which has been taken off to lay the base of the lantern which crowns it, presents consequently the form of a spherical zone or of a spherical segment with two bases ; it is required to determine its convex area, its height being 9 metres and diameter of the sphere of which it forms a part 20 mètres ?

Ans. $20 \times 3.1416=$circ. 62.832 and $62.832 \times 9 = 665.488$ square metres.

(151) REM. If the radius or diam. of the sphere of which a zone forms a part is not known and that the only data be the radii or diameters of the inf. and sup. bases of the segment and the heightor perpendicular distance which separates them, paragraph **(574 G.)** may furnish the method of arriving at the required radius ; but one may attain it quite as well and in a more expeditious manner and accurately enough in practice by a simple graphic process which would allow of to determining at once the required radius or diam. of the sphere, with the same scale made use of to fix on the paper the relative proportions and positions of the data, the centre of the circle being then easily found by repeated trials on the perpendicular (prolonged if necessary) which unites the centres of the two given chords.

5. The diameters of the inf. and sup. bases of a roof in the form of the segment of a sphere measure respectively 16 and 12 metres and the height 2 metres ; what is the area of the zone forming the lateral or convex surface of the roof ?

Ans. We obtain (**574 G.**) either by calculation or by construction the diameter 20 of the sphere of which the segment forms part. This diam. gives for circumference 62.832, this circ. × 2, height of the roof, gives for its convex area 135.664 square metres.

6. What is the convex area of a segment $21\frac{2}{5}$ inches high taken off from a sphere 6 feet diam ?

Ans. 4840.577 square inches.

7. If the diameter of the earth considered as a perfect sphere is 7970 miles, the height of the frigid zone must be 252.361283 miles ; what is its area ?

Ans. $7970 \times 3.1416 \times 252.36128{,}761 = 6{,}318{,}761$ square miles.

8. What is the area of one of the 10 component sections or compartments of a vault or dome in the form of a spherical segment, the interior diameter of the segment or of its base being 40 feet and its height 10 feet ?

Ans. The remainder of the diameter of the sphere of which the height 10 of the segment forms a part is (**539 G.**) $(\frac{1}{2}40)^2 \div 10 = 40$ and the entire diameter consequently $= 40 + 10 = 50$, the circumference $= 50 \times 3.1416 = 157.08$ and the entire area of the segment $= 157.08 \times$ the height $10 = 1570.8$; therefore the area of the proposed section is $1570.8 \div 10 = 157.08$ square feet.

(152) RULE II. *Divide the surface to be computed into zones of equal breadth, and proceed afterwards in the manner of par.* **(126 T.)**.

Ex. 1. The infer. circumference of a spherical zone, or which appears to be one, measures 260 feet, its sup. circumference 213 feet, and two intermediate circumferences equidistant 250 and 234 feet, the length of the generating arc is 15 feet, and the developed breadth of one of the three component zones is consequently 5 feet; what is the area of the entire zone?

Ans. $\frac{1}{2}260 + 250 + 234 + \frac{1}{2}213 = 720.5$, this sum × 5 = 3602.5 square inches nearly.

2. The vault or arched ceiling of a circular room in the form of a spherical segment has for its inf. diam. 186 decimetres, and for the intermediate diam. of five component zones 154, 119, 82 and 42 decimetres, the length of the generating curve, that is, the curvilineal distance from the centre of the vault to its spring is 103 decimetres 28 millimetres; what is its concave area?

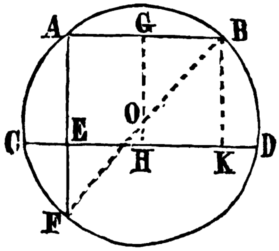
Ans. 103.28 decimetres $\div 5 = 20.656$ = breadth of one of the component zones, $\frac{1}{2}$ inf. diam. $= 186 \div 2 = 93$, $93 + 154 + 119 + 82 + 42 = 490$, $490 \times 3.1416 = 1539.384$ sum of the circumferences to enter into the computation, now, $1539.384 \times 20.656 = 31{,}797.5$ square decimetres or 317 square metres 97½ square decemetres, since the square metre is $10 \times 10 = 100$ square decimetres and that by putting the decimal point 2 places back we divide by 100.

PROBLEM XXXV.

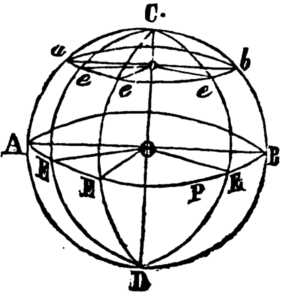
To determine the solidity of a spherical segment or of any spherical zone.—(See the models of the *tableau*.)

(153) REM. 1. The spherical segment *a e b* C or *a e b* D (See the figure of the paragraph (**156**) may be smaller or greater than a hemisphere or equal to a hemisphere if the plane of section passes through the centre O of the sphere. In every case the general formula gives its accurate solidity. In the same way, the spherical zone may be lateral, central or excentric. We will call it lateral when it is the zone of a hemisphere like the one which in the figure is comprised between the planes of section, parallel circles AEB, *a e b*. It may be central if its planes of section, opposite or limiting bases, are equidistant from the centre O of the sphere, and excentric, if these bases are unequally distant from the centre.

(154) REM. II. To obtain in the spherical segment the diameter of the central or half-way section, it suffices to remember (**539 G.**) that the half-chord *a o* (see the figure of paragraph (**156**) is a mean proportional between the versed-sine *o* C height of the segment and the remainder *o D* of the diameter. Let therefore AEB–C any segment of a sphere, and *a e b* its half-way section, that is such that we may obtain $Oo=oC=\frac{1}{2}OC$; then as OC, height of the solid, is known, we will obtain $oC=\frac{1}{2}OC$ and we will find *oa* or $ob=\frac{1}{2}\,ab=\sqrt{o\,C \times o\,D}$. Let again *a e b* D the segment to be measured, and AEB its half-way section passing through a point O half-way between *o* and D. Knowing *o* D and consequently $OD=\frac{1}{2}\,o\,D$, we will obtain OB or $OA=\frac{1}{2}AB=\sqrt{OD \times OC}$, or by measuring directly the required diameter of the body to be computed.



(155) REM. III. To obtain in the spherical zone ABDC, for instance, the diam. of its intermediate section ; we will at first find, if it is not already known, the radius OB or OF of the sphere of which the zone to be computed forms a part. To this effect (**574 G.**) the plane figure ABCD being the vertical section of the spherical segment in question, we will obtain $EF=\overline{CE \times ED}\div AE$, then $AF \div AE$ or $GH+EF$ and diam. $BF=\sqrt{AB^2+AF^2}$. The radius OB being now known$=\frac{1}{2}BF$, we will obtain $OG=\sqrt{OB^2-BG^2}$ or $OH=\sqrt{OC^2CH^2}$, or after having found OG or OH we will obtain $OH=GH-OG$ or $OG=GH-OH$; now, if we suppose the line GH prolonged on both sides to the circumference at XY, we will obtain $GX,=OB-OG$ and $HY=OB-OH$ and thence we will easily obtain, as in **REM. II**, the intermediate diam. half-way between AB and CD.



(156) RULE I. *Multiply* (**1088 G.**) *the half-sum of the areas of the parallel bases by the height of the segment ; add to this product the solidity of a sphere the diameter of which is equal to the height of the segment: the sum of these two solidities willl be the required volume.*

REM. When the segment has but one single base, the other is considered =0.

RULE II. *To the sum of the areas of the inf. and sup. bases of the segment, add 4 times the area of a* *section equidistant from those bases, and multiply the whole by the sixth part of the height; the result will be the required solidity* (**135 T.**)

Ex. 1. What is the solidity of a segment forming part of a sphere the diameter of which is 40, the respective distances from the centre to each of the planes of section being 16 and 10 ?

Ans. We must first determine the areas of the parallel bases of the given segment ; but, the diameters of these bases are parallel chords of a great circle of the sphere, distant from the centre of the circle, the one by 16 and the other by 10 units of measure, the segments of the diameter of the great circle perpendicular to these chords are respectively, of one of them, 16+20=36 and 40—36=4, of the other, 10+20=30 and 20—10=10 ; now we have (**540 G.**) 36 × 4=144 = the square of one of the half-chords and 30 × 10=300 = the square of the other half-chord ; these squares multiplied each by .7854 and by 4 or at once by 3.1416, give 452.3904 and 942.48 for the required areas of the parallel bases. The sum of these areas =1394.8704, this sum × 3, the half-height (16—10) of the segment, or the half-sum of these areas × 6=4184.6112=part of the required solidity ; the remainder of the required solidity = 6^3 × .5236=113.0976 =sol. of a sphere the height of which is 6. These two solidities united give 4297.7088 for the solidity of the proposed segment.

2. The same example by Rule II gives for area half-way between the parallel bases 33 × 7=231=the square of the radius of the base or intermediate section, this square × 4 gives the square of the diam. of such base or section, and this last square × .7854 gives its area=725.7096, 4 times this area=2902.8384 to which adding the sum of the areas of the bases we obtain 4297.7088 for the required solidity, for $\frac{1}{6}$ height=1 and multiplying by 1 does not alter the value of the multiplicand.

3. How many cubic feet of liquor may a hemispherical boiler 10 feet diameter contain ?

Ans. We have seen (**134. T.**) that in the hemisphere the area of the intermediate section equidistant from the base and apex of the solid is equal to the $\frac{3}{4}$ of the area of the base or of a great circle of the sphere ; now, we have for area of the sup. base of the boiler 10 × 10 × .7854=78.54 square feet ; but 4 times $\frac{3}{4}$ =3 and three times 78.54 + 78.54=4 times 78.54=314.16 and 314.16 × $\frac{1}{6}$ height= 314.16 × 5÷6=261.8 cubic feet.

(157) REM. In the case of the hemisphere, as in the entire sphere, Rule II does not offer any advantage, and on the contrary, it gives more work, since it is more simple to arrive at the required result, to cube at once the diameter, multiply this cube by .5236 and take half the product for the solidity of the hemisphere.

4. How many gallons of water may find room in a reservoir in the form of a spherical segment with a 100 feet diameter, and 20 feet deep, at $7\frac{1}{2}$ gallons per cubic foot ?

Ans. By the first rule, we obtain the required vol. = area of the base of the segment (that is, the sup. area of the reservoir) × the height (vertical depth of the reservoir) ÷ 2, plus the vol. of a sphere having for its diameter such height; that is, the required vol. = $(100 \times 100 \times .7854 \times 20 \div 2 = 78540) + (20 \times 20 \times 20 \times .5236 = 4188.8) =$ 82,728.8 cubic feet × 7.5 = 620,466 gallons.

Ans. By the second rule, we have first (**540 G.**) for the remainder of the diam. of the sphere or of the great circle of which the height of the reservoir forms a part $(\frac{1}{2}100)^2 \div 20 = 125$, $125 + 10$ (half-distance from the surface to the bottom) $= 135$, $135 \times 10 = 1350$ = rectangle of the segments of the diam. = square of the half-diam. of the intermediate section, this square × 3.1416 = 4241.16 = area interm. section, 4 times this area + the area of the base of the segment = 24.818.64, this sum × 20 ÷ 6 = 82,728.8 cubic feet, as before.

(**158**) **REM.** The choice to be made between the two rules for the solution of this problem will sometimes depend on the nature of the data, but especially on the doubt that might exist as to the particular kind of the figure to be computed, and the use of this formula will dispense with the necessity of enquiring first of all as to the exact nature of the proposed solid. Thus, if the reservoir to be measured were the segment of a spheroid, hyperboloid, or any other figure resembling nearly those just enumerated, rule II would in any case give its accurate solidity (**127 T.**), or very nearly, while if we treated as part of a sphere proper a figure which were not such and calculated it by the rule applicable to the sphere, we might be deeply mistaken in the result.

5. A basin the form of which seems to be that of spherical segment, has for its sup. diam .15 inches, for half-way diam., 12 inches, and for depth or height 7 inches ; what is its capacity in gallons of 231 cubic inches ?

Ans. Sup. area = 15 × 15 × .7854 = 176.715 square inches, intermediate area = 12 × 12 × .7854 = 113.0976, area base + 4 intermediate area = 629.1054, this sum × 7 ÷ 6 = 734 cubic inches nearly ; dividing by 231 we obtain 3.18 or $3\frac{1}{5}$ gallons nearly for the capacity of the proposed vessel.

6. The vacuity or space under a dome or arched ceiling of a circular room, presents the aspect of the segment of a sphere with parallel bases the diameters of which measure respectively 19.9 metres and 8.718 metres; the diameter of the dome equidistant from its bases is 17.32 metres; required the number of cubic metres of air to be heated, the height being 8 metres?

Ans. $(19.9)^2 \times .7854 = 396 \times .7854 = 311.02$, $(8.718)^2 = 76$ and $76 \times .7854 = 59.69$, $(17.32)^2 = 300$ and $300 \times .7854 \times 4 = 942.48$, the sum 1313.19 of these areas $\times 8 \div 6 = 1750.92$ cubic metres, or which is the same thing and more simple $\overline{(19.9)^2 + (8.718)^2 + 4\ (17.32)^2} \times .7854 \times 8 \div 6 =$ sol.

7. A vessel having the form of the frustum of a cone is terminated by a bottom having the appearance of a spherical segment. The inferior diameter of the vessel is 12 feet, the intermediate diameter of the segment is 8.72 feet, and its height 2 feet; how much must be added to the content of the body of the vessel to obtain its whole capacity?

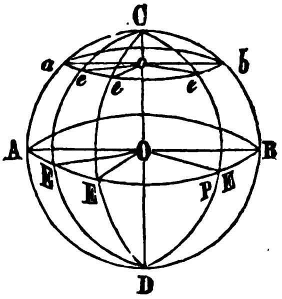
Ans. $(12)^2 + 4\ (8.72)^2 \times .7854 \times 2 \div 6 = 117.3$ cubic feet, (where we have taken $(8.72) = \sqrt{76}$; for, 6^2 or $(\frac{1}{2}12)^2 \div 2 = 18$, $18 + 2 =$ diam. of the sphere of which the segment forms a part. Now the intermediate half-diameter $= \sqrt{19 \times 1}$ or the diam. $= \sqrt{4 \text{ times } 19} = \sqrt{76} = 8.72$ nearly) and $117.3 \times 7\frac{1}{2} = 889$ gallons nearly.

PROBLEM XXXVI.

To determine the solidity of a spherical ungula, and the area of the lune which forms its base.

(See the *tableau.*)

REM. The spherical ungula is a part of the solid sphere comprised between two half great circles ADC, EDC meeting at any acute, right or obtuse angle AOE and its solidity is evidently to that of the entire sphere as its angle AOE is to 360° or as its arc AE to the entire circ. whence it evidently follows that its solidity will immediately be obtained by the following:



RULE I. *Multiply 4 times the area of the sector of a circle AOE by $\frac{1}{6}$ of the diameter CD, length or height* *of the spherical segment ;* for the areas of the opposite bases or ends C and D, are both equal to zero.

RULE II. *Get first* (**1075 G.**) *the area, then the solidity of the entire sphere of which the ungula forms a part. Divide then that area and solidity by the ratio between the angle of the ungula and* 360° ; *the result will be the required area and solidity.*

Ex. I. Required the area and solidity of the ungula of a sphere the angle of which is 60° and the diameter 10 ?

Ans. The area of the entire sphere$=$(**144 G.**) $10 \times 3.1416 \times 10=$ 314.16 units, the ratio of 60° to 360°$=\frac{1}{10}$, therefore $314.16 \div 10=$ required area$=31.416$.

The solidity of the entire sphere$=$(**149 G.**) $10^3 .5236=523.6$, this solidity divided by the ratio $\frac{1}{10}$ just established, gives 52.36 for the solidity of the proposed ungula.

2. One of the compartments of the interior vault or of the exterior roof of a dome, presents the figure of a spherical semi-lune, the diameter of the dome is 100 feet, and its circumference is divided into 16 parts or sections by ribs drawn from the apex to the springing ; required the area of one of the component semi-lunes ?

Ans. The entire area of the sphere of which the dome forms a part$=100 \times 3.1416 \times 100=100^2 \times 3.1416=10000 \times 3.1416=31.416$ square feet, this area divided by 32, since there are 32 semi-lunes in the entire area, gives for the required area $981\frac{3}{4}$ square feet.

(**160**) **RULE III.** *Multiply the length of the arc which measures the breadth of the lune by the diameter of the sphere of which it forms a part ; the product will be the required area. The area thus obtained (or as established by the first rule) multiplied by one third the radius will give the required solidity.*

Ex. 1. How many square metres of silk are there in one of the component sections of a spherical balloon of which the diameter is 10 metres, and the number of the component breadths 36 ?

Ans. The whole circumference of the balloon being $10 \times 3.1416 =31.416$ metres and the number of compartments 36, it follows that the breadth of the gore will be $31.416 \div 36=.872\frac{2}{3}$ metres, then, $.872\frac{2}{3} \times$ diam. $10=8.72\frac{2}{3}$ square metres$=$required area.

2. There is to be replaced one of the 10 component ungulae of a wooden ball 30 inches diameter, required the solidity and convex area of the ungula.

Ans. The circ. of the ball$=30 \times 3.1416=94.248$ whence it follows that the breadth of the ungula$=94.248 \div 10=9.4248$, this breadth $\times$

diam. 30 gives for the area of the ungula 282¾ square inches. The solidity=the area × one third the radius=282.744 × 15÷3=282.744 × 30÷6=282.744 × 10÷2=2827.44÷2=1413¾ cubic inches or 1413.72 ÷1728 (number of cubic inches in a cubic foot)=.82 nearly of a cubic foot, let the four fifths of a cubic foot.

3. Required the number of toises (87 cubic english feet per toise) of masonry in one of the 8 compartments of a hemi-spherical vault of which the int. diameter is 30 feet and the thickness of the vault 3 feet ?

Ans. It is plain (**1083**)' that we will obtain the required solidity by taking the difference of the component semi-ungulae of the interior and exterior hemispheres of the proposed vault. Now, the int. diameter being 30, the solidity of the sphere=30^3 × .5236= 14137, the sol. of the ext. sphere=36^3 × .5236=24429, the difference (24429—14137=10292) of these solidities divided by the number (16) of the component semi-ungulae of the entire sphere, gives for the solidity of the compartment 643¼ cubic feet, dividing this latter number by 87 we get 7 toises 34¼ cubic feet.

Or, approximatively, by multiplying the half-sum of the ext. and int. areas of the compartment by the thickness of the vault; we have area of the int. sphere 30 × 30 × .7854 × 4 or 30^2 × 3.1416 =2827.44 of which the half 1413.72 is the interior area of the entire vault, the area of the ext. sphere=36^2 × 3.1416=4071.5136 of which the half 2035.7568 is the exterior area of the entire vault, the sum 3449.4768 of these areas÷8 is the sum of the ext. and int. areas of the section of the vault to be measured, and this latter sum 431.1846 × 1½ (half-thickness of the vault) or half that sum multiplied by the entire thickness of the vault, gives for the cubical content of the compartment 646¾ cubic feet, or 7 toises 37¾ cubic feet.

(161) REM. We say "approximately," and in fact, the solid to be measured is nothing but the frustum of a spherical pyramid, comprised between parallel bases. The spherical pyramid, like the ordinary pyramid, has for its solidity (**1082 G.**) the third of the product of its base by its height; but, were it true that one could arrive at the solidity of a frustum of a pyramid by multiplying the half-sum of its parallel bases by the height of the frustum, it would happen also that one would correctly obtain the solidity of the entire pyramid equal to the half product of its base by its height ; for if it be supposed that the height of the frustum increases indefinitely, this height must at last become equal to that of the entire pyramid, and its superior base will by the fact cease to exist or become equal to 0 ;

in that case the half-sum of the opposite bases will be the half-base of the pyramid, and the rule would then give for the solidity of the pyarmid, the half-product of its base by its, height ; but the solidity of the pyramid is on the contrary the third of the product of its base by its height ; and the difference between $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{1}{6}$; therefore the error of the approximative method might in an extreme case reach $16\frac{2}{3}$ per cent. In the above example the error in excess is but $3\frac{1}{4}$ feet for 643 feet or $\frac{1}{2}$ per cent nearly, and would be still less if the the diameter of the vault were greater relatively to its thickness, or which is the same thing, if the height or thickness of the frustum to be measured formed a smaller part of the entire height of the pyramid of which the frustum forms a part.

PROBLEM XXXVII.

To find the solidity of a spherical sector.

(See the models of the *tableau.*)

(162). REM. I. The sector or spherical cone is, as indicated by its name, any part of the solid sphere comprised between and having for its base a spherical segment and for its lateral wall the surface generated by the revolution of the radius of the sphere about the circumference of the smaller circle of the sphere serving as a base to the segment. We may consider the spherical cone under two aspects and measure it in consequence ; 1° as a sperical sector or as an infinitary spherical pyramid to obtain its solidity by adding to the area of its spherical base 4 times the area of the imaginary spherical base parallel to the first and situated half-waw between the exterior surface and the centre or otherwise ; between the base and apex, as for an ordinary pyramid or cone. Now it is evident that as in the case of the pyramid and cone properly so called or with plane bases, the area of the half-way section is equal to the fourth part of that of the base, which reduces the method of cubing the cone or the spherical sector to that enunciated in the above given rule. 2° as a compound of a spherical segment and a right cone which may be measured separately by the rules already given to take afterwards the sum of the component solidities. This method evidently dispenses with the necessary knowledge to arrive at the convex area of the base of the sector to be measured and reduces the whole work to that of obtaining the respective areas of the circle answering as a common base to the component cone and segment and of the circle parallel to this latter and situated half-way between it and the apex of the segment.

(163) REM II. Let us say also, respecting the spherical cone that if it were required to cube any frustum of a spherical cone comprised between parallel bases, such as the section of a shell for instance or the vault of a circular room of uniform depth, we would arrive at it, as in the case of the frustum of an ordinary cone by adding to the sum of the convex aud concave areas of its parallel bases 4 times the area of a section parallel to the bases and half-way between them, and afterwards multiplying the whole by the sixth part of the height of the frustum; or by computing the respective solidities of the component spherical cones, then to take their difference.

(164) RULE. *After having established by the method of problem* 34 *the area of the base of the sector , we must multiply* (**1077 G.**) *that area by one third of the radius to get the required solidity.*

Ex. 1. The height of the segment, forming (**975 G.**) the base of a spherical sector, is 1½ metres, and the radius of the sphere of which the sector forms a part is 5 metres; what is the solidity of the sector ?

Ans. The area of the base=circ. of a great circle × the height of the segment, the circ.=diam. 10 × 3.1416 = 31.416 × 1.5=47.124 square metres, this area × ⅓ radius or by 5÷3=78.54 cubic metres.

2. What is the solidity of a buoy having the form of a spherical sector, the length of the side being 10 feet and the diameter of the base 5 feet ?

Ans. With these data we obtain first the height of the segment= $10-\sqrt{10^2-2.5^2}=10-9.6825=.3175$ of a foot, the circ.=diam. 20 × 3.1416=62.832 which × .3175=19.94916 square feet=area of the convex base, this latter × 10÷3=66.497 cubic feet.

3. A circular tower of which the int. diam. is 30 feet, has for its cut stone vault the frustum of a sector with parallel bases, the thickness of which is 5 feet, the height of the cap of the vault is 10 feet; what is the concave area and the solid content of the vault ?

Ans. The solidity of the frustum is (**1083 G.**) equal to the difference of the component entire and partial sectors=ext. area or of the extrados × ⅓ R, less the int. area or of the intrados × ⅓ *r* where R. and *r* are the respective radii of the ext. and int. spheres of which the sectors of the same name form a part; now, we first obtain (**540 G.**) for the remainder of the diameter of the great circle of which the height of the vault forms a part and of which the diameter of the vault is a chord, $15^2 \div 10$ (the square of the semi-chord÷the versed sine, that is, the diam. of the vault÷its height) $=225\div10\div22.5$; then we obtain the diameter$=22.5+10=32.5$ and the radius$=16.25$, and the depth of the vault being 5 feet, we obtain for the radius of the extrados $16.52+5=21.25$; now, we will obtain the interior area of the vault by making the circumference 102.102 $(=3.1416\times32.5)$ and by multiplying it by the height 10, which will give 1021 square feet for the required area.

We will obtain (**1074.2° G.**) the area of the extrados by making $r^2 : R^2 ::$ int. area : ext. area or $16.25^2 : 21.25^2 :: 1021 : x$, let $264 : 452 :: 1021 : x=1748$; finally the required solidity=ext. area $\times\frac{1}{3}$ R— int. area $\times\frac{1}{3}r=(1748\times21.25\div3)-(1021\times16.25\div3)=12382-5530=6852$ cubic feet of cut stone.

REM. I. The approximate rule in question in the *rem.* to last problem, would give in the actual case $\frac{1}{2}(1718+1021)+5=6922$ that is an excess of 70 cubic feet, the error being consequently $1\frac{1}{20}$ per cent.

4. A reservoir, the lateral wall of which is the zone of a sphere and the bottom a plane surface, is lined in all its concave surface with a thickness of eight inches brick masonry radiating to the centre of the sphere of which the reservoir is a segment. The superior diameter of the reservoir, which is at the same time that of the sphere is 100 feet and the depth of the reservoir or height of the zone is 20 feet. Required the number of bricks in the frustum of a spherical sector formed by the lateral lining of the basin ?

Ans. The circ. of the int. sphere or of a great circle is $100\times3.1416=314.16$, this circ. × the height 20 of the interior zone, gives for the area of that zone 6283.2 square feet, and the solid sector of which that zone is the base or convex area is $6283.2\times\frac{1}{3}r=6283.2\times50\div3=104{,}720$ cubic feet, the area of the ext. zone of the brick lining is obtained **1074. 2° G.**) by making $100^2 : 101\frac{1}{3}^2 :: 6283.2 : 6451.8687$, this latter $\times\frac{1}{3}$ R or by $\frac{1}{3}(101\frac{1}{3}=108{,}964.894$ cubic feet = vol. of the ext. solid sector, the difference 4244.894 of the int. and ext. sectors is the solidity of the lining in cubic feet, multiplying by 20 we obtain 84,898 for the number of bricks employed in the work.

REM. II. In this latter example, the sum of the ext. and int. parallel areas of the lining is 12735.0687, this sum × the half-thickness, 4 inches, or by $\frac{1}{3}$ of a foot, gives 4245.0229 cubic feet, $\times 20=84900\frac{1}{2}$ bricks, or a difference de $2\frac{1}{2}$ bricks only in the result; proving thus, as already stated, that with a very small thickness relatively to the radius, we obtain very nearly the solidity of the frustum of a spherical sector, by multiplying its height by the half-sum of its parallel bases. However, with regard to the amount of work entailed by the two modes of calculation, the second method offers no advantage over the first which it is therefore better to employ in all cases.

REM III. We may also in practice (and this is what is sometimes done) when the thickness of a vault is uniform and its radius of curvature relatively great, simplify the operation and arrive at a sufficiently approximate result by multiplying at once the int. or ext. area of the vault by its thickness. In the last example this manner of proceeding gives, by using the area of the intrados of the brick lining, 6283.2 × 8 inches or by the $\frac{2}{3}$ of a foot = 4188.8 cubic feet × 20 = 83776, this result is deficient by 1122 bricks or $1\frac{1}{4}$ per cent. If on the contrary we take the ext. area 6452 × $\frac{2}{3}$ we have 4301 cubic feet, or 86,020 bricks, result which is in excess of the truth by 1122 bricks or $1\frac{1}{4}$ per cent as before.

PROBLEM XXXVIII.

To find the area of a spherical triangle[1]

(See the *tableau.*)

(165) RULE I. *Compute first the area of the sphere of which the triangle forms a part, and divide the area by* 8 *to obtain* (**1193 G.**) *that of the tri-rectangular triangle.*

Compute afterwards (**1200 G.**) *the sum of the three angles, from it subtract* 180° *and divide the remainder by* 90° *; multiply then the quotient by the tri-rectangular triangle and the result will be the required area.*

RULE II. *Multiply, as for the triangle with a plane surface, the developed length of the base by the developed height perpendicular to that base ; the result will be the area nearly of the proposed triangle.*

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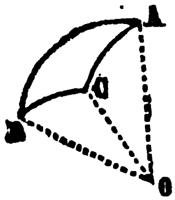
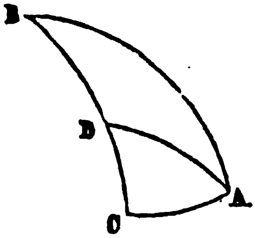
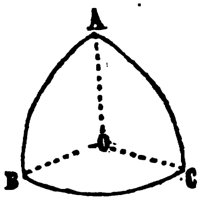
[1] We will find among the models of the tableau, pyramids and frusta of spherical pyramids the bases of which present the acute-angled, right-angled and obtuse-angled spherical triangle, including the tri-rectangular spherical triangle in question.







Ex. 1. Required the area of a triangle described on a sphere the diameter of which is 30 feet, the angles being 140°, 92° and 68° ?

Ans. The area of the entire sphere=diam. 30 × 30 × .7854 × 4= 30^2 × 3.1416=2827.44 square feet of which ⅛=353.43=area of the tri-rectangular triangle which must enter as an element into the calculation to be made. The sum of the three angles is 300°, 300°—180= 120°, 120° ÷ 90°=1⅓ and 1⅓ times the area 353.43 of the tri-rectangular triangle gives 471.24 the required area.

2. The angles of an equilateral spherical triangle are each of 120°, and the diam. of the sphere of which the triangle forms a part is 20 metres ; what is the area of the triangle ?

Ans. 20^2 × 3.1416÷8=157.08=tri-rect. triangle area, the sum of the angles=360°, 360° — 180°=180, 180° ÷ 90°=2 and 157.08 × 2= 314.16 square metres.

3. One of the 8 compartments of the surface of a dome or vault in the form of a hemisphere is an isosceles spherical triangle of which each of the angles at the base is a right angle, and the angle of which at the apex=360°÷8=45°, the length of the arc measuring the breadth of the compartment at the springing of the dome is 39.27 and the whole circumference is consequently =39.27 × 8=314.16, whence the diam. is 100 ; what is the area of the compartment ?

Ans. The whole area of the sphere of which the semi-lune to be computed forms a part=100^2 × 3.1416=31416 square units, the tri-rect. triangle=31416÷8=3927, the sum of the angles exceeds by 45° two right angles, 45°÷90=½, therefore the area or solidity=3927 ÷2=1963½=required area.

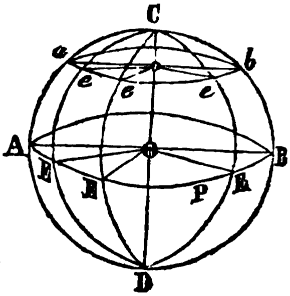
Besides, in this example where the triangle to be computed forms a known aliquot part of the entire sphere, the calculation becomes simplified and reduced to that of obtaining the area of the sphere, afterwards to take its 16th part. The example has nevertheless the advantage of showing the accuracy of the rule (the area of the entire sphere 31446 divided by 16 giving as before 1963¼ for the convex area of the proposed ungula) and indicates the manner of proceeding in any other anologous case.

4. The sum of the three angles of a triangle traced on the surface of the terrestial sphere, exceeds (**1416 G.**) by one second (1″) 180°, what is its area, supposing the earth to be a perfect sphere with a diameter of 7912 english miles ?

Ans. The area of the earth$=(7912)^2 \times 3.1416=196,663,355.75$, dividing by 8, we obtain for the area of the tri-rect. triangle 24.582, 929.47 square miles ; now $1'' \div 90° = 1'' \div 324,000$ (number of seconds in $90° = 90° \times 60' \times 60''$) $= \frac{1}{324000} = .00000308642$ nearly and the area of the tri-rectangular triangle $24,582, 919.47 \times .00000308642$ or, which is the same thing, divided by its converse 324000 gives 75.87321 for the area of the proposed triangle in square miles.

(**166**) **REM. I.** It is plain from the rule that the area of every spherical triangle of the same radius, that is, of every triangle traced on the same sphere has a direct relation to the excess of the sum of its three angles on 180°. For instance, if the spherical excess were 10 seconds instead of one, the area of the triangle would be 758.7321 square miles instead of 75.87321 ; in the same way if the excess of the three angles on 180° was but of one tenth of a second, the area of the triangle to be computed would be but one tenth of what it is for one second, viz : 7.587321. An excess of one minute would give for the area of the triangle to be computed a number of miles 60 times greater than that given by a second, that is, the 5400th part of the tri-rect. triangle, since $324000 \div 60 = 5400$ or that $90° \times 60 = 5400$; so 1° would give the 90th part of the tri-rect. triangle and so on ; whence it evidently follows that in every geodesical survey of part of the terrestrial sphere, it will suffice, after having established the area corresponding for instance to a second, or to one 10th, 100th 1000th, &c., of a second, to multiply this area by the number of seconds or tenths of a second, &c., in the excess of the sum of the three angles of any triangle on 180°, to obtain at once the area of such triangle, and we have seen (**1415** 3° **G.**) the manner of establishing if required such spherical excess.

REM. II. It is plain that if the spherical triangle ACP, for instance, is the tri-rectangular triangle or the 8th part of the sphere and it be divided into any number of equal parts, that is, of isosceles spherical triangles equal to each other, ACE, ECE, &c., all the angles at C will be equal to each other and the arcs AE, EE also equal to each other ; the areas of all these triangles will consequently be equal. Let then the angle ACE at the apex or



pole C=1° and because the space all around C is divided into 360°, the area of the bi-rectanglar spherical isosceles triangle ACE, or of which the sides comprising the angle of 1° at the apex or pole C are equal to each other and each of them to one quarter of a circumference, this area, say we, must evidently be the 360th part of that of the hemisphere AEBA—C or which is the same thing the 90th part of that of the quarter of a hemisphere or tri-rectangular triangle. If the angle at C is but 1′ or the 60th part of 1°, the area of the triangle ACE will be but the (90° × 60)′ 5400th part of that of the semi-quarter of the sphere. If the angle at C is but a second, the same area will be but the (90° × 60′ × 60″ 324,000th part of that of the half-quarter of the sphere, or as shown in the above example, of 75.87321 square english miles ; whence it is plain, that an angle C or ACE of .1″ would give 7.587321 square miles ; an angle ACE of .01″ would give .7587321 of a square mile ; an angle ACE of .001″ an area of .07587321 of a square mile and so on, or, as just said, .07587321 for each thousandth of a second. Now 1 square mile=5280 × 5280 english feet=27,878,400 square feet and multiplying by .07587321 we obtain 2,115,223.4 feet which then corresponds also to the area of a spherical triangle in which the excess of the sum of its angles on 180° is of .001″ or of the thousandth part of a second ; whence, if the excess it but of 0001″ the corresponding area of the triangle will be 211.522.34 ; if the spherical excess is of .00001″ or of one hundred thousandth part of a second, the spherical area will be but of 21,152.234 and finally if the spherical excess is of .000001″ or of the millionth part of a second, the area of the spherical triangle corresponding to such an excess will be of 2115,2234 square feet or of an extent of ground not exceeding a square of 46 feet in the side. Whence :

RULE I. *To determine the spherical area of any triangle described on the surface of the terrestrial spheroid* [1] *(sphere flattened at the poles by nearly one three hundredth* $(\frac{1}{300})$ *of its diam.) we have but to multiply each millionth of a second of the excess of the sum of its three*

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[1] As the areas of similar figures are to each other as the squares of their homologous sides, we will arrive at the area of a spherical triangle described on a sphere of any radius by making the required proportion.

angles on 180° per 2115 square feet, or each thousandth of a second by 2115,223 square feet or by .07587321 square miles, or each .01″ (hundredth of a second) by .7587321 square miles, or each .1″ (tenth of a second) by 7.587321 square miles or each 1.″ (second) *by* 75.87321 *square miles, reducing to that effect the degrees (°) and minutes (′) in the given excess of the sum of the three angles of such spherical triangle over 2 right angles, into seconds, and multiply afterwards these seconds by 75.87321 and the fractions of seconds as has just been said.*

And Conversely, *to determine the spherical excess of the sum of the three angles of any spherical triangle on 2 right angles, we may divide the area previously obtained in an approximate manner (by considering* (**165**, **R. 2**,) *the lengths of the arcs constituting its sides as those of the sides of a rectilineal triangle) by 2215 square feet to obtain the number of millionths of a second (000,001″) contained in such excess, or by 2,115,223 square feet or .07587321 square miles to obtain the number of thousandths of a second (.001″) contained in said excess, by .7587321 square miles for the hundredths of a second* (01″), *by 7.587321 square miles for the* tenths of a second (.1″), finally by 75.587321 square miles for the *seconds (1″) and the seconds if required reduced into minutes by dividing by 60, and the minutes into degrees by dividing by 60, will still give the spherical excess required.*

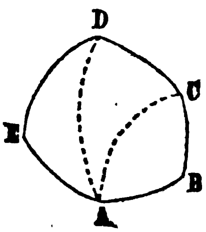
REM. III. The spherical triangle ACE on which we have argued is, as stated bi-rectangalar at A and E, that is, the angles at A and E are, and evidently are, right; whence it follows that the angle at C at the apex or at the pole is the spherical excess or the quantity by which the 3 angles exceed 2 right angles and in the same way as this spherical excess furnishes the area in the case of the bi-rectangular isosceles triangle, (**1200 G.**) so does that excess afford the means of arriving at the required area, or the area at the required excess in any other spherical triangle.

PROBLEM XXXIX.

To determine the area of a spherical polygon.

(See the *tableau.*)

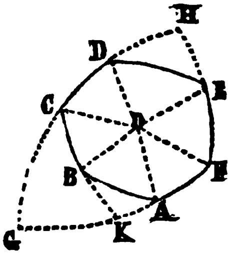
(**167**) **RULE.** *Find as in the last problem the area of the tri-rectangular triangle* (**1201 G.**)· *From the sum of all the angles of the polygon subtract asmany times 2 right angles as there are sides less two. Divide the remainder by 90° and multiply the tri-rect. triangle by the quotient thus obtained : the product will be the required area.*



Ex. 1. What is the area of a regular polygon of eight sides described on the surface of a sphere of which the diameter is 30, each angle of the polygon being 140° ?

Ans. 140° × 8=1120°=sum of the angles of the polygon, 180° × 6=1080°=as many times 2 right angles as sides less two, 1120—1080=40,40÷90=$\frac{4}{9}$; the area of the proposed polygon will then be the $\frac{4}{9}$ of that of the tri-rect. triangle, the area of the sphere = 30 × 30 × 3.1416=3.1416 × 900=2827.44 which÷8=353.43 area of the tri-rect. triangle, this latter × 4÷9 = 167.08 the required area of the polygon.

2. Required the area of an irregular polygon of 7 sides described on a sphere of 8½ metres radius, the sum of the angles being 1080° ?



Ans. Area of the sphere=17^2 × 3.1416 =907.9224 of which the eighth part 113. 4903 is the area of the tri-rect. triangle 1080°—5 times 180°=180°, 180°÷90=2 and 113.4903 × 2 = 226.9806 area of the proposed polygon.

3. The sum of the 15 angles of a polygon of a geodesical triangulation is 2340° 1′ 50″, what is the area of the polygon in square miles, supposing the diameter of the earth at the survey to be 7912 english miles, that is supposing the trigonometrical operation to have taken place on a sphere of that diameter.

Ans. We obtain as in the last problem, for the area corresponding to an excess of 1″, 75.87321 square miles, and we have seen that the area to be measured is in direct relation with the number of units in the given excess ; now the sum of the angles is in this example 2340° 1′ 50″ which diminished by 13 times 180° or by 2340° leaves for the excess 1′ 50″ or 110″ ; the required area will then be 110 times 75.874321 that is 8346.17531 square miles.

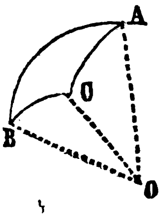
(168) REM. The supposition just made seems to indicate that the earth is not in all its extent of the same curvature, that is of the same radius or diameter, or that it is not a perfect sphere, and in fact the terrestrial globe is a spheroid of which the flattening towards the poles is nearly $\frac{1}{300}$ of the diam. at the equator or about 26 miles ; now the areas of two spheres of different radii or of any two homologous parts of the spheres are to each other (**1074 G.**) as the squares of the radii of those spheres.

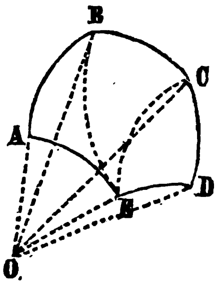
Let it be required then to find the relation of the areas of two similar figures traced on the terrestrial sphere, one at a point where the osculatory diameter is 7912, the other in a latitude where this diameter is 7930 miles, we will make $7912^2 : 7930^2 : 1 : 1.0045552$; multiplying by this last number the 8346.0531 square miles of the last example, we obtain 8384.071 square miles for the area of the same polygon at a point where the diameter of the earth would be 7930 instead of 7912, that is a difference of 38 square miles, quantity which though relatively small, considering the total area of the extent of territory comprised in the survey, is none the less very large in itself, equivalent as it is to that of a city or canton of 6 miles diameter; which shows the importance of considering the relative dimensions of each part of the terrestrial sphere in the operations to be performed to determine its area.

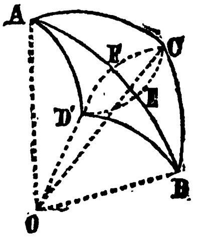
PROBLEM XL.

To determine the solidity of any spherical pyramid.

(See the *tableau.*)







(**196**) **RULE**. *Find first by the preceding rules the area of the base of the given pyramid; multiply then* (**1082 G.**) *this area by one third the height of the pyramid; that is by one third the radius of the sphere of which the pyramid forms a part and the result will be the required solidity.*

Ex. I. What is the solidity of a spherical pyramid the base of which is 10 square metres and the height 30 metres?

Ans. 100 cubic metres.

2. Among the component parts of a polyhedron to be cubed, is a spherical pyramid or a part of a sphere bounded by planes meeting in the centre of the sphere of which the pyramid forms a part; what is its solidity, the radius being 15 inches and the area of the triangle or polygon constituting its base 100 inches ?

Ans. 500 cubic inches.

3. There is to be made a vault or part of a vault of which the radius of the intrados is 30 feet, the depth of the vault 3 feet and its form that of an irregular polygon of which the int. area or superficies is 100 square feet; what is its solidity ?

Ans. The solid to be computed is the frustum of a spherical pyramid with parallel bases; this solidity is equal (**1083 G.**) to the difference of the solidities of the component entire and partial or ext. and int. pyramids. We will then obtain for the required solilidy, the expression (ext. area × $\frac{1}{3}$ R)—(int. area × $\frac{1}{3}$ r); we have therefore to find the ext. area which must enter into the calculation to be made; to that effect we obtain (**1074**, 2° **G.**) $30^2 : 33^2 :: 100 : 121 =$ area of the extrados; now, $(121 \times 11) - (100 \times 10) = 1331 - 1000 = 331$ cubic feet of masonry.

4. What is the weight of the fragment of a shell or bomb of which the int. diam is 10 inches, the thickness 5 inches, and the int. and ext. or concave and convex areas 60 and 240 square inches, the planes of section of the fragment being directed towards the centre of the sphere of which the solid to be measured forms a part, and the weight of the cast-iron being 480 pounds to the cubic foot ?

Ans. $(240 \times 10 \div 3) - (60 \times 5 \div 3) = 800 - 100 = 700$ cubic inches, the cubic foot $= 12 \times 12 \times 12 = 1728$ cubic inches, whence we obtain the required weight by making $1728 : 480 :: 700 : 194\frac{1}{2}$ pounds.

PROBLEM XLI.

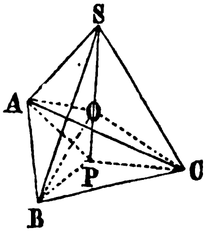
To find the area or solidity of any regular polyhedron.

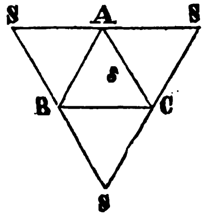
(See the 5 regular polyhedrons of the *tableau.*)

(169) RULE 1. For the area : *calculate the area of one of its component faces, and multiply* (**1118 G.**) *that area by the number of faces in the proposed polyhedron.*

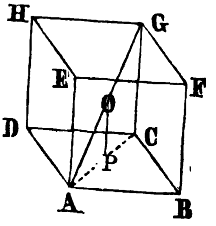
For the solidity : *Multiply* (**1124 G.**) *the area of the polyhedron by the third of the radius* OP *of the inscribed sphere, that is by the third of the perpendicular let fall from the centre to one of the faces of the solid, the product will be the required solidity.*

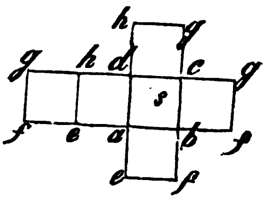
REM. We have seen (**1132** and **1134 G.**) that to determine in the case of the Dodecahedron and Icosahedron, the radius of the inscribed sphere, we must first find the angle formed by two of the adjacent faces of these solids, and we have indicated the manner of establishing this angle. We may also by means of the same angle,





calculate the perpendicular in each of the three other polyhedrons (of which that of the hexahedron is equal to the half-side of that body) or obtain that perpendicular by the method of par. (**1128 G.**) or (**1131** G.) as the case may be.





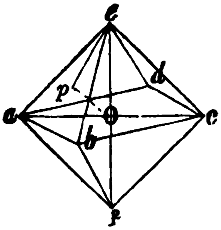
(**170**) It is well to calculate and dispose under the form of a table, as at (**27 T.**) for the regular polygons, the areas and solidities of the five polyhedrons having for their sides the unit, in order afterwards to make use when required of those areas and solidities, to determine the area or the solidity of any other regular polyhedron of the same name.

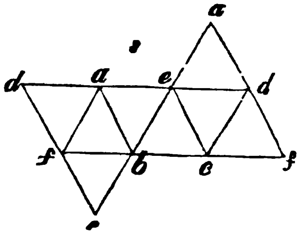
Table of the regular polyhedrons of which the side is 1.

NAME.	N° OF FACES.	ANGLE OF THE FACES.	AREA.	SOLIDITY.
Tetrahedron	4	70° 31′ 42″	1.7320508	0.1178513
Hexahedron	6	90°	6.0000000	1.0000000
Octahedron	8	109° 28′ 18″	3.4641016	0.4714045
Dodecahedron	12	116° 33′ 54″	20.6457288	7.6331189
Icosahedron	20	138° 11′ 23″	8.6602540	2.1816950

(**171**) **RULE II.** 1° **For the area** : *Square the side of the given polyhedron and multiply this square be the area of the polyhedron of the same name of which the side is* 1.

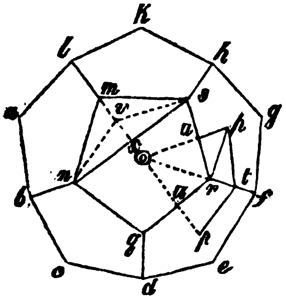
For the surfaces of similar polyhedrons are composed of a like number of similar polygons, and the areas of these polygons or their sums are to each other (**556, G.**) as the squares of their homologous sides.

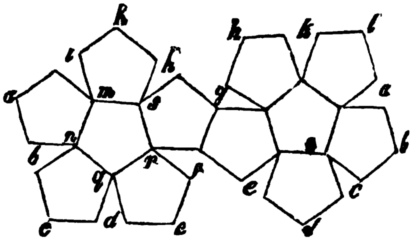




2° **For the solidity**: *cube the side of the given polyhedron and multiyly that cube by the solidity of the polyhedron of the same name of which the side is* 1.

For, similar polyhedrons are composed of a like number of similar pyramids and the solidities of these pyramids or their sums are to each other (**1070 G.**) as the cubes of their homologous sides.





Ex. 1. What is the area of a tetrahedron the side of which is 12?

Ans. 12 × 12 × 1.7320908=249.4153152.

2. The area of a hexahedron or cube the side of which is 30?

Ans. 5400.

3. Required the area of an octahedron the side of which is 10?

Ans. 10 × 10 × 3.4641016=346.41016.

4. Determine the area of a dodecahedron the side of which is 3?

Ans. 3^2 × 20.6457288=185.8115592.

5. What is the area of an icosahedron the side which is 20?

Ans. 8.660254 × 20^2=3464.1016.

6. What is the solidity of a tetrahedron the side of which is 15 ?

Ans. $15^3 \times 0.1178513 = 397.748$.

7. The solidity of a cube the side of which is 12 ?

Ans. 1728.

8. If the side of an octahedron is 10, what is its solidity ?

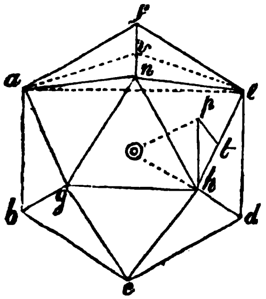
Ans. 471.4045.

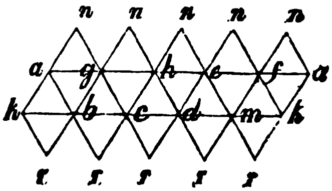
9. The side of a dodecahedron is 2 : what is its solidity ?

Ans. 61.3049512.

10. What is the solidity of an icosahedron the side of which is 20 ?

Ans. 17453.56.





11. A monument has been terminated by a ball or a crowning cut stone having the form of a dodecahedron of which the edge or side measures 13½ inches : required the solidity of the block of stone in cubic feet and its area in square feet ?

Ans. The area $= 13.5 \times 13.5 \times 20.6457288 = 3762.6840738$ square inches. We would equally obtain this area without using that of the table by computing separately by the method of par. (**28**, **T.**) the area of one of the component polygons and multiplying afterwards by 12 the element thus obtained. Thus the area of a pentagon the side of which is $1 = 1.7204774$, multiplying by 182.25 (square of the given side) we obtain for the area of one of the faces of the proposed polyhedron 313.55700615 square inches ; then, multiplying by 12 (number of faces of the dodecahedron) we obtain as before 3762.6840738 square inches, which proves also the accuracy of the tabular multiplier. Now we have but to divide the number of inches just found by 144 (the square inches in a square foot) to obtain 26 square feet 18.684 square inches, the required area.

Ans. The solidity $= 13.5 \times 13.5 \times 13.5$ or $(13.5)^3$ or $2640.375 \times 7.6331189 = 18780.3349$ cubic inches, dividing by 1728 (number of cubic inches in a cubic foot) we obtain 10.87 cubic inches nearly.

PROBLEM XLII.

Being given the diameter of a sphere, to find the side of any of the regular polyhedrons, which may be inscribed in the sphere, circumscribed about the sphere, or which is equal to the sphere.

(See the 5 regular polyhedrons of the *tableau.*)

(172) RULE. *Multiply the given diameter by the number which, in the following table, answers to the question, and the product will be the side of the required polyhedron.*

It suffices from what has already been said concerning the regular polyhedrons (pages 423 to 427) to show immediately how this table has been calculated.

The diameter of a sphere being 1 the side of a	*Capable of being inscribed in the sphere, is*	*Capable of being circumscribed about the sphere, is*	*Equal in solidity to that of the sphere, is*
Tetrahedron....	0.8164966	2.4494897	1.6439480
Hexahedron....	0.5773503	1.0000000	0.8059958
Octahedron.....	0.7071068	1.2247447	1.0356300
Dodecahedron..	0.3568221	0.4490279	0.4088190
Icosahedron....	0.5257309	0.6615845	0.6214433

Ex. It is required to recast in the form of a perfect cube of equal solidity, a cannon ball of which the diameter is 10 inches; what will be the length of the side of the required hexahedron ?

Ans. 0.8059958 × 10=8.059958 inches.

2. By how much will the weight of a stone sphere 5 feet diameter be diminished, by reducing it to the greatest regular polyhedron of 20 sides that may be got out of it, the weight of the stone being supposed equal to 150 pounds per cubic foot ?

Ans. The solidity of the given sphere$=5^3 \times .5236=65.45$ cubic feet or 65.55 × 150=9817½ pounds in weight. The side of the required icosahedron will be, from the rule, 0.5257309 × 5=2.6286545; cubing this latter number, we obtain 18.163 and multiplying this cube by the solidity 2.181695 of the polyhedron of the same name the side of which is 1, we obtain for the solidity of the sphere reduced to an icosahedron 39.626 cubic feet or 39.626 × 150=5943.9 pounds in weight; the difference 3873.6 pounds is the required weight.

PROBLEM XLIII.

Being given the side of one of the five regular polyhedrons, to find the diameter of a sphere that may be inscribed in the polyhedron, circumscribed about the polyhedron or equal to it in solidity.

(See the 5 regular polyhedrons of the *tableau.*)

(**173**) **RULE.** *Make the following proportion: As the respective number of the above table, under the title "inscribed," "circumscribed," "equal," is to 1, so is the side of the given polyhedron to the diameter of the inscribed, circumscribed or equal sphere, as the case may be.*

In other words: as the side of the inscribed, circumscribed or equal polyhedron (as the case may be) of the table, is to the diameter 1 of its inscribed, circumscribed or equal sphere, so is the side of the given polyhedron to the diameter of its inscribed, circumscribed or equal sphere.

Ex. 1. The side of an icosahedron is 2.62865, it is required to reduce it to a sphere of the greatest possible diameter, what will its diameter be ?

Ans. .6615845 : 1 :: 2.62856 : 3.973, nearly the required solidity. The area of the given icosahedron is (**28 T.**) 2.62865 × 2.62865 × .4330127 × 20=59.842355, this area × 3.973 ÷ 6 (that is by the sixth part of the diameter or the third of the radius of the inscribed sphere) gives for the solidity of the icosahedron 39.6259 cubic feet or 39.626, as in example 2 of the preceding problem, each of the two results being in this way a verification of the accuracy of the other and at same time a proof of the accuracy of the factors of the table.

2. Required the diameter of a cannon ball that can be obtained by recasting a mass of iron under the form of an octahedron 12 inches side ?

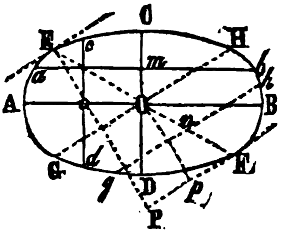
Ans. 1.03563 : 1 :: 12 : 11.58715, that is, the diam. of the ball will be 11.6 inches nearly.

PROBLEM XLIV.

To find the solidity of any spheroid.

(See on the tableau, the flattened, elongated and 3 axed spheroids.)

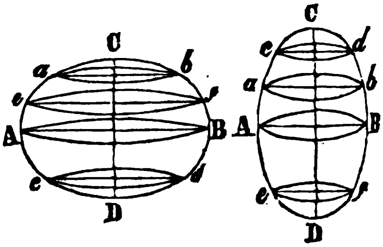
(**174**). **RULE I.** *Multiply the fixed axis by the square of the revolving axis (or by the rectangle or product of the two axes of revolution, as the case may be) and the product by .5236 : the result will be the required solidity.*



REM. It is plain that this rule is in every way analogous to the one given (**1086 G.**) for establishing the solidity of a sphere ; and in fact, the spheroid, like the sphere, is equal to the $\frac{2}{3}$ of its circumscribed cylinder ; for it is demonstrated in "conics" that if we have A*o* : *o*O in the ellipsis :: A*o* : *o*O in the circle ; we will also obtain *oc* : OC in the ellipsis :: *co* : OC in the circle ; thence, since we may (**1009 G.**) consider the sphere and the spheroid as each composed of an infinity of thin slices or superposed surfaces generated by the revolution of the same number of ordinates *oc* perpendicular to the fixed axis AB of the two solids, and that these component surfaces are to each other as the squares of the generating radii, it is evident that the two solids will also be to each other as the squares (**104 G.**) of those ordinates, or which is the same thing, as the areas of the bases or corresponding sections of the cylinders of equal height AO circumscribed about those solids.

What we have just said of the elongated or prolate spheroid AB and of its circumscribed sphere, is equally understood of the flattened or oblate spheroid CD and its inscribed sphere, for, whatever the relation of O*m* to *m*C may be in each of these two last solids, we will have between *am* and AO of the one the same relation as between *am* and AO of the other.

(**175**) **RULE II.** *Multiply* (**127 T.**) 4 *times the area of any section* (AB, CD, GH, &c.) *passing through the centre* (*O*) *of the spheroid, by* $\frac{1}{6}$ *of the perpendicular height* (CD, AB *or* EP, &c.) *of the solid corresponding to such section.*



For, **in the first place,** in the case of the spheroid generated by the revolution of the semi-ellipsis ACB about its axis AB, the factors in the two rules are reduced to the same. In fact the first rule gives for the solidity AB × CD × CD × .5236 and the second rule gives CD × CD × .7854 × 4 × $\frac{1}{6}$ AB ; if these expressions are equal or equivalent, we must get (neglecting the factors AB, CD, common to the two formulas) .7854 × 4 × $\frac{1}{6}$ = .5236 ; or .7854 × 4 = 3.1416 ÷ 6 = .5236 ; **therefore &c.**

In the second place, The section AB of the same spheroid is an ellipsis equal in everything to the ellipsis ACBD and its area is (**57 T.**) = AB × CD × .7854 ; if the second rule be correct, we will then obtain AB × CD × .7854 × 4 × ⅙ CD = AB × CD × CD × .5236 ; and in fact by eliminating the factors AB, CD and CD common to the two expressions, there still remains .7854 × 4 × ⅙ = .5236 ; therefore, &c.

In the third place, it is to be demonstrated that 4 area section GH × ⅙ EP is still equal to $\overset{2}{\text{CD}}$ × AB × .5236 ; now, the "conics" show that whatever may be the axes or conjugate [1] diameters GH, EF made use of, the parallelograms circumscribed to the ellipsis and of which the sides are parallel to these conjugate axes, are all equal in area to the rectangle AB × CD ; but (**8, T,**) the area of the parallellogram having for its sides GH, EF is GH × EF × nat. sin. angle EOC or EFP = GH × EP. The area of the section GH = (for any section of a spheroid is an ellipsis) GH × CD × .7854 and we have just seen that GH × EP = AB × CD ; therefore GH × CD × .7854 × 4 × ⅙ EP = AB × CD × CD × .5236, CD being common to both formulas, AB × CD = GH × EP and .7854 × 4 × ⅙ = .5236 ; therefore, &c.

REM. In the case of the flattened spheroid generated by the revolution of the semi-ellipsis DAC round the axis CD, the proof is analogous to the one just given.

Ex. 1. What is the solidity of an ellipsoid of which the axis of revolution is 60, and the fixed axis 80 ?

Ans. 60 × 60 = 3600, 3600 × 80 = 288000, 288000 × .5236 = 150796.8 units of solidity.

2. With the same data, what will be the solidity of the flattened spheroid ? **Ans.** 80 × 80 = 6400, 6400 × 60 = 384000, 384000 × .5236 = 201062.4 units of solidity.

3. A prolate spheroid has for its axis 100 and 200; what is its solidity ?

Ans. $\overset{2}{100}$ × 200 × .5236 = 1,047,200 = the required solidity. Now let EF in this example any diameter = 166, we will obtain its conjugate GH $=\sqrt{AB^2+CD^2-EF^2}$ (for it is demonstrated in "conics" that the sum of the squares of any pair of conjugate diameters is equal to the sum of the squares of the major and minor axes) = 149.81322, 4 area GH = GH × CD × .7854 × 4 = 47065.3212. Since AB. CD = EF.GH × nat. sin. EOG, we obtain nat. sin. EOG = AB.CD ÷ EF. GH = 20000 ÷ 24869 = .8042141 = 53° 32′, and .8042141 × 166 = EP = 133.

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[1] The diam. GH, conjugate of EF, is the one parallel to the tangent PF to the ellipsis at the point F, where the diameter EF meets the curve.

49954, and 47065.3212 × 133.49954 ÷ 6 = 1,047,199.8, the difference .2 between the two results being due to the decimals neglected in the calculation.

4. If the two axes of the earth are to each other as 304 and 303, what will be the solidity of the spheroid (it is flattened, the polar diam. being less than the equatorial diam.) and by how much will this solidity differ from that of a sphere on the great axis.

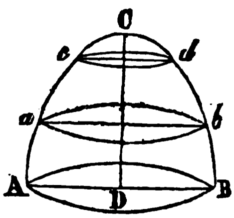
Ans. The solidity of the spheroid = 304 × 304 × 303 × .5236 = 14661872.3328 the solidity of a sphere on the greater axis = 14710261 .3504 and the difference of these solidities is 48389.0176.

PROBLEM XLV.

To determine the solidity of any segment of a sphere or spheroid with a single base or of any frustum with two parallel bases, perpendicular or not to the axes of the solid.

(See segments and solid zones of *tableau.*)

(176). RULE. *To the area of the base of the segment or to the sum of the areas of the bases of the frustum, add* 4 *times the area of a section half-way between the base and apex or between the parallel bases, as the case may be, and multiply the whole by* $\frac{1}{6}$ *of the height of the solid: the product will be the required solidity.* [1]



In the first place, as to what concerns the semi-spheroid (of which we may besides obtain the solidity by computing that of the entire spheroid and taking the half thereof) we have seen (**174 T.**) that area section *cd* : area section CD in the spheroid :: area section *cd* : area section CD in the sphere ; now it has been demonstrated (**184, T.**) that in the sphere, area *cd* half-way between A and O = $\frac{3}{4}$ area CD ; therefore also in the spheroid, area *cd* = $\frac{3}{4}$ area CD ; therefore area CD + 4 area *cd* = 4 area CD, and by the last problem, solidity ACD = 4 area CD × $\frac{1}{6}$ AO ; therefore solidity ACD = (area CD + 4 area *cd*) × $\frac{1}{6}$ AO.

Now, for the semi-spheroid of which the base AB is an ellipsis = ACBD and section *ab* also an ellipsis similar to the base (for all parallel sections whatever of the spheroid are similar ellipses) we again obtain area *ab* : area AB :: area *ab* : area AB in the the sphere ; for as *ab* : AB :: *ab* : AB in both solids and (**104. G.**)

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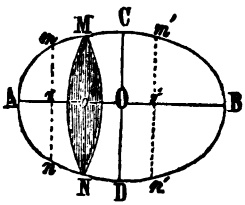
[1] The figure of this paragraph represents any segment ACB or *aCb* or *cCd* of a sphere or spheroid, or any frustum AB—*ab* or *ab*—*cd*, or AB—*cd* of a sphere or spheroid ; but the demonstration takes place by the figure at the top of page 136.

$ab^2 : AB^2 :: ab^2 : AB^2$ in both solids, and the areas of similar ellipses, as of all other similar figures, are to each other as the squares of their diameters or other homologous lines ; therefore area ellipsis $ab = \frac{3}{4}$ area ellipsis AB ; now, solidity half-spheroid ACB=by the last problem 4 area AB × $\frac{1}{6}$ CO ; therefore also the same solidity=(area AB + 4 area *ab*) × $\frac{1}{6}$ CO.

REM. I. It is also a property of the ellipsis that every diam. EF of that figure bisects every chord or double-ordinate *gh* parallel to the conjugate diameter GH, which gives $nh = ng$ and we demonstrate that in the same way that we obtain (conics) $AB : CD :: \sqrt{Ao.oB} : oc$, and $CD : AB :: \sqrt{Cm.\ m\,D} : mb$, so we also obtain $EF : GH :: \sqrt{En.\ nF} : nh$ and consequently that $nh : OH :: mb : OB :: oc : OC$ when O*n*, O*m* and O*o* have to OF, OC and OA or to *n*F, *m*C & *o*A the same relation. We then obtain area section $gh = \frac{3}{4}$ area section GH and as it is already demonstrated that solidity semi-spheroid GFH$=\frac{1}{2}$ (4 area GH × $\frac{1}{6}$ EP) we will also obtain solidity GFH=(area GH + 4 area *gh*) × $\frac{1}{6}$ O*p* or by $\frac{1}{6}$ EP ÷ 2.

(**177**) **In the second place**, as to any segment of a spheroid other than the semi-spheroid, it will suffice after what we have just said and the demonstration given at par. (**135 T.**) of the accuracy of the rule in the case of any segment of a sphere, to show its equal accuracy in the actual case ; which will also dispense from adding unnecessarily to the already voluminous dimensions of this treatise.



Ex. 1. What is the solidity of a segment MNA of a spheroid with one base MN perpendicular to the fixed axis AB, the height A*o* of the segment being 10 units and the lengths of the axes AB=100, CD =60 ?

Ans. $AB : CD :: \sqrt{Ao.\ oB} : oM$, whence $oM = 18$ and MN=36 ; $rm = Ar.rb \times CD \div AB = 13.0766985$ and $mn = 26.153397$; area MN + 4 area $mn = (MN^2 + 4\,mn^2) \times .7854$, $MN^2 = 1296$, $mn^2 = 684$ very nearly, (1296+4 times 684) × .7854=3166.7328, multiplying by $\frac{1}{6}$ A*o*, or by $\frac{1}{6}$ 10, we obtain 5277.888 units of solidity in the proposed segment.

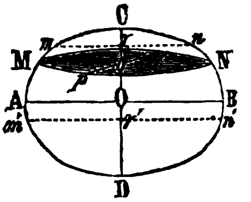
2. Required the solidity of a segment MNB of a spheroid by a plane MN perpendicular to the fixed axis AB, *o*B being=90 and AB, CD 100 and 60 respectively ?

Ans. If $m'r'$ is not given we find it$= Ar'.r'B \times CD \div AB$ (since $AB : CD :: \sqrt{Ar'.r'B} : r'm'$ or $100 : 60 :: \sqrt{55 \times 45} : r'm'$)=29.8496208 or

$m'n'=59.6992416$, $MN^2+4m'n'^2 \times .7854 \times \frac{1}{6}\,oB$=solidity MNB = 183218.112; the sum 188,496 of these solidities is the solidity of the entire spheroid ACBD, for (**174 T.**) $60 \times 60 \div 3600$, $3600 \times 100=360{,}000$ and $360{,}000 \times .5236=188{,}496$, which also proves the accuracy of the rule of this problem.

REM. II. In the two last examples we have supposed the axes AB and CD to be known : but that knowledge is in no way essential, since the intermediate diameters *mn*, *m'n'* are considered known or besides since they may be directly obtained by measuring, in practice, those diameters ; and it is one of the advantages of the rule of this problem, that it does not require our knowing what spheroid the segment to be measured belongs to.

3. A segment MNC of a spheroid by a plane MN perpendicular to the axis of revolution CD, and of which the base is consequently an ellipsis, has for its height *o*C 12 units, the axes AB, CD being respectively 100 and 60 : what is the solid content of the segment ?



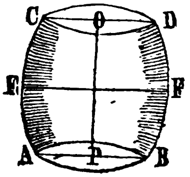
Ans. $CD : AB :: \sqrt{Co.oD} : oM$ or $60 : 100 :: \sqrt{48 \times 12} : 40$, and because the parallel sections MN, AB are similar, we have $AB : CD :: MN : pq$ conjugate diameter of the elliptical base MN of the given segment : therefore $pq = 60 \times 80 \div 100=48$ and area $Mp\ Nq=80 \times 48 \times .7854$; since $oC=12$ we obtain $r\ C=6$ and $rD=54$, $60 : 100 :: \sqrt{54 \times 6} : 30=rm$, the conjugate diam. of $rm=18$ (for $100 : 60 :: 30 : 18$) and the area of the section $mn=60 \times 36 \times .7854$; that admitted, we obtain solidity MNC=(area MN + 4 area *mn*) $\times \frac{1}{6}\,oC=MN^2+4\,mn^2 \times .7854 \times 2=19603.584$ units of solidity.

4. What is the solidity of the other segment of the same spheroid ?

Ans. We have $r'D=oD-\frac{1}{2}\,oC=24$, and $r'C=36$, whence we obtain as before $m'\ n'=97.9796$; the other diameter or axis of the ellipsis $mn=58.78776$; whence area $m'n'=4523.904$ and area MN + 4 area $m'n'=21111.552$, this sum $\times \frac{1}{6}$ 48 or by $8=168892.416$ the required solidity.

The two segments united give 188,496 which is in fact the solidity of the entire spheroid as it has been seen at the 2d example.

5. What is the solidity of a central frustum AD of a spheroid the parallel bases of which are equal circles of 40 inches diameter, the greatest diameter of the frustum=50 inches and the height or distance between the parallel bases 18 inches ?



Ans. (area AB + area CD + 4 area EF) $\times \frac{1}{6}$ = OP = ($40^2 + 40^2$ (or twice 40^2) + 4 times 50^2) $\times .7854 \times 3 = 31101.84$ cubic inches or 18 cubic feet nearly.

6. The respective diameters of the parallel bases of the frustum of a spheroid are 10 and 20, the diameter of a section equidistant to those bases is 30 and the height of the frustum is 40 : what is its solidity ?

Ans. ($10^2 + 20^2 + 4$ times 30^2) $\times .7854 \times 40 \div 6 = 3220.14 \times 40 \div 6 = 64402.8$ cubic inches.

7. One of the component parts of a cul-de-lamp bearing against a wall, presents the form of the semi-segment or frustum of a spheroid with elliptical parallel bases. The diameters of the ellipses or rather of the inf. and sup. semi-ellipses measure respectively 30 and 39 inches, the intermediate diameter is 36 and the three conjugate semi-diameters or projections of the cul-de-lamp measure 10, 13 and 12 inches, the height of the frustum is 18 inches ; what is its solidity ?

Ans. ($30 \times 10 + 30 \times 13 + 4$ times 36×12) $\times .7854 \times 3 = 59729.67$ cubic inches or 3.4 cubic feet nearly.

8. It is desired to know how many gallons there are (231 cubic inches to a gallon) in a cask of wine the length of which is 40 inches and the diameters at the centre and at each end 32 and 24 inches ?

Ans. (twice $24^2 + 4$ times $32^2 \times .7854 \times 40 \div 6 = 27478.5$ cubic inches, dividing by 231 we obtain 119 gallons minus a half pint nearly.

9. In an inclined vessel, the form of which seems to be that of a semi-spheroid, is found a quantity of liquor, the greatest depth of the liquor is 15 inches, the respective diameters of its elliptical area are 48 and 36 inches and the corresponding diameters of the intermediate parallel ellipsis between the surface and the bottom are 30 and 22½ inches ; what is the quantity of liquor in the vessel ?

Ans. ($48 \times 36 + 4$ times 30×22.5) $\times .7854 \times 2.5 = 8694.378$ cubic inches, say 37⅔ gallons nearly.

PROBLEM XLVI.

To determine the solidity of the frustum of a spheroid with non parallel bases. [1]—(See frusta of *tableau.*)

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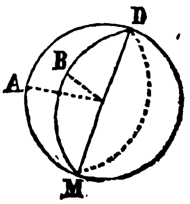
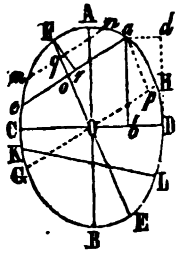
[1] We will obtain with accuracy, as in the case of the sphere, the solidity of any ungula ADBMA of a flattened or elongated spheroid or of a spheroid with 3 axes, provided the common intersection DM of the containing or limiting planes DAM, DBM passes in any direction through the centre O of the solid of which the ungula forms part ; and if the edge DM of the ungula does not pass through the centre O of the spheroid, we will none the less obtain very nearly the soli-



(178) RULE. *Compute the solidity of the segment of a spheroid with a single base of which the given frustum forms part, calculate also the solidity of the segment which is wanting to given frustum to complete the segment; the difference of these solidities will be that of the proposed frustum.*



Ex. 1. Let it be required to find the solidity of the part CD*ae* of a spheroid comprised between a plane CD passing through the centre perpendicularly to AB and any other plane *ea* not parallel to the first.

Ans. We must for this purpose determine the unknown axis AB of the spheroid of which the height AO of the segment CDA forms part. Having measured any ordinate *ab* and the abscissae C*b*, *b*D or rather *d*D=*ab*, *ad*=*b*D and C*b*=CD—*b*D, we will make (conics) $\sqrt{Cb.bD} : ab :: CD : AB$ and we will obtain the solidity of CDA=4 area CD × $\frac{1}{6}$ AO. We will then measure *ae*, OH parallel to *ae*, *o*O drawn from the centre to the middle point *o* of *ae* (*o*O forming part of the diameter EF the conjugate of GH) and the abscissa *p* H of the ordinate *a p* parallel and equal to *o*O ; with these data, we will make $\sqrt{Gp.pH} : ap :: GH : EF$; we will then obtain *o*F and consequently the perpendicular F*r*, as at par. (**175**, **T. Ex. 3**). It remains to establish the diam. *mn* of an intermediate section between *ae* and the apex F of the segment *ae* F; now we will obtain *mq* or *nq* half (**176**, **T.**, **REM. 1.**) of *mn* by making EF : GH :: $\sqrt{Eq.qF} : mq$. Finally we will obtain the required solidity CD*ae*=(4 area CD × $\frac{1}{6}$ A*o*) minus (area *ac* + 4 area *mn* × $\frac{1}{6}$ F*r*).

2. If the solid to be measured was the frustum KL*ae*, we would operate for the segment KLB as it has been done for *ae*F and the sum of the solidities of these segments deducted from that of the entire spheroid ACBD, would leave the solidity of the proposed frustum.

PROBLEM XLVII.

To find the solidity of a right or oblique paraboloid or of any frustum or segment of a paraboloid comprised between parallel bases, perpendicular, or not, to the axis of the solid.

(See models of the *stereometrical tableau.*)

(179) RULE. *To the sum of the areas of the opposite bases, add*

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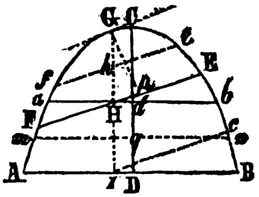
dity of the proposed ungula by multiplying the product of 4 times the area of its half way section AOBA (a kind of sector of a circle or ellipsis) by the sixth part of the length DM of the ungula.

4 times the area of an intermediate section half-way between them ; multipiy the whole by ⅙ of the height of the body to be measured and the product will be the required solidity.



In fact, the generating parabola ABC is a curve such that the abscissae are proportional to the squares of the ordinates, that is : we always have $cd : CD :: db^2 : DB^2$, and it is the same for every other pair or system of axes or conjugate diameters EF, GH which still give $Gh : GH :: fh^2 : FH^2$; then if Cd=½ CD, db^2 will be=½ DB^2 and in the same way if Gh = ½ GH, we will have $eh^2 = ½EH^2$; now it is demonstrated that the solidity of the paraboloid is equal to half its circumscribed cylinder, that is, this solidity = area base AB × ½ CD, or solidity FEG=area base EF × ½ Gp ; but if $bd^2 = ½ BD^2$ we have area *ab*= ½ area AB (since the parallel sections *ab*, AB are circles and the similar figures are to each other as the squares of their homologous lines) and area AB + 4 area *ab*=3 area AB ; therefore area AB × ½CD=3 area AB × ⅙ CD=(area AB + area *ab*) × ½ CD. Similarly area elliptical base EF=2 area similar elliptical base *ef* and area EF × ½ Gp= (area EF + 4 area *ef*) × ½Gp.

In the second place, Let AB *ba* any segment of a paraboloid with parallel bases, we demonstrate that the solidity is obtained by multiplying by the height *d*D of the frustum, the half-sum of the areas of its parallel bases ; now on account of $Cd : Cq : CD :: db^2 : qn^2 : DB^2$, it is plain that the intermediate area *mn* is an aritmetic mean between area AB and area *ab* ; whence it follows that area AB + area *ab* + 4 area *mn*=6 area *mn* ; therefore the solidity of AB *ba*= (area AB + area *ab* + 4 area *mn*) × ⅙ *d*D.

(180) REM. In the case of the paraboloid or frustum of a paraboloid properly so called, it is plain that this rule offers no advantage and on the contrary it is more simple to arrive immediately at the desired solidity by computing the product of ½ CD by area AB, or of ½ G*d* by area EE, or of ½ *d*D by the sum of the areas of AB and *ab*, as the case may be ; but in practice it is rare that the solids to be measured are perfectly geometrical, and were they, we would not know it without much preliminary labour which we might as well devote at once to the calculation of the required solidity according to the rule here given ; whilst if (**137, 146, T.**) we took for a par aboloid, a solid which on the contrary were a segment or the frustum of a spheroid or of a hyperboloid, or which only resembled those solids without being identifiable with any of them, the rule of this problem is that one which would offer the guarantee of an accuracy very near the truth.

Ex. 1. What is the solidity of a right paraboloid the height of which is 84, and the radius of the base 24 ?

Ans. diam. 48 × 48 × .7854 × ½84 = 76001.5872 the requir. solidity.

2. What is, in gallons of 231 cubic inches, the capacity of a parabolic boiler the depth of which is 36 metres and the diameter 60 inches ?

Ans. $60^2 \times .7854 \times 18 \div 231 = 50{,}893.92$ cubic inches $\div 231 = 220.32$ or 220⅓ gallons nearly.

3. A vault which appears to be parabolic, is 60 inches high, the diameter of its base is 40 metres and its intermediate diameter is 28 metres 285 millimetres ; what is the solidity of the included space ?

Ans. $(40^2 + 4$ times $28.285^2) \times .7854 \times 60 \div 6 = 37{,}699.2$ cubic metres.

4. In an inclined vessel which may be a paraboloid or the segment of a spheroid, is a quantity of liquor the surface of which is consequently an ellipsis having for its diameters 50 and 30 inches, the greatest depth of the liquor is 18 inches, and one of the diameters (the least) of the elliptical section taken at the middle of that depth is 22.5 inches : what is the content ?

Ans. The intermediate section being similar to the base or surface we will obtain its greater diameter by making 30 : 50 :: 22.5 : 37.5 ; the solidity = $(50 \times 30 + 4$ times $22.5 \times 37.5) \times .7854 \times 3 = 11486$ cubic inches.

5. One of the component parts of a solid to be measured, seems to be the frustum of a parabolic conoid with parallel bases, the respective circumferences of its two circular bases and of a section halfway between them, are 182.2, 94¼,145.15 inches and the height is 48 inches ; what is its solidity in cubic feet ?

Ans. Dividing each of the circumferences by 3.1416 we obtain for the diameters of the respective sections 58, 30 and 46.2 inches, which gives for the solidity $(58^2 + 30^2 + 4$ times $46.2^2)$ $.7854 = 10054.5$, and $10054.5 \times 48 \div 6$ that is by $8, = 80{,}436$ cubic inches, $\div 1728 = 46.55$ cubic feet.

PROBLEM XLVIII.

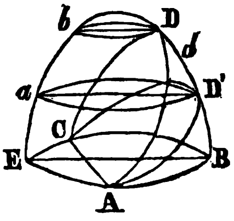
To determine the solidity of any frustum ABEF [1] of a right paraboloid ABC, with non parallel bases.

(See the *tableau.*)

(**181**) **RULE**. *Get, by the last problem, the respective solidities of the entire paraboloid* ABC *of which the frustum forms part, and of the partial paraboloid or segment* EFG *wanting in the given frustum to complete the entire paraboloid: the difference of these solidities will be the required solidity.*

Let ABEF (figure of the last problem) a section of the given frustum by a plane perpendicular to the centre D of its base; take on the axis Dd of the section any length Dd, measure DB, db and since (**179, T.**) we have $\mathrm{CD} : \mathrm{C}d :: \mathrm{DB}^2\, db^2$, make (**96, div. G.**) $\mathrm{CD} - cd : cd :: \mathrm{DB}^2 - db^2 : db^2$, or which is the same thing $\mathrm{DB}^2 - db^2 : db^2 :: \mathrm{D}d : d\mathrm{C}$, which will give for the height of the generating parabola $d\mathrm{D} + d\mathrm{C} = \mathrm{DC}$. Now, through the middle point H of EF, draw HG parallel to DC (for in the parabola the centre is infinitely distant and any diameter GH, that is any bisectrix GH of the chords or parallel double ordinates EF, *ef*, *rc*, is consequently parallel to the axis CD), measure any Hr, HE and rc and make, as before, $rc^2 - \mathrm{HE}^2 : \mathrm{HE}^2 :: \mathrm{H}r : \mathrm{HG}$; with HG and the angle GHp or GHE, we easily find (**175 T.**) the perpendicular height Gp of the segment FGE, to compute afterwards the respective solidities of the entire and partial conoids and their difference, which will resolve the problem.

(**182**) **REM.** If the **frustum** to be measured ABEF (See figure to problem L and suppose it to be the frustum of a paraboloid) is that of **an oblique paraboloid**; draw from A to F any straight line Bb parallel to FE, bisect in h', H$'$ these double ordinates and draw Gh' H$'$ which will pass through the apex G$'$ of the segment FEG$'$; draw afterwards Ee parallel to AB, bisect these pa-



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[1] We may also obtain as for the ungula of the prism, pyramid, cone or cylinder, the solidity very nearly of the ungula ABC-D, ABC-D', ACD'-D of a right or inclined parabolic-conoid, and that, absolutely in the same manner as for those various solids. (See problem XXXI, &c.)

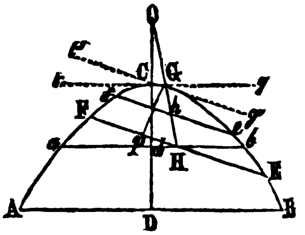
rallels in H′, *h* and draw the diameter G*h* H which will meet the apex G of the oblique paraboloid ABG ; we will calculate, as before, the heights GP, G′*p* of the entire and partial conoids, with the aid of the angles G′*h*′E, GHB and of the straight lines G*h* G*h*′ of which we will establish the lengths as already stated, and we will obtain the solidity of the frustum = solidity ABG—solidity FEG′ = area AB $\times \frac{1}{3}$ GP—area EF $\times \frac{1}{3}$G′*p*. To obtain if required, CD, we will draw from any point between A and F a straight line *m o n* (suppose a straight line *m o n* perpendicular to the parallels G′*h*, GH and of which *o* is the middle point) perpendicular to GH or to G′H′, the perpendicular CD, where *m o* = *o n* will be the required axis.

PROBLEM XLIX.

To find the solidity of a right or oblique hyperboloid, or of any frustum of a hyperboloid, comprised between parallel bases, perpendicular or not, to the axis of revolution.

(See the *tableau.*)

(183) RULE. *To the sum of the areas of the opposite bases of the solid, add 4 times the area of a section half way between them, multiply the whole by $\frac{1}{6}$ of the height and the product will be the required solidity.*



In the case of the right hyperboloid ABC or of the frustum AB*ba* of a right hyperboloid with parallel bases, this rule is, in other terms, the very one given by the " differential and integral calculus " and since the intermediate diam. is here essential to the calculation to be made, it is to be demonstrated how it can be obtained when it is not among the necessary data. The hyperbola is such that its centre O is outside the circumference of the curve, and as in the circle, the ellipsis and parabola, so in the hyperbola any diameter produced OC, OG bisects the chord or double ordinate AB, *ab*—EF, *ef* parallel to the tangent *tg*, *t′g′* drawn through the point C or G where such diameter meets the curve. It follows then that to determine the centre of the hyperbola, it suffices to draw and bisect in D, *d*,—H, *h*, any two pairs of parallels AB, *ab*,—EF, *ef*, and to produce outside the figure the straight lines D*d*, H*h* uniting the points of section, to their meeting at O which will be the required centre; or, if the direction OD of the axis is known, the intersection of that axis by the straight line Hh produced will determine the required centre. Now, by the nature of the hyperbola, we demonstrate in "conics" that $2OC.CD + CD^2 : 2\ OC.Cd + Cd^2 :: DB^2 : db^2$, or that $2OG.GH + GH^2 : 2OG.Gh + Gh^2 :: HE^2 : he^2$; this is then the manner to obtain the intermediate diam. *ab* or *ef* by taking $Cd = dD$ or $Gh = h$ H' as the case may be.

Ex. 1. The height CD of a right hyperboloid ABC is 10 inches, and AD the radius of its base is 12 inches, the intermediate diameter *ab* is 15.8745 inches; what is its solidity ?

Ans. $(24^2 + 4$ fois $15.8745^2) \times 10 \div 6 = 2073.454691$ cubic inches.

2. A vessel which seems to be a right hyperbolic conoid, has for its height or depth 50 inches, for sup. diam. 104 inches and for intermediate diam. 68 inches : what is its capacity in wine gallons.

Ans. $104^2 \times .7854 = 8494.8864 =$ area of the superior base, 4 times $68^2 \times .7854 = 68^2 \times 3.1416 = 14526.7584$, the sum of these areas is 23021.6448, this sum $\times \frac{1}{6}$ 50 or, which is the same thing $\times 50$ and the product $\div 6 = 191847.04$ cubic inches, $\div 231 = 830\frac{1}{2}$ gallons.

3. How many cubic metres of space are there under a vault which appears to be hyperbolic and the height of which is 15 metres, the diameter of the base 32 metres and the intermediate diam. 20 metres ?

Ans. 5152.224 cubic metres.

4. A boiler in the form of a hyperboloid, contains a quantity of liquor; it is asked how many more gallons would be required to fill it, the part of the vessel to be filled having consequently the form of the frustum of a hyperboloid with parallel bases; the diameters of these bases are 24 and 32 inches, the inter. diam. 28.1708 and the height of the frustum 20 inches ?

Ans. $(24^2 + 32^2 + 4$ times $28.1708^2) \times .7854 \times 20 \div 6 = 12499\frac{1}{4}$ cubic inches or 54.108 gallons, *or* 7.2334 cubic feet.

5. One of the component parts of a cul-de-lampe or other object to be measured, presents the appearance of the frustum of a hyperboloid of which the height is 12 inches, the smaller diam. 6 inches the greater diam. 10 inches and the interm. diam. $8\frac{1}{2}$ inches : what is its solidity ?

Ans. 667.59 cubic inches.

(184) REM. For the oblique hyperboloid or the segment of a right hyperboloid by a plane not perpendicular to the axis, the "calculus" shows how to obtain the solidity by making the following proportion : $GH + 2GO :: \frac{2}{3}\ GH + 2GO :: \frac{1}{2}$ cylindroid of the same base and height : required solidity.

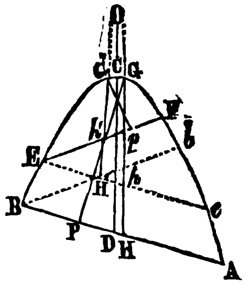
6. Let it be required to cube the hyperboloid EFG of which the greater axis EF of the elliptical base measures 78 units and its conjugate diameter 69.2, let OG=41.7, GH=19.8, G*p*=15.8 ; we have seen that to find *ef* there is to be made $2GO.GH + GH^2 : 2\,GO.Gh + Gh^2 :: HE^2 : he^2$, or (which (**73, Ax. G.**) is the same thing) $:: EF^2\ ef^2$, that is, in figures, $2043.36 : 923.67 :: 6084 : 2750.18 = ef^2$, whence *ef*= 52.44217 ; now the parallel section *ef* being a similar ellipsis to EF, we will obtain area *ef* by making $EF^2 : ef^2$ :: area EF : area *ef* or 6084 : 2750.18 :: 4239.275 : 1916.3=area *ef* ; finally (area EF + 4area *ef*) × $\frac{1}{8}$ G*p*=solidity EFG, or (4239.275 + 4 times 1916.3) × 15.8 and ÷ 6 = 31, 348.4508 units of volume in the solid to be measured. For proof, the rule given in the remark preceding this example, gives 103.2 : 96.6 :: 33490.2723 : 31,348.4525, the difference $\frac{17}{318484525}$ ou .00000005 being due to the neglected decimals.

PROBLEM L.

To determine the solidity of any frustum ABEF of a hyperboloid with non parallel bases AB, EF.[1]

(See the *tableau.*)

(**185**) **RULE.** *Get separately the respective solidities of the entire hyperboloid* ABG *and of the partial hyperboloid* EFG, *and take the difference of these solidities which will be the required solidity.*



Draw B*b* parallel to EF, E*e* parallel to AB, bisect these two pairs of parallels and through the points of bisection draw the straight lines HO,H′O of which the intersection at O will be the centre of the generating curve. Through the points of intersection G, G′ draw the perpendiculars GP, G′*p* to the bases AB, EF and the required solidity will be (area AB + 4 area intermediate section between AB and G) × $\frac{1}{8}$GP, minus (area EF + 4 area inter. sect. between EF and G′) × $\frac{1}{8}$ G′*p*.

REM I. To fix the direction of the axis CD of revolution : from the centre O with any radius, intersect the opposite sides of the curve, join these intersections by a straight line, and OD drawn perpendicular from the centre O to this latter will be the required direction.

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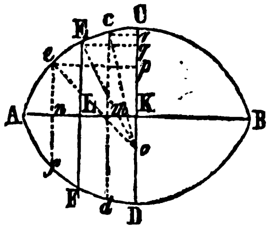
[1] We may also obtain as for the ungula of a prism, pyramid, cone or cylinder, the solidity nearly of the ungula of a right or inclined hyperbolic conoid, and that absolutely in the same manner as for those various solids. (See the problem XXXI, &c

REM. II. To find the points G and G′, that is, the factors GP, G′*p* and the other elements necessary to the calculation of the areas of the intermediate sections and of the solidities of the entire and partial solids, we have seen (**183 T.**) that $2\,OG.GH + GH^2 : 2OG.Gh + Gh^2 :: AB^2\, e\, E^2$, which gives (**96 div. G.**) $(2OG.GH + GH^2) — (2\,OG.Gh + Gh^2) : 2\,OG.Gh : Gh^2 :: AB^2 — eE^2 : eE^2$. In this proportion, we know $(2\,OG.GH + GH^2) — (2\,OG.Gh + Gh^2) = 2\,Hh.\; hO + Hh^2$ (as a simple diagram of $2OG.Gh + Gh^2$ superposed to $2OG.GH + GH^2$ shows at once); we also know $AB^2 — eE^2$ and eE^2; that is, 3 terms to find the fourth $2OG.Gh + Gh^2$; now (**359 G.**) $hO^2 — (2OG.Gh + Gh^2) = GO^2 \sqrt{GO^2} = GO$, $HO — GO = GH$ and with GH and the angle GHB we determine GP, &c., &c.

PROBLEM LI.

To determine the solidity very nearly of any spindle, either circular, elliptic, parabolic or hyperbolic.

(**185**) **RULE.** *Divide the semi-spindle* (ACD or BCD) *into two parallel sections or slices* (AEF, ECDF) *of thickness or height* (AL, LK) *equal or nearly equal, by planes perpendicular to the axis of revolution* (AB) *of the generating curve* (ACB *or* ADB*;*) *get separately the solidity of each of these slices, by adding to the sum of the areas of its parallel or opposite bases, 4 times the area of a section* (*ef*, *cd*) *equidistant from those bases, and multiply the whole by $\frac{1}{6}$ of the height of the slice ; make afterwards the sum of the solidities of the two component slices and double the result for the solidity nearly of the proposed spindle.*

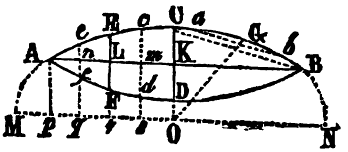


(**187**) **Ex 1. Required the solidity nearly of a circular spindle** (that is generated by the revolution of the arc of a circle) of which the length AB is 48, and the diameter CD 36 ?

Ans. If the intermediate diameters EF, *ef*, *cd* are not given or may be directly obtained by the measurement of the solid to be computed, it will be easy to determine them by calculation ; thus we will obtain immediately the radius *o*C of the arc ACB by the method of par. (**540 G.**) : $24^2 \div 18 = 32 =$ the remainder of the diam. of which CK forms part, the diam.—$32 + 18 = 50$ and the radius consequently$=25$. Now we will obtain *op*, *oq* and *or* respectively equal to the square roots of the differences between the square of the radius and the squares of *ep* E*q* and *cr*, which is evident; now, if we suppose AL = KL we will have A*n*=*n*L = L*m* = *m*K, or $cr = 6$, $Eq = 12$, ep 18, $op^2 = oe^2 - ep^2 = 625 - 324 = 301$ of which the √ is 17.349352 from which subtracting $ok = 7 = 25 - 18$ there remains K*p* or *en* = 10.349352 and consequently *ef* or 2 *en*=20.698704 or say 20.6987, for, as the difference of volume according to this rule is always in excess we may neglect at least the last decimals ; in the same manner we find diam. EF=29.863 and *cd* = 34.5386.

The solidity of $EC = (DC^2 + 4\,cd^2 + EF^2) \times .7854 \times \frac{1}{6}$ KL or by 2 $= 10931.82$, the solidity of $efA = (EF^2 + 4\,ef^2) \times .7854 \times 2 = 4092.72$, these solidities added together and the whole × 2, gives for the solidity of the entire spindle 30049 cubic units.

REM. The accurate solidity of the spindle of the last example is 29916.6714, that is the approximate solidity exceeds by $\frac{132}{29916}$ or .0044 (less than the half hundredth) the real solidity, which in practice is generally equivalent to perfect or at least sufficient accuracy, considering the additional labour which must be devoted to the calculation by the ordinary rules ; and besides as already stated (**1137 G, 140 T.**) we may with the rule given here carry the precision to any degree required by a subdivison of the semi-spindle into more numerous slices and of which the sides come nearer the straight line.



(**188**) **Ex 2. To find the solidity nearly of an elliptical spindle** (that is generated by the revolution of the arc of an ellipsis round a chord perpendicular to one of its axes) of which the length AB is 80 decimetres, the greater diameter CD 24 decimetres and a diam. EF equidistant from A and CD 18.99094 decicimetres ?

Ans. Let AECGB the generating curve ; to find its centre, draw (**176 T, R. I.**) any two parallel chords BC, *ab* and through the middle points of these chords draw a straight line GO which will intersect CD, produced if necessary, in O centre of the ellipsis. Let now CO = 30, we have a diameter of the ellipsis=2CO, an ordinate AK or KB=½AB=40, an abscissa CK or segment of the diam.=12 and consequently the other segment$=2\,CO-CK=60-12=48$, to find (**176 T. R. I.**) the other diameter MN of the ellipsis by making $\sqrt{CK\times(2\,CO-CK)} : KB :: 2\,CO : MN$ or $\sqrt{12\times 48} : 40 :: 60 : 100=MN$, MN being the smaller or the greater diam of the ellipsis, according as the rectangle of the segments is greater or smaller than the square of the ordinate or perpendicular KB.

To obtain *ef*, we will first make the proportion $MN : 2CO$ or (which is the same thing) $MO : Co :: \sqrt{Mq.qN} : qe$ or $50 : 30 :: \sqrt{20\times 80} : eq=24$ and as $nq=KO=CO-CK=30-12=18$, we will obtain $en=24-18=6$ and diam $ef=2en=12$; we will equally find $cs=29.39412$, $cs-ms=1139412=cm$ and $2\,cm=$diam. $cd=22.78824$. If EF were not given it could be equally determined.

Diam. EF$18.99094^2 = 360.6558$
4 Diam. *cd* $22.78824^2 = 2077.2155$
Diam. CD $24.00000^2 = 576.0000$

Sum$=3013.8713$
$\times\ .7854$

Product$=2367.0935$
$\times\ 40$

$\div 6)\ 94683.74$
Quotient$=15780.62$
$=2$ vol. ECDF

Diam. EF $18.99094^2 = 360.6558$
4 Diam. *ef*. $12.00000^2 = 576.0000$

Sum$=936.6558$
$\times\ .7854$

Product$=735.649456$
$\times\ 40$

$\div 6)29425.9786$
Quotient$=4904.32976$
$=2$ vol. EFA

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sol. 2 FC $=15780.62$
sol. 2 EFA$=\ 4904.33$

sol. AB $= 20684.95$

The sum 20,684.85 of these solidities is that of the proposed spindle and differs but by 57 units in excess, or the fourth part of 1 per cent, from the accurate sol. 20628.34 of which the calculation by ordinary rules requires as much more labour, and offers in consequence of the diversity of the operations to be performed (as detailed in an enunciation of 15 lines of text) many more chances of error, as of course it is always the case, more or less, when the process to be followed is not so simple and direct that we may easily account, by following the details of the calculation, for each of them. This is the enunciation in question.

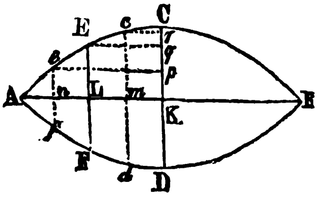
1° *From three times the square of the diameter at the centre* (CD) *subtract four times the square of the diameter* (EF) *between the middle and the end ; also, from four times this last diameter, subtract three* *times the diameter at the centre ; and one fourth of the quotient proceeding from the division of the first difference by the last, will give the central distance* (OK.)

2° *Find by the method of par.* (**176, T. R. I.**) *the axis of the ellipsis and by the method of par.* (**61 T.**) *the area of the generating segment* (ACB.)

3° *Divide three times the area thus found by the length* (AB) *of the spindle, and subtract from the quotient the diameter at the centre ; multiply then the remainder by four times the central distance, and take this product from the square of the diameter at the centre ; this latter remainder multiplied by one third the length of the spindle, and the product again by* 1.57079, *will give the solidity of the spindle.*

The calculation to be made from this enunciation is at least of two or three pages, and that without even comprehending the details of the multiplications, divisions &c. ; whilst all that is essential to the enunciation of the rule given here is resumed in these words : *Multiply the sixth part of the height of each of the component slices by the sum of the areas of its bases plus four times the area of a section half-way between them ; the sum of the solidities thus found will be the solidity of the spindle very nearly ;* and as, in practice, the necessary diameters *ef*, EF, *cd*, CD, are generally obtained by a direct measurement of those diameters or of their respective circumferences, all the calculation to be made is reduced, excepting the multiplications, to the one just indicated at the bottom of last page.

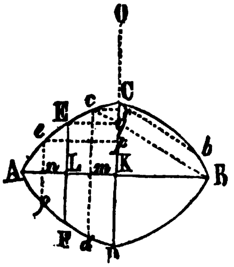
(**189**) **Ex. 3. To find the sol. of a parabolic spindle,** (that is generated by the revolution of a parabola ACB or ADB about a double ordinate AKB perpendicular to the axis CK) of which the length AB is 60 and the greatest diam. CD 34 ?



Ans. In the case of the parabola and equal distances A*n*, *n*L, L*m*, *m*M, the intermediate diameters *ef*, EF, *cd*, if not given, are very easy to be determined, since as seen (**179, T.**) the abscissae or segments C*p*, C*q*, C*r*, of the axis are sa the squares of the corresponding ordinates *ep*, E*q*, C*r* and when these ordinates are equal multiples or sub-multiples of one another, the segments or abscissae are also simple multiples or sub-multiples of the entire axis CK ; now (**215 G.**) on account of $\mathrm{E}q=\frac{1}{2}$ AK we will obtain $\mathrm{E}q^2=\frac{1}{4}\mathrm{AK}^2$ and consequently $\mathrm{C}q=\frac{1}{4}$ CK, we will equally obtain $\mathrm{C}r=\frac{1}{4}\mathrm{C}q$ or $\frac{1}{16}$ CK and $\mathrm{C}p=\frac{9}{16}$ CK since *ep* : AK :: 3 : 4 and that $3^2 : 4^2$ :: 9 : 16 ; we will then find in this manner $Cq = 17 \div 4 = 4.25$, Cr $4.25 \div 4$ or $17 \div 16 = 1.0625$, $Cp = \frac{9}{16}\,17 = \frac{1}{2} + \frac{1}{16} = 8.5 + 1.0625 = 9.5625$, from which we obtain diam. $ef = 2pK = 14.875$, $EF = 2Kq = 25.5$, $cd = 2\ Kr = 31.875$; now sol. $AEF = (\text{area } EF + 4 \text{ area } ef) \times \frac{1}{6}\ AL = (EF^2 + 4\,ef^2) \times .7854 \times \frac{1}{6}\ AL =$ $(25.5^2 + 4 \text{ times } 14.875^2) \times .7854 \times 2\frac{1}{2}$ or at once by 5 (since there are two conoids or equal segments in the spindle to be cubed) $= 6033.1$ cubic units ; the solidity of the frustum $FC = (34^2 + 4 \text{ times } 31.875^2 + 25.5^2) \times .7854 \times 2\frac{1}{2}$ or by 5 to obtain $2FC = 23052.7$ cubicc units ; the sum 29085.8 of these solidities is the solidity of the proposed spindle ; it differs from the accurate solidity 29053.4 but by 32 units, that is, $\frac{32}{29053}$ or .0011, say $\frac{1}{9}$ of 1 per cent in excess.

REM. However complicated ordinary rules for the solidity of the circular and elliptic spindles may be, the rule for the parabolic spindle is on the contrary very simple; it consists only in multiplying the square of the central diameter by the length of the spindle and the product again by .418879 ($= 3.14159 \div 7\frac{1}{2}$) ; but there is always this to be considered that if the spindle were not properly parabolic, this last rule might be pretty far from giving an accurate result, whilst with the general rule found here for all the elementary solids, we have not first to consider the nature of the solid to be measured, except however when we have to determine by calculation the required intermediate diameters.

(190) Ex. 4. A spindle ABCD havingthe appearence of being **hyperbolic** (that is, generated by the revolution of a hyperbola ACB or ADB, about a cord or double ordinate AKB perpendicular to its axis CK or KD) and of which the greater diameter $CD = 71$ inches, measures 106 inches in length AB, and its intermediate diameters taken at 3 places m, L, n, equidistant from each other and each distance equal to the fourth part of the half length AK of the spindle, are respectively $ef = 26.8$, $EF = 49$, $cd = 65.4$: what is its solidity ?



Ans. $(CD^2 + 4\,cd^2 + EF^2) \times .7854 \times \frac{1}{6}$ LK and $(EF^2 + 4\,ef^2) \times .7854 \times AL$, or which is the same thing, since $AL = LK$, sol. $= (CD^2 + 4\,cd^2 + 2EF^2 + 4\,ef^2) \times .7854 \times \frac{1}{6}$ LK or AL, or by $\frac{1}{3}$ LK or AL to obtain at once the solidity of the entire spindle $= (71^2 + 4 \text{ times } 65.4^2 \times \text{twice } 49^2 + 4 \text{ times } 26.8^2) \times .7854 \times 53 \div 6 = 206{,}914$ cubic inches or **119.742 cubic feet.**

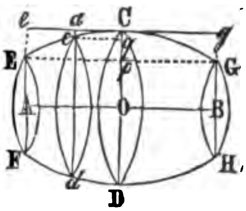
To find Op or Cp and consequently $pK = CK - Cp = en = \frac{1}{2}$ inter. diam. ef, we first obtain $AK^2 : ep^2 :: 2OC.CK + CK^2 : 2OC.\ Cp + Cp^2$, then, as we said (**REM. II.**) $(2OC.CK + CK^2) - (2OC.Cp + Cp^2) = 2Kp.pO + Kp^2$; now it is plain (**339, G.**) that $2Kp.pO + Kp^2 + pO^2 = KO^2$; whence, $pO^2 = KO^2 - (2Kp.pO + Kp^2)$ or $pO^2 = KO^2 - (\overline{2OC.CK + CK^2 - 2OC.Cp + Cp^2})$ and $Op = \sqrt{\overline{Op}^2}$. We will then equally obtain qo by finding first $2OC.Cq + Cq^2 = \frac{(2OC.CK + CK^2) \times Eq^2}{AK^2}$ and by extracting afterwards the square root of the difference or remainder $KO^2 - (\underline{2OC.CK + CK^2 - 2OC.Cq} + Cq^2)$, then there will come $Or = \sqrt{KO^2 - (2OC.(CK + CK^2 - 2OC.Cr + Cr^2)}$, and consequently the other necesssary diameters EF, *cd*. We have already shown that to find the centre O, and consequently OC or OK it suffices to draw and bisect any two parallel chords *c*B, C*b* of the generating curve and then unite the points of bisection by a straight line the prolongation of which will intersect the axis of the curve (produced if necessary) in a point which will be the required centre.

(**191**) **REM.** If we have devoted to the study of the spindle considerable space, it is not because this solid properly so called offers itself very often to the valuation of the measurer; but it is with the view of arriving at the consideration of the frustum of a spindle which forms the subject of the following problem and which presents itself every day under the thousand and one forms of casks, barrels, tuns, puncheons, quarters, &c., such as are used to contain and transport tobacco, sugar, flour, pork, oils, molasses, beer, brandy, liquors in general and a thousand other substances capable of adapting themselves to the form of such vessels.

PROBLEM LII.

To determine the solidity very nearly of the central frustum of any spindle, that is of the frustum of a spindle of which the opposite and parallel bases EF, GH are equidistant from a plane CD parallel to the bases and passing perpendicularly through the centre O of the axis of the spindle of which the frustum forms part.

(**192**). **RULE** *To the area of one EF of the equal bases, add that of a parallel section CD taken at the centre of the frustum and* 4 *times the area of an intermediate parallel section cd equidistant from the* *centre o and base A, and multiply the whole by $\frac{1}{6}$ of the height, length or depth of the frustum ; the result will be the required solidity.*



REM. It is hardly necessary to say that to obtain by calculation the intermediate diameter *cd*, we have C*p* equal to the half-difference between the diameters CD, EF and that we afterwards find C*q* and consequently *cd*==CD—2C*q*, in the same manner as in the various cases of the last problem. If we have to measure the capacity of a cask we will obtain its interior diameter CD by introducing through the bung hole a scale of inches or other equal parts. We will obtain the intermediate diam. *cd* by measuring the distance *ac* between the cask and a rectilineal rod *eg* tangent at C, and afterwards make *cd* ==CD—2*ac*. From the whole length measured outside, the sum of the thicknesses of the two ends must then be subtracted, for the interior length or height to be taken into the calculation. To obtain *eg* tangent at C, it is plain that it will suffice to see that *e*E—*g*G or A*e*=B*g* ; finally *eg* exterior length of the cask is the distance intercepted on the straight line *eg* by two other straight lines H*g*, F*e* resting on the parallel bottoms so as to meet *eg*. We could also arrive (**31, G.**) at the required areas of the respective sections CD, *cd*, EF by measuring on the outside of the cask the circumferences of those sections from which should be deducted the double thickness of the staves multiplied by 3.1416 or by $3\frac{1}{7}$.

Ex. I. What is the solidity of the central frustum of a circular spindle the length of which is 40 inches, the greater diameter 36, the smaller 16, and the intermediate diam. 31.826 inches ?

Ans. (area CD + 4 area *cd* + area EF) $\times \frac{1}{6}\, 40 {=\!=} (36^2 + 4$ times $31.826^2 + 16^2) \times .7854 \times 40 \div 6 -{=} 29{,}340$ cubic inches, exceeding but by .002 8or a little more than one fourth of 1 per cent the accurate solidity 29,257.3 cubic inches==$126\frac{2}{3}$ gallons nearly.

2. The length of the frustum of a circular spindle is 3 feet 4 inches, the diameter at the centre 2 feet 8 inches, the extreme diameter 2 feet and the intermediate diam. 30.0588 ; what is its solidity ?

Ans. 27,301 cubic inches, to $27{,}287\frac{1}{2}$ cubic inches the accurate solidity, or an excess of .0005 or $\frac{1}{20}$ of 1 per cent.

3. Required the capacity of a vessel having the form of the frustum of a circular spindle, the length 50 inches, the smaller and greater diameters 25 and 35 inches and the interm. diam. 32.574 ?

Ans. 39,887 cubic inches÷1728=23.083 cubic feet, to 39,782 cubic inches or 23.022 cubic feet, say an excess of .0026 or nearly one fourth of 1 per cent.

4. The central zone of a circular spindle measures 3 feet in length, the extreme diameters are 2 feet and 16 inches and the calculated interm. diam. is 22.0722 : what is its solidity ?

Ans. 13,104 cubic inches or 7.58327 cubic feet, the accurate solidity according to the general rules being 13.090.4 cubic inches or 7, 57546 cubic feet, say an error in excess of .00103 or $\frac{1}{10}$ of 1 per cent.

5. What is the capacity of a hogshead the length of which is 5 feet, the extreme diameters 50 and 30 inches and the interm. diam. 45.394 ?

Ans. 91,439.89 cubic inches, to 91, 302.75 the accurate solidity, the difference in excess being .0015 or $\frac{1}{7}$ of 1 per cent.

6. A cask appearing to form part of an elliptical spindle is 28 inches long, its greatest diam. is 24 inches, the diam at the head 21.6 and the interm. diam. 23.40909 inches : what is its capacity in wine gallons of 231 cubic inches to the gallon ?

Ans. $(24^2 + 21.6^2 + 4 \text{ times } 23.40909^2) \times .7854 \times 28 \div 6 = 11{,}855.2$ cubic inches, to 11,854.75 the accurate sol. the excess being in this case but of .000005 which shows that the proposed cask is very nearly the frustum of a spheroid, the rule in such case giving as seen (**176, G.**) the accurate solidity. The required capacity in gallons is 51.316.

7. How many gallons may be contained in a ton of elliptic curvature the greater diameter of which is 32 inches, the smaller diameter 24 inches, the diam. at 10 inches from the head 30.16756 inches and the length 40 inches ?

Ans. 27,425.7 cubic inches or (÷231) 118.726, say 118$\frac{3}{4}$ gallons nearly ; the accurate capacity is 27,419.6 cubic inches, the difference in excess being but 6 cubic inches or one 40th of a gallon.

8. The central zone of a parabolic spindle is 36 inches long, its diameter at the centre is also 36 inches, that of the apex 20 inches and the interm. diam. 32 inches; what is its solid content in cubic feet ?

Ans. 27,294 cubic inches, to 27.233.9 accurate sol. or an excess of .0022 ; say an error in excess of $\frac{1}{5}$ of 1 per cent. In cubic feet the solidity is 15.795 to 15.76.

9. Determine the capacity of a tun the length of which is 40 inches, the great and small diameters 32 and 24 inches and the intermediate diameter 30 inches ?

Asn. ($32^2 + 24^2 \times 4$ times 30^2) × .7854 × 40 ÷ 6==27,227.2 cubic inches or 117.87 gallons nearly ; the accurate solidity is 27.210.5 cubic inches, say an error of .00062 or $\frac{1}{16}$ of 1 per cent, equivalent to $\frac{1}{14}$ of a gallon or a little more than half a pint.

10. How many cubic feet are there in a hogshead the diameter of which at the centre is 5 feet, at the head 3 feet, its intermediate diameter 4.5 feet and length 7 feet ?

Ans. 105.3745, to 105.19124 the accurate solidity, or an excess of $\frac{1}{8}$ of 1 per cent.

11. How many gallons of salt can be put into an empty flour barrel the height of which is 25 inches, the inf. or sup. diam. 17 inches, the greater diam. 20 inches and the intermediate diam. between the bottom and the centre 19.3 inches ?

Ans. ($17^2 + 20^2$. + 4 times 19.3^2) or 2179 × .7854 × 25 ÷ 6==7130 cubic inches, dividing by 231 we obtain 30 gallons 2 quarts and 3 pints nearly or (÷2339) 3 $\frac{1}{20}$ bushels nearly.

12. There are three varieties of casks in which the extreme diameters are 24 and 32 inches, in one the interm. diam. is 30.2 inches, in another this diam. measures 30 inches and in third 29.2 inches, the length is 42 inches, what is the content of each cask in imperial gallons of 277.274 cubic inches per gallon ?

Ans. ($24^2 + 32^2 + 4$ times 30.2^2) × .7854 × 42 ÷ 6 ÷ 277.274=104.06, to 104, diff.=$\frac{1}{20}$ of a gallon.

($24^2 + 32^2 + 4$ times 30^2) × .7845 × 42 ÷ 6 ÷ 277.274=103.106, to 103, diff.=$\frac{1}{10}$ of a gallon.

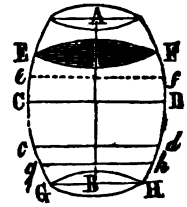
($24^2 + 32^2 \times 4$ times 29.2^2) × .7854 × 42 ÷ 6 ÷ 277.274=99.35 to 99.3, diff.=$\frac{1}{20}$ of a gallon.

PROBLEM LIII.

To find the solidity nearly, of any frustum of a spindle EFHG or cd GH, with parallel bases perpendicular to the axis of the spindle.

(See the *tableau.*)

(193) RULE. *Compute separately the solidity of each of the slices* EFDC, GHDC *situated at opposite sides of the centre on greater diam. CD of the given frustum, by adding to the sum of the bases* CD, EF, CD, GH *of each of them, four times the area of an intermediate section ef, cd, and multiply these sums by one sixth of the height of the respective slices ; the sum of those solidities will be the required solidity.*



REM. It is plain that if the frustum is lateral as *cd*HG or does not extend beyond the centre CD, we will have but one operation to perform to determine its solidity = (area *cd* + area GH + 4 area *gh*) × $\frac{1}{6}$ *o*B.

Ex 1. One of the component parts of a cul-de-lamp presents the form of the lateral frustum of a spindle. Its three diameters are 24,30 and 32 inches and its height 21 inches : what is its solidity ?

Ans. $(24^2 + 32^2 + 4 \text{ times } 30^2) \times .7854 \times 21 \div 6 = 14.294$ cubic inches or 8.272 cubic feet.

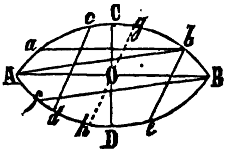
2. A tun placed on end and the height of which is 42 inches and sup. diam. 24 inches, contains wine to the three fourths of its height ; the entire capacity of the tun is 104 imperial gallons (277.274 cubic inches per gallon) how many gallons are there remaining in the tun ?

Ans. Here, since the entire solidity of the frustum of the spindle to be measured is known, then, instead of computing separately the solidities of the two component slices of the frustum to get their sum, it will suffice to cube the empty part of the tun and afterwards subtract its solidity from that of the entire tun. The diam. of the tun at the height to which the wine reaches is 30.2 inches and the intermediate diam. between this latter and the top is 27.6. Then the sol. of the frustum to be deducted is $(24^2 + 4 \text{ times } 27.6^2 + 40^2) \times .7854 \times \frac{1}{4}42 \div 6 = 6233$ cubic inches $\div 277.274 = 22\frac{1}{2}$ gallons nearly, there remains then in the tun $104 - 22\frac{1}{2} = 81\frac{1}{2}$ gallons.

PROBLEM LIV.

To determine the approximate solidity of any segment of a spindle (Adc, aCb Acb) with a single base parallel or not at the axis (AB) of the spindle or to its diameter (CD ; or the solidity of a frustum (ABba, dcbe, AbBf) with parallel bases inclined or not to the axes of the solid ; or the solidity of any ungula ABC—D, ABC—D′, AEC—D of a spindle.

(194). RULE. *To the sum of the areas of the parallel or opposite bases (if there is but one base, we consider the other = 0) of the frustum, add* 4 *times the area of a section equidistant from those bases and* *multiply the whole by one sixth of the height of the segment, frustum or ungula as the case may be.*



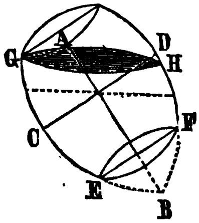
REM. If the given frustum contains the centre O of the spindle of which it forms part, draw through the centre a section gOh parallel to the bases and calculate separately each of the component parts of the frustum and add then together; but if the parallel bases Ab, fB are equidistant from the centre O, then it is plain that we will have but one operation to perform and afterwards double the result.

PROBLEM LV.

To determine the solidity nearly of any frustum of an ungula with non parallel bases.

(See the *tableau.*)

(**195**) **RULE 1.** *Decompose the frustum EFHG by an imaginary plane parallel to either GH, EF of its bases and passing through the point F or H (as the case may be) the nearest to its other base, into the frustum of a spindle with parallel bases and an ungula; then compute separately their solidities and add them together.*



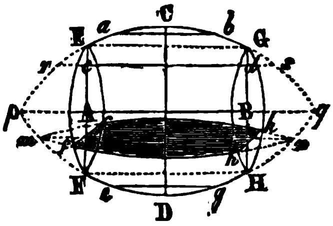
(**196**) **RULE II.** *Compute by the last problem the respective solidities of the two segments of a spindle with a single base* GHB, EF'B *of which the given frustum forms part; the difference of these solidities will be the required solidity.*

REM. An inclined ton or cask containing liquor and which one would not displace to facilitate its gauging will sometimes present to the valuation of the measurer a solid of this kind.

PROBLEM LVI.

To value the content, nearly, of a ton or cask laying on its side and being but partly full. (See the *tableau.*)

(**197**) **RULE I.** *After having obtained the chords and versed-sines of the segments of a circle forming the opposite bases* fFf hHh *of the frustum* FH *to be valued, multiply the sum of those bases plus* 4 *times the area of a central section parallel to those bases, by* $\frac{1}{6}$ *height or length* AB *of the cask, or to be still nearer the accurate content, operate only on the half of the frustum and afterwards double the result.*



(**198**) **RULE II.** *If the liquor in the ton does not reach the heads or ends* EF, GH *of the vessel, as at eg or* FH, *we will value its content by the method of the last but one problem, and the same if the liquor outreaches those ends, as at* EG *or ab, we will calculate in the same manner the segment* ECG *or a*C*b, to diminish by as much the content of the entire ton. If the area of the liquor is at* AB, *axis of the ton it is plain that we will obtain the content* AFDHB=CEFD=½FG. *If on the contrary the surface is at cd or fh, we will first calculate by the method of the last but one problem, the frustum of a spindle rsqDpr or mnD (as the case may be) of which the corresponding segment of the ton forms part, to subtract afterwards the ungulae, or kinds of pyramids hnh*H, *fmf*F *or the solids crpF, dsq*H *composed of the semi-conoids* AF*p*, BH*q and of the frusta rc*A*p, sd*B*q of which we will pretty correctly obtain the solidities by the general rule of the sum of the parallel bases p*A, *rc* + 4 *times the intermediate section, multiplied by one sixth of the height.*

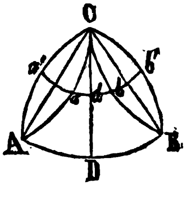
PROBLEM LVII,

Determine the solidity nearly of a convex or concave conoid Aa′ Cb′B, AaCbB or of the segment of a spindle terminated by a convex or spherical base ADB.

(See the *tableau.*)

(**991**) **RULE I.** *Decompose the solid by a plane passing through AB into the segment of a spindle or conoid ABC and the segment of a sphere ADB which you will value separately by the rules alreadg given and ufterwards take the sum of the component solidities.*

(**200**) **RULE II.** *To the area of the convex base (ADB) add* 4 *times that of a parallel convex section a′db′ equidistant from the base and apex and multiply the sum by one sixth of the height CD of the solid.*



What has been already stated (**1077 G, 137, T.**) concerning the spherical sector and ordinary conoid, suffices to show immediately that we must by that rule arrive at a pretty correct determination of the solidity of the proposed solid.

REM. If the height CD were unequal to AC or BC, that is, greater or less than AC or BC we would evidently have to increase the sol. of the proposed conoid, or to diminish it by the difference of the solidities of the respective segments ADB having for radii CD = AC or BC and CD < or > AC or BC, as the case may be.

The same if the base of the conoid were concave, we would compute the sol. of the corresponding conoid with plane base, and afterwards deduct from it the sol. of the hollow segment.

(**201**) **RULE II**. *Compute the solidity of the component spherical sector* ABB–C, *then the solidity of the continued frustum of a prism of which the generating segment* AoCa′A *or* BoCb′B *is the section; the sum of these solidities will be the required solidity.*

PROBLEM LVIII.

To determine the solidity of any vault of which the thickness is not uniform.

(**202**) **RULE**. *Calculate separately (page* 431 G.) *by the preceding problems, the volumes of the component exterior and interior solids (that is of the prisms or cylinders, hemi-spheres, hemi-spheroids or conoids, or of the segments of those solids) and afterwards take their difference which will be the solid content of the proposed vault.*

PROBLEM LIX.

To determine the solidity of any prismoid or cylindroid.

(See the numerous and varied prismoids and cylindroids of the *tableau.*)

(**203**) **RULE**. *To the sum of the areas of the two parallel bases, add four times the area of a section equidistant from those bases, and multiply the whole by one sixth of the height of the body; the result will be the required solidity.*

REM 1. It is said (**1102 G.**) that "the prismoid is a solid having for its parallel bases, any plane figures with parallel sides." This definition does not exclude the equality of the parallel sides; therefore, *any prism or cylinder (infinitary prism) is at the same time a prismoid.*

Neither does the definition exclude the proportionality of the parallel sides; therefore, *any frustum of a pyramid or cone (frustum of an infinitary pyramid) with parallel bases, is a prismoid.*

Nor does the definition assign limits to the inequality of the parallel sides; therefore, each of the sides of one of the parallel bases may diminish indefinitely, even so as to become finally$=0$; that base will therefore also be reduced to zero or to a single point, as in the case of the pyramid; therefore, *the pyramid or cone* (*infinitary pyramid*) *is also a prismoid.*

If the sum of all the sides, but one, of one of the bases, becomes equal to the side thus excepted, that base will be but a line or edge parallel to the plane of the other base, as in the case of the wedge; therefore, *any wedge or other solid having for one of its bases any plane figure and for the other base a line parallel to the first, is still a prismoid.*

(**204**) It would appear that in this manner of reducing to a simple line any plane figure, we have neglected the necessary paralllelism of the opposite sides; but it is not so, for if the base to be reduced is a rectangle for instance, the two sides perpendicular to the excepted side become each of them$=0$; the sum of the sides less one, is the side of the rectangle parallel to the excepted side, and which, when the perpendicular sides become null, ends by approaching the excepted side so as to form with this latter but one and the same line or edge. If the base to be reduced is any polygon, there will be, or not, in the perimeter of that base a side parallel to the excepted side; if there is a side parallel to it, this side may diminish or increase so as to become of equal length to that of the excepted side, and all the other sides becoming each$=0$, the two parallel sides will unite to form but one; if there is not in the base on which we operate a side parallel to the excepted side, we will interpose between two of the sides of that base, a side which shall be the required parallel; for in the same way as one of the sides of the prismoid may, without affecting the definition, become equal to 0, so may a side at first equal to zero acquire any development, and that, in any proportion as in any direction.

(**205**) It is plain that if one of the two bases of any figure, may become a line, it is the same of the other base which may also, from any figure, become a line. If the two lines which now form the opposite bases are parallel to each other, it is plain that the solid will have ceased to exist or shall have become equal to zero or to a mere surface; but if the lines or edges answering as opposite bases to the body in question are not parallel to each other, though in planes parallel to each other, the solid will not have ceased to exist; therefore *a prismoid may be such as that its opposite bases be both of them simple lines or edges.*

(**206**) Let us say, to sum up, that : *a prismoid may have for its parallel bases : any two figures equal or similar, any two figures unequal or not similar ; any figure and a line parallel to the plane of that figure, any figure and a point, any two lines not parallel, but situated in planes parallel to each other ;* say, for instance : two equal or unequal squares ; a square and any rectangle ; any two rectangles or parallelograms ; any two equal or similar, unequal or dissimilar polygons, of which the sides of the one correspond either to parallel sides or to angular points of the other ; a square, rectangle or other polygon and a circle or ellipsis (infinitary polygon) ; a circle and any ellipsis or any two ellipses (this latter prismoid with curvilineal parallel bases is sometimes known under the name of *cylindroid*) a square, rectangle, parallellogram, polygon, circle, ellipsis and a line ; a square, rectangle, parallelogram, polygon, circle, ellipsis and a point ; two lines of any lengths not parallel but situated in planes parallel to each other. [1]

(**207**) **REM II.** There is now to be considered the kind or nature of the figure answering as an intermediate section between the opposite bases of the prismoid to be measured. Thus, it is plain that if the opposite bases are rectangles with parallel sides, the intermediate parallel section will also be a rectangle or a square ; if the two bases are parallellograms with parallel sides, the section will also be a parallellogram ; if the bases are a square, rectangle, parallelogram and line parallel to one of the sides of such rectangle, &c., the section will still be, in the first case a rectangle, in the second case a rectangle or a square ; in the third case a parallelogram ; if the bases are any figure and a point, the section will be a figure similar to the base and equal (**131, T.**) in area to the fourth of the base ; if the two bases are lines (**907**, **G.**) the one perpendicular to the direction of the other, the section will be a square or a rectangle ; if the bases are lines not perpendicular the one to the direction of the other, the section will be a lozenge or parallelogram.

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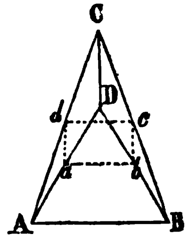
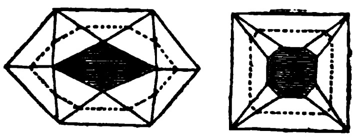
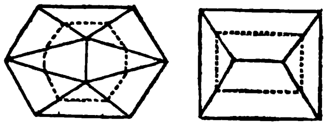
[1] All these forms are met with in practice, and more especially among the diversified roofs of buildings of all kinds. A tower or square turret for instance, will be often terminated by a roof crowned by a circular or octagon platform, or the tower will have for ground plan a circle, and for platform to roof a square or other polygon, or again there may be two squares of which the sides of the one are parallel to the diagonals of the other, that is for prismoids of which the parallel bases are any figures. If a building of which the horizontal section is a square, rectangle, or polygon, is covered with a roof terminated by a ridge long or short, we will obtain the prismoid of which one of the bases is any figure and the other base a line. The wedge is also a solid of the kind. Neither is it rare to find among the component parts of a roof or other object to be measured, prismoids of the kind of that of par (**208, T.**) that is, of which the bases AB, CD are both simple lines, without the area of the intermediate section *abcd* having any the less for that a real and easily determined value. In this latter case the factor "4 area *abcd*" = BA × CD (**208, T.**) whence, the sol. = AB × CD × height ÷ 6.



(**208**) Nothing is easier in all these cases than to determine the area of the intermediate section of which the multipliers or factors are each an arithmetical mean between the parallel sides of the opposite bases or between the sides or edges and opposite points or apices as the case may be. For instance, in the prismoid AB–CD where each of the bases is a simple line or edge AB, CD and of which the area is consequently null, we obtain for intermediate section the square, rectangle or parallelogram *abcd* in which $ab = \frac{1}{2}$ AB + D or $= \frac{1}{2}$ AB, since D$=0$, the same, $dc = \frac{1}{2}$ AB$= ab$, and $ad = \frac{1}{2}$ CD$= bc$; whence area section *abcd* $= ab \times ad$ or $\frac{1}{2}$ AB $\times \frac{1}{2}$ CD if it is a rectangle, or (**8**, **T.**)$= ab \times ad \times$ nat. sin. angle *bad* if it is a parallelogram.



(**209**) If one of the bases is any polygon and the other base also any polygon, and if all the lateral faces of the prismoid are triangles, that is if each of the sides in one of the bases corresponds to a point in the other base, the number of sides in the intermediate section will be equal to the sum of the numbers of sides in the two bases.



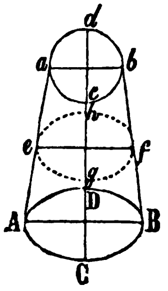
(**210**) If one of the bases is any rectilineal figure, and the other base a line not parallel to the sides thereof the number of sides in the int. section will be equal to the number of sides of the base plus 2 ; and if the line or one of the sides, or more than one, of the figure forming one of the bases, is parallel to one or to more than one of the sides of the other base, the number of sides of the interm. section may vary indefinitely, as the case may be, but will nevertheless be always easy to determine with the help of a simple sketch of the figure.

The summary just made may still be simplified, abridged or translated as follows, to wit : *The prismoid or cylindroid (infinitary prismoid) is a solid with parallel bases of which the planes (lateral faces) pas-*

sing through the sides or edges of one of the bases, are terminated by points or by parallel sides or edges in the other base.

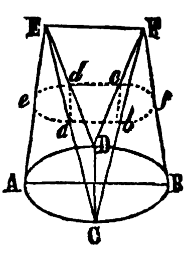
In other terms : *The prismoid or cylindroid is such that all its lateral faces are, or its lateral surface may be decomposed into triangles or rectilineal trapeziums, that is, with plane surfaces, or parts capable of being developed into plane surfaces.*

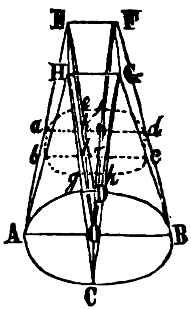
Let us add that every prismoid may be decomposed, if one of its bases is (fig. to par. (**212 T.**) any figure and the other base a line, into two pyramids and an elementary prismoid having for its bases lines, (fig. to par. **208, T.**); if its two bases are any figures, into four pyramids having their bases two and two in the opposite bases of the solid, and a prismoid having lines for its bases ; or, at pleasure, into pyramids, wedges, &c., and prismoids with lineal bases, according to the manner of operating the division of the solid by planes of which the number and position may be varied.



(**211**) If one of the bases is for instance a circle or ellipsis and the other base an ellipsis, the interm. base will also be an ellipsis of which we will obtain the diam. *ef*=½ (AB + *ab*) and the diam. *gh*=½ (CD + *cd*).

(**212**) If one of the bases is a circle or an ellipsis AB and the other base a line EF, the, interm. base *abfcde* will be a mixtilineal figure of which the parts *ab* and *cd* will be straight lines parallel to EF, and the parts *aed*, *bfc* similar figures (**1033, G.**) to CDA, CBD. To calculate the area of the interm. section, we have (**1033, 520 G.**) *ab* and *dc* each=½ EF, *ad* and *bc* each=½ CD, and as the component parts ACD–E, BCD–F of the prismoid are evidently pyramids with mixtilineal bases, we obtain (**131 T.**) the area *aed*= ¼ area ADC, area *bfc*, =¼ area BCD ; that is we will obtain area section *ef*=*ab* or ½ EF × *ad* or *bc* or ½ CD + ¼ area AB, and if EF is not parallel to AB or perpendicular to the direction of DC, we will moreover multiply (**8 T.**) the product *ab*, *bc* by the nat. sin. of the angle *bad* or substitute for the factor *ad* or *bc* the perpendicular breadth of the parallelogram *ab cd*.





(213) If one of the bases is a circle or an ellipsis AB and the other a square or rectangle EG, the given prismoid will be decomposed into: 1°, a prismoid EFGH—CD (having for its bases (**207 T.**) a square or rectangle and a line, and for interm. section a rectangle *efgh* where *ef*=*gh* =½ EF or GH, and *eg*=*fh*=½ EH + ½ CD); 2° two prismoids AO–EH and BO—FG (having each for its bases lines AO, EH and BO, FG and (**207 T.**) for interm. base a rectangle *ablk* where *ab*=*kl*=½ EH and *ak* or *bl*=½ AO, and *nrcd* where *nr* or *cd* =½ FG and *nd* or *rc*=½ OB); 3° four pyramids AOC -H, AOD–E, BOC–G and BOD–F (having each for its interm. section a figure *blg*, *aek*, *rch* and *ndf* respectively equal in area to one fourth of the corresponding base AOC, AOD, BOC and BOD, or their sum equal in area to one fourth of the base AB.

(214) It is plain from the prismoids or cylindroids just mentioned that those bodies may indefinitely vary their forms, but the preceding considerations will suffice to indicate the manner of proceeding in each case to the determination of the intermediate area to enter as an element into the calculation of the solidity to be established; or if necessary, to determine immediately whether the proposed solid is, or not, such a prismoid or cylindroid that its solidity may be measured by the general rule here given.

Ex 1. A tent or tester of which the sup. base is a circle or ellipsis of one metre in area, and its inf. base a rectangle of 3 metres area, has for its intermediate section a mixtilineal figure of which the area is two metres, height or perpendicular distance between the parallel bases 2½ metres; required the solidity of the space or air comprised between the curtains?

Ans. (1 + 3 + 4 lines 2) x 2.5 + 6 = 5 cubic metres.

2. A camping tent of which the top or sup. base is a ridge, that is a simple line or edge 2 yards long, and of which the inf. base is composed of a rectangle of 2 x 3 yards and two semi-circles of 3 yards diam. is 2 yards in height; what is its solidity?

Ans. In this example it is plain that the prismoid to be cubed is composed of a wedge with equal edges (that is (**1100, G.**) of a triangular prism) and two semi-cones. The area of the base of the tent is composed of that of the rectangle=2 x 3=6 square yards, and of that of two semi-circles, that is of a circle 3 yards diam. =3 x 3 x

.7854=7.0686 square yards, in all 13.0686 square yards ; the area of the intermediate section is equal to half the rectangle at the base plus one fourth (**212, T.**) of the two semi-circles, and has consequently the value of 3 + 1.76715=4.76715. The area of the sup. base being null in the actual case, the sol.=(area base + 4 interm. area) × height÷6,=(13.0686 + 4 times 4.76715) × $\frac{1}{6}$ height=32,1372 × 2÷6=10.7124 cubic yards.

REM. If the ext. surface of the tester or of the camping tent of the two last examples, instead of being taut, that is plane or capable of being developed (**1140 G.**) into a plane surface, were concave or not taut, we would equally obtain the required solidity, at least very nearly (**139, 140, T.**) by the same rule (**203, T.**)

Ex 3. An observatory of which the ground plan is an octogon of 100 metres area, is crowned with a roof terminated by a circular platform of which the area is 25 metres, the area of the intermediate section is 56 metres ; what is the solidity of the space occupied by the roof the height of which is 6 metres ?

Ans. 100 + 25 + 4 times 56=349 cubic metres.

PROBLEM LX.

To determine the accurate solidity of any irregular body of small dimensions or of a body composed of several elementary parts with different dimensions and forms.

(**215**) **RULE. If it is the capacity of any vase or vessel which we want to measure,** *the idea generally suggests itself of arriving at the result by determining the number of times which such a vessel can give place to or contain the contents of any other vessel of an elementary form of which we know the capacity.*

(**216**) **But if it is the solidity of the substance itself of the vessel, &c., which we desire to measure,** the manner of operating does not immediately present itself to the mind of any one wishing to obtain the result.

(**217**) **RULE.** If **the solidity to be measured is that of a non absorbent substance,** *we immerse it in a vessel full of water or any other liquid of which we will measure the displacement by means of another vessel of known capacity ; or if the first vessel is large enough and its form rectangular or cylindrical and of easy gauging, we will first put in it enough liquid to cover the object to be measured ; having afterwards observed the height of the level of the water in the vessel, we will im-*

merse in it the object in question and observe again the level of the liquid ; if now we suppose that each fraction of a metre, inch, line or any other unit of the height of the containing vessel corresponds to a cubic metre, foot, inch, or line,, &c., we will have but to count the number of such units in the height of the displaced level of the water to obtain immediately the solidity of the proposed object.

(218) **If the body is absorbent,** *we may for instance use sand or any other fluid substance, of the kind, that we can level the surface of by means of a rod with a rectilineal edge.*

In this manner we would arrive at the solidity of the most deversified bodies of the animal, vegetable or mineral kingdom and of the thousand and one raw or manufactured objects which we have constantly under our eye and of which it would often be impossible to measure the solidities by the ordinary rules of geometry.

It is well to remind also that we may arrive by a simple proportion at the solidity of a body by comparing its weight with that of another body of the same substance and of determined solidity, that is by the system of specific gravities which shows at the same time how to obtain the solidity of a body from its weight : which will form the subject of the next problem.

Ex. 1. The weight of an irregular block of stone is 13 pounds 7 ounces : required to determine with the help of the given piece the weight nearly of a cubic foot of such stone ?

Ans. First cube the block of stone ; to that effect get a rectangular vessel, say 10 inches square or 100 inches in horizontal area, and the height of which is divided into inches and hundreths of an inch ; having poured into the vessel water enough to cover the stone to be cubed, I note the height of the water which I find 8.53 inches, I then immerse the stone in the vessel and I note again the height of the water whichis now 9.89 inches ; the difference of these heights is 1.36 inches. Since the vessel is 10 × 10 inches, it is plain that every inch of its height corresponds to 100 cubic inches and consequently, each hundredth of an inch of such a height to one cubic inch ; therefore the observed height 1.36, of the displaced level of the water corresponds to 136 cubic inches ; therefore the solidity of the stone is 136, and we will now obtain the weight of the cubic foot by making 136 : 215 ounces (weight of the stone) :: 1728 cubic inches (that is a cubic foot) : 2732 ounces, or, dividing by 16, 170¾ pounds, the required weight.

2. In a cylindrical vessel such that each inch of its height corresponds to 1 cubic inch of space or solidity, we have immersed a piece of silver which has displaced by 73 hundreths of an inch the level of the liquid in the vase ; required the solidity of the ingot of silver ?

Ans. .73 of a cubic inch.

3. Having filled with water any vessel, we have immersed in it an object the solidity of which we want to know ; we have gathered in another vessel, the water overflown, the quantity of which is 3 gal. 2 quarts and $\frac{1}{2}$ pint ; what is the solidity of the proposed object, the gallon made use of being 231 cubic inches ?

Ans. 1 gallon $+ 2$ quarts $+ \frac{1}{2}$ pint $= 231 + 115\frac{1}{2} + 14\frac{7}{16} = 360\frac{15}{16}$ cubic inches.

4. Required the solidity of an absorbent substance placed in a vessel one foot square filled with sand ; after having removed the object to be measured, we find that the uniform height of the sand in the vessel, first levelled to that effect, is .3 of a foot, the height of the vessel being 1.5 feet ?

Ans. $1.5 - .3 = 1.2$ feet = height of the displaced level of the sand, and as the vessel is 1 square foot in horizontal section, it follows that the solidity of the object is 1.2 cubic feet.

5. In a vessel having the form of the frustum of a cone is a quantity of liquid of which the diameter at the surface is 10 inches ; we immerse in it an object which increases by 9 inches the height or depth of the liquid in the vessel and which gives to its displaced surface a diameter of 14 inches ; required the solidity of the proposed body ?

Ans. The volume of water displaced which is at the same time that of the object, is that of the frustum of a cone of which the parallel bases measure respectively 10 and 14 inches and of which the height is 9 inches ; this sol. = (**112, T.**) $(10^2 + 14^2 + 4$ times $12^2) \times .7854 \times 9 \div 6 = 872 \times .7854 \times 1.5 = 684.8688 \times 1.5 = 1027.3032$ cubic inches.

THEOREM LXI.

To determine the solidity or weight of any body or substance, by comparing the volume or weight of such body with that of a body or substance of the same nature of which we know beforehand the weight and volume.

(**219**) **REM.** The weight of a cubic foot of water at the temperature of 40° Fahrenheit (at which water nearly reaches its greatest density) is 1000 *ounces avoir du poids* nearly, or $62\frac{1}{2}$ pounds (english weight) and we denominate weight or specific gravity of any body or substance, the weight of a volume of such body or substance equal to that of the water taken for comparison ; whence it results that if in advance we know the weight of a cubic foot, for instance, of each of the different substances that we may be called on to measure or value, as stated in table X, we will at once determine by a simple proportion the volume of any other weight or quantity of the same substance or the weight of any other volume of such substance, by the following rules.

(220) **RULE. To determine the solidity of a body from its weight** ; *make the proportion : the specific weight of the proposed body is to* (:) *its weight in ounces or pounds, &c. as* (::) 1 *cubic foot or* 1728 *cubic inches, is to* (:) *the solidity of the body in feet or inches, as the case may be.*

Ex. 1. The weight of a shell or cast iron ball or of any fragment of such a solid is 45 pounds : required the solidity of the proposed body ?

Ans. It is seen by table X of specific gravities that the weight of cast iron is 450 pounds nearly, per cubic foot ; we will then obtain the required solidity by making 450 pounds : 1728 cubic inches :: 45 pounds : 172.8 cubic inches.

2. Required the volume of a marble statue the weight of which is 1000 pounds, the specific gravity of the marble from which the statue is drawn being 170 pounds nearly to the cubic foot ?

Ans. 170 pounds : 1 cubic foot : : 1000 pounds : 5.9 cubic feet nearly.

3. A quantity of sand weighs 13 pounds : what is its solidity ?

Ans. From table X, the specific gravity of sand is 1.520, that is, 1.520 times the weight of an equal volume of water or 1520 ounces to the cubic foot (since the weight of a cubic foot of water is 1000 ounces) ; we will therefore make 1520 ounces : 1728 cubic inches :: (13 × 16=) 208 ounces : $x = \frac{1728 \times 208}{1520} = 236\frac{1}{2}$ cubic inches.

4. The weight of a tusk or tooth of an elephant is 25 pounds ; what is its solidity ?

Ans. Ivory is 1825 ounces to the cubic foot ; we will therefore obtain the solidity of the tooth by making 1825 : 1 :: (25 pounds or) 400 ounces : .22 nearly of a cubic foot, or 1825 ounces : 1728 cubic inches :: 400 ounces : 378.74 cubic inches.

It is required to determine in advance the probable weight st iron grating which must be cast according to a carved model of pine wood the weight of which is 7 pounds ?

Ans. We will first obtain the solidity of the pine model by making, as per rule (the pine being considered in this case as of 25 pounds to the cubic foot) 25 pounds : 1 cubic foot :: 7 pounds : .28 of a cubic foot. Now, as the solidity of the cast iron will also be = .28 of a cubic foot and the weight of cast iron is 450 pounds per cubic foot, we will obtain the weight of the proposed grating=450 × .28= 126 pounds.

(221) RULE. To determine the weight of a body from its volume; make the proportion : as one cubic foot is to (:) the volume of the proposed body, so is (::) its specific gravity to (:) its weight.

Ex. 1. The volume of a heap of snow on the roof of a building is 7000 cubic feet, the weight of a cubic foot of this snow, made heavy by rain, &c. is 30 pounds : required the total weight which bears on the roof ?

Ans. 7000 × 30=210,000 pounds.

2. What is the weight of a piece of pure cast gold the dimensions of which are 3 inches by ¾ × ⅓ inches ?

Ans. The solidity=3 × ¾ × ⅓=2¼ cubic inches; the specific gravity of pure gold is 19.258 ; the rule gives : 1 cubic foot or 1728 cubic inches : 2¼ cubic inches :: 19.258 : x $=\frac{19.258 \times 2.25}{1728}$=25.07552 ounces

3. One desires to know the weight of a firkin of butter the volume of which obtained from the rule to article (112), is 1830 cubic inches ?

Ans. The specific weight of the butter is .940 of that of water, that is, of 940 ounces to the cubic foot ; we will therefore obtain the required weight$=\frac{1830 \times 940}{1728}$=995½ ounces, ÷ 16=62 pounds 3½ ounces.

4. What is the weight nearly of a stick of english oak half-dry, the volume of which is 150 cubic feet ?

Ans. The half-dry oak, from the table, is 66 pounds nearly per cubic foot, whence the required weight, is 150 × 66=9900 pounds.

5. What is the weight nearly of a box of bound books the volume of which is 15 cubic feet ?

Ans. 15 cubic feet × 43 pounds nearly=645 pounds.

PROBLEM LXII.

To determine the specific gravity of any body or substance.

(**222**) **RULE. I.** *Cube and weigh the proposed body, and afterwards make this proportion ; as the solidity of the body is to* (:) *its weight in ounces, so is* (::) *a cubic foot of such body to* (:) *the weight of one foot of it in ounces ; that is, by cutting off three figures for deci- its specific gravity.*

Ex. 1. What is the specific weight of seasoned black walnut, if a sample of this wood the dimensions of which are 11 × 7 × .9 inches, weighs 24 ounces ?

Ans. 11 × 7 × .9 = 69.3 cubic inches = sol. of the proposed body ; now, from the rule 69.3 inches : 24 ounces :: 1728 inches : 598 ounces or 37.4 pounds ; the required specific gravity is therefore .598 of that of water the weight of which is 1000 ounces to the cubic foot.

2. An irregular piece of chalk of which the solidity has been obtained, = 432 cubic inches, by the method of example 4 of the last but one problem, weighs 43½ pounds : required the specific gravity of that substance.

Ans. 432 inches : 1728 inches :: 43½ pounds : 174 pounds : whence, the required specific gravity is 174 × 16 = 2.784 times the weight of an equal volume of water.

3. A bateau or pontoon of 100 feet by 20 × 10 feet and the total volume of which is consequently 20,000 cubic feet, required in its construction 5000 feet of white pine half-seasoned, the weight of which is estimated at 40 pounds to the cubic foot, 500 cubic feet of elm computed at 50 pounds to the cubic foot, and 5000 pounds weight of iron spikes : required the draughft of water of the proposed body ?

Ans. The weight of the pine = 5000 × 40 = 200,000 pounds, the weight of the elm = 500 × 50 = 25000, the iron 5000 pounds ; the total weight of the bateau is consequently 230,000 lbs ; the average weight or the specific gravity of the pontoon is 230,000 pounds ÷ 20,000 cubic feet = 11.5 pounds to the cubic foot, that is 11.5 × 16 = 184 ounces per cubic foot, say .184 of the weight of an equal volume of water. The height of the poontoon is 10 feet, therefore the draught will be .184 of the height of the pontoon or 1.84 feet, that is 1 foot 10 inches and .96 of an inch = 1 foot 11 inches nearly.

4. By what quantity can the bateau or poontoon of the last example be loaded without causing it to founder or sink beyond its deck or superior surface ?

Ans. Since water weighs 62.5 pounds to the cubic foot and the total volume of the pontoon is 20,000 cubic feet, the total weight of the water which the pontoon must displace before sinking to the level of the water is 20,000 × 62.5 = 1,250,000 pounds; now the weight of the boat is but 230,000 pounds; whence it follows that we might still without causing the bateau to founder load it with a weight equal or nearly equal to the difference between 1250,000 pounds and 230,000, that is 1020,000 pounds.

(**223**) **RULE II. If the body to be computed is heavier than water**; *first weigh the body in air, then in water, by means of a hydraulic balance; the difference between the results will be the weight lost in water, or the weight of a quantity of water equal in volume to that of the body. Make now the proportion: as the weight lost in water (:) is to the weight of the body in air (::) so is the specific gravity of water (:) to the specific gravity of the body.*

Ex. 1. A piece of tin weighs 183 pounds, its weight in water is but 158 pounds: what is the specific gravity of tin?

Ans. 183—158=25 : 183 :: 1000 : 7320=required specific gravity.

2. A block of granite weighs 21 ounces in air and only 13 ounces in water: what is the specific gravity of the granite?

Ans. 2625.

(**224**) **RULE III. If the body to be computed is lighter than water**; *tie to the proposed body by a thread the weight of which is relatively null, another body heavier than water, so that both of them taken together may penetrate or sink in the water; having first weighed each body in air, and the heavier in water, weigh then in water the compound body, and from the weight lost by the compound body, substract the weight lost by the heavier body as weighed alone; the remainder is the weight lost by the light body. Then: as the weight lost by the light body in water, (:) is to the weight of that body in air, (::) so is the specific gravity of water (:) to the specific gravity of the body.*

Ex. 1. To a piece of elm which in *air* weighs 15 grains, we have tied a piece of copper the weight of which is 18 grains *in air* and 16 grains *in water*, and the compound in water weighs but 6 grains: what is the specific gravity of the elm?

Ans. 18—16 =2 =the number of grains lost by the copper *in the water.*

18 + 15— 6=27=the number of grains lost by the compound *in the water.*

27—2 =25=the number of grains lost by the elm *in the water.*

25 : 15 :: 1000 : 600=the specific gravity of the elm.

2. A piece of copper, weighing in air 27 ounces and in water 24 ounces, is tied to a piece of cork weighing in air 6 ounces, and the compound weighs in water but 5 ounces : what is the specific gravity of cork ?

Ans. 0.240.

PROBLEM LXIII.

To determine the quantity of each ingredient or element in a compound of two substances or elements.

(225) **RULE.** *Find first the specific weight of the compound, mixture or alloy, and of each of the component elements and multiply the difference of every two of these three specific weights by the third. Make then : the greatest product,* (:) *is to each of the other products,* (::) *as the weight of the alloy,* (:) *is to the weight of each ingredient.*

Ex. 1 A mass of gold and silver weighs 62 ounces, and its specific gravity is 16126 ; what is the quantity of each ingredient, the specific gravity of gold being 19640, and that of silver 11091 ?

Ans. (19640—11091) × 16126 = 137,861,174. Alloy.
(19640—16126) × 11091 = 38,973,774. Silver.
(16126—11091) × 19640 = 98,887,400. Gold.

137,861,174 : 98,888,400 :: 63 : 45 ounces, 3 penny weights, 19 grains of gold.

137,861,174 : 38,973,774 :: 63 : 17 ounces, 16 penny weights, 5 grains of silver.

2. A mass of copper and gold weighs 48 ounces, and its specific gravity is 17150, the specific gravity of gold is 19640 and that of copper 9000 : what is the quantity of each element of the mixture ?

Ans. Gold = 42 ounces 2 penny weights 2 $\frac{20179}{45619}$ grains, copper = 5 ounces, 17 penny weights 21 $\frac{25449}{45619}$ grains.

3. An alloy of silver and copper weighs 60 ounces, its specific gravity being 10535 : required the weight of each ingredient, their respective specific gravities being 11091 and 9000 ?

Ans. 46 ounces 7 penny-weights 9 $\frac{1232397}{1468579}$ grains silver, 13 ounces 12 penny-weights 14 $\frac{238182}{1468579}$ of copper.

4. An alloy of copper and tin weighs 112 pounds and its specific gravity is 8784, what is the quantity of each of the ingredients of the mixture, their respective specific gravities being 9000 and 7320 ?

Ans. 100 pounds copper, 12 pounds tin.

5. Required the weight of gold, in a compound of quartz and gold the specific gravity of which is 3500, that of gold being 19640 and that of quartz 3000 ?

Ans. 19640—3000=16640, 16640 × 3500=58,240,000=
Factor for the compound body.
19640—3500 = 16140, 16140 × 3000=48,420,000=
Factor for the quartz.
3500—3000=500, 500 × 19640=9,820,000=
Factor for the gold.

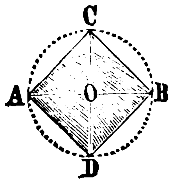
58240000 : 9820000 :: 100 : 16.86.8612638—ounces of gold; if this result be correct, the weight of the quartz must be equal to the difference between the weight of the gold and that of the alloy, and in fact 58240000 : 48420000 :: 100 : 83.1387362 + ounces of quartz; the sum of these numbers=100; therefore, &c.

PROBLEM LXIV.

To determine the solidity of the largest piece of squared timber that may be got out of a round log, or out of felled or standing tree.

(**226**) **RULE.** *Multiply the diameter of the tree or log by the half-diameter, and this product by the length: the result will be the required solidity.*

In fact, it is plain that the diam. AB multiplied by the half-diameter OC (or ½ AB) gives for product the area of the inscribed square ABCD, that is, the area of a section, of the timber to be computed, by a plane perpendicular to its length, and that area multiplied by the length of the log gives (**78, T.**) the required solidity.



REM. This rule supposes that the diam. of the tree is every where the same or that we make use of a mean diameter, as taken at the middle of the length, and this is generally done when there is not too much difference between the diameters of the opposite ends; but to be precise (**148, T.**) we must as already stated (**91, T.**) add to the sum of the areas of the ends of the log or tree to be measured four times the area of a section taken at the centre and multiply the whole by the sixth part of the length, or which is the same thing, multiply the sum of the areas by the whole length and take the sixth part of the result.

Ex. 1. The circumference of a log, the length of which is 12 feet, is 6.28 feet, deduction being made of the bark if necessary: how many cubic feet of wood will there be in the stick of squared timber to be got out of the log?

Ans. The circ. 6.28 corresponds to a diam. 2, the section of the timber will therefore be 2 × 1 =2 square feet in area, and as the length is 12, the solidity will be 24 cubic feet.

2. A tree the height of which is 50 feet, has for its sup. diam. 30 inches, and for its inf. diam. 36 inches, for its interm. diam. 33 inches ; what is the solidity of the piece of square timber that may be got out of it.

Ans. Area small end=2½ × 1¼ feet = 3.125 sup. feet, area large end=3 × 1½=4,5 sup. feet, intermediate area=2.75 × 1.375=3.78125, 4 intermediate area = 15.125, the sum of the areas=22.75 and that sum × 50 ÷ 6=189.6 cubic feet.

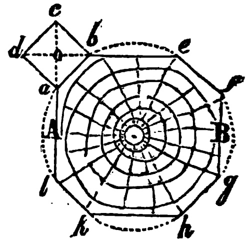
3. We have measured at 5 places nearly equidistant by means of a thickness compass, the diam. of an irregular tree just felled ; these diameters are respectively 39, 39½, 38, 37½ and 36 inches, and the length of the tree 40 feet ; what will its solidity be after it has been squared.

Ans. The sum of the diameters 190 inches ÷ 5=38 inches= mean diam.=3⅙ feet, 3.166 × 1.583=5.012 nearly=area of the section ; multiplying this latter by the length 40, we get 200½ cubic feet.

PROBLEM LXV.

To cube a stick of timber AB which is but partly squared, or of which the edges or angles are wanting, called " waney timber."

(**272**) **RULE.** *Square the diam.* AB *of the timber, and from such square subtract that of the diam. ab of the sapwood, the difference of these squares multiplied by the length of the timber, will be the required solidity.*



In fact, it is plain that the surface wanting at each of the four angles, corners of edges of the timber, to complete the square A B, is the triangle *abo*, or a triangle equal to *abo*, when as it is supposed, *ef*=*gh*=*kl*=*ab* ; now the square on *ab* is worth 4 *abo* ; therefore, &c.

REM. I. If the sides *ab*, *ef*, &c. are not equal to each other, we may take one fourth of the sum of these four sides for a mean diameter *ab*, or for greater accuracy, we will make separately the squares of *ab*, *ef*, &c., and the fourth of the sum of those squares will be, or the sum of the fourths of those squares will be the quantity, nearly, to be subtracted from the square AB to obtain the net area of the section of the timber.

REM. II. Let us observe as in the last problem that if the timber is not throughout its entire length of equal size, its section must be taken at about the middle of its length, and this is generally what is done (**148 T.**) or, we will determine several sections of the timber and then take their mean, or finally we will make the sum of the areas of the opposite ends plus four times that of the intermediate section and afterwards multiply the whole by the length and take the sixth part of the result.

REM. III. We must also observe that we may arrive at the area of any regular or symmetrical octagon or of the kind here illustrated by subtracting from the square of the perpendicular distance AB which separates any two of its parallel sides, the square of one *ab* of the sides adjacent to the first.

Ex. I. An eight sided pillar is 3 feet wide or thick AB, the side *ab* of the chamfer *aob* is 6 inches : what is the solidity of the pillar, its length or height being 10 feet ?

Ans. $(3+3-(.5\times .5)=8.75$ superficial feet, and $8.75\times 10=87.5$ cubic feet=required solidity.

2. A log of timber the edges of which are waney, measures 30 inches square and 30 feet long, the average of the sides *ab*, *ef*, &c. of the wane is 9 inches ; what is the solidity of the timber ?

Ans. (30×30) minus $(9\times 9)=919$ square inches=area of the section of the timber=6.382 feet very nearly, and $6.382\times 30=191.46$ cubic feet.

3. We have reduced to 30 inches square at the large end a tree the diam. of which was at that point 36 inches; at the small end the diam. 30 inches has been reduced to 25 inches; the wane, sapwood or defect from a true square *ab* is from 7 to 6 inches respectively at the two ends, such as obtained by a direct measurement of the piece of wood to be cubed, or by means of a sketch made from a scale of equal parts : what is the solidity of the timber, its length being 60 feet ?

Ans. Area at the large end $=(30\times 30)-(7\times 7)=851$ square inches, area at small end$=(25\times 25)-(6\times 6)=589$ sq. f., the intermediate area $\left(\frac{30+25}{2}\times\frac{30+25}{2}\right)-\left(\frac{7+6}{2}\times\frac{7+6}{2}\right)=(27\frac{1}{2}\times 27\frac{1}{2})-(6\frac{1}{2}\times 6\frac{1}{2})=27.5^2-6.5^2=756.25-42.25=714$; $851+859+4$ times $714=4296$ square inches, dividing by 144 we obtain 29.833 square feet, multiplying by $\frac{1}{6}$ of the length or by 10 we obtain 298.33 cubic feet

Ans. Area section at the centre=714 square inches, 714÷144= .4983 square feet, 4.9583 × 60=297.498 cubic feet, that is, equal to the accurate solidity by less than one foot nearly, or by less than one 300th nearly, or by less than one third nearly of 1 per cent, sufficient accuracy (**148 T.**) in practice.

REM. IV. A comparison of the two answers of the last problem indicates sufficiently that the ordinary practice of cullers, who take the dimensions of a log at the middle of its length, and afterwards multiply the area of the section at that place by the length of the timber, to obtain thus its solidity, is, considering all things, (**148, T.**) sanctioned by circumstances.

TABLES

OF

I. Squares and Square Roots of numbers from 1 to 1600

II. Circumferences and areas of circles of diameter $\frac{1}{64}$ to 150, advancing by $\frac{1}{8}$.

III. Circumferences and areas of circles of diameter $\frac{1}{10}$ to 100, advancing by $\frac{1}{10}$.

IV. Circumferences and areas of circles of diameter 1 to 50 feet, advancing by 1 inch.

V. Sides of Squares equal in area to a circle of diameter 1 to 100 advancing by $\frac{1}{4}$.

VI. Lengths of circular arcs, to diameter 1 divided into 1000 equal parts.

VII. Lengths of semi-elliptic arcs to transverse diameter 1 divided into 1000 equal parts.

VIII. Areas of the segments of a circle to diameter 1 divided into 1000 equal parts.

IX. Areas of the zones of a circle to diameter 1 divided into 1000 equal parts.

X. Specific gravities or weights of bodies of all kinds solid, fluid, liquid and gazeous.

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
1	1	1.0000000	61	3721	7.8102497	121	14641	11.0000000
2	4	1.4142136	62	3844	7.8740079	122	14834	11.0453610
3	9	1.7320508	63	3969	7.9372539	123	15129	11.0905365
4	16	2.0000000	64	4096	8.0000000	124	15376	11.1355287
5	25	2.2360680	65	4225	8.0622577	125	15625	11.1803399
6	36	2.4494897	66	4356	8.1240384	126	15876	11.2249722
7	49	2.6457513	67	4489	8.1853528	127	16129	11.2694277
8	64	2.8284271	68	4624	8.2462113	128	16384	11.3137085
9	81	3.0000000	69	4761	8.3066239	129	16641	11.3578167
10	100	3.1622777	70	4900	8.3666003	130	16900	11.4017543
11	121	3.3166248	71	5041	8.4261498	131	17161	11.4455231
12	144	3.4641016	72	5184	8.4852814	132	17424	11.4891253
13	169	3.6055513	73	5329	8.5440037	133	17689	11.5325626
14	196	3.7416574	74	5476	8.6023253	134	17956	11.5758369
15	225	3.8229833	75	5625	8.6602540	135	18225	11.6189500
16	256	4.0000000	76	5776	8.7177979	136	18496	11.6619038
17	289	4.1231056	77	5929	8.7749644	137	18769	11.7046999
18	324	4.2426407	78	6084	8.8317609	138	19044	11.7473401
19	361	4.3585989	79	6241	8.8881944	139	19321	11.7898261
20	400	4.4721360	80	6400	8.9442719	140	19600	11.8321596
21	441	4.5825757	81	6561	9.0000000	141	19881	11.8743421
22	484	4.6904158	82	6724	9.0553851	142	20164	11.9163753
23	529	4.7958315	83	6889	9.1104336	143	20449	11.9582607
24	576	4.8989795	84	7056	9.1651514	144	20736	12.0000000
25	625	5.0000000	85	7225	9.2195445	145	21025	12.0415946
26	676	5.0990195	86	7396	9.2736185	146	21316	12.0830460
27	729	5.1961524	87	7569	9.3273791	147	21609	12.1243557
28	784	5.2915026	88	7744	9.3808315	148	21904	12.1655251
29	841	5.3851648	89	7921	9.4339811	149	22201	12.2065556
30	900	5.4772256	90	8100	9.4868330	150	22500	12.2474487
31	961	5.5677644	91	8281	9.5393920	151	22801	12.2882057
32	1024	5.6568542	92	8464	9.5916630	152	23104	12.3288280
33	1089	5.7445626	93	8649	9.6436508	153	23409	12.3693169
34	1156	5.8309519	94	8836	9.6953597	154	23716	12.4096736
35	1225	5.9160798	95	9025	9.7467943	155	24025	12.4498996
36	1296	6.0000000	96	9216	9.7979590	156	24336	12.4899960
37	1369	6.0827625	97	9409	9.8488578	157	24649	12.5299641
38	1444	6.1644140	98	9604	9.8994949	158	24964	12.5698051
39	1521	6.2449980	99	9801	9.9498744	159	25281	12.6095202
40	1600	6.3245553	100	10000	10.0000000	160	25600	12.6491106
41	1681	6.4031242	101	10201	10.0498756	161	25921	12.6885775
42	1764	6.4807407	102	10404	10.0995049	162	26244	12.7279221
43	1849	6.5574385	103	10609	10.1488916	163	26569	12.7671453
44	1936	6.6332496	104	10816	10.1980390	164	26896	12.8062485
45	2025	6.7082039	105	11025	10.2469508	165	27225	12.8452326
46	2116	6.7823300	106	11236	10.2956301	166	27556	12.8840987
47	2209	6.8556546	107	11449	10.3440804	167	27889	12.9228480
48	2304	6.9282032	108	11664	10.3923048	168	28224	12.9614814
49	2401	7.0000000	109	11881	10.4403065	169	28561	13.0000000
50	2500	7.0710678	110	12100	10.4880885	170	28900	13.0384048
51	2601	7.1414284	111	12321	10.5356538	171	29241	13.0766968
52	2704	7.2111026	112	12544	10.5830052	172	29584	13.1148770
53	2809	7.2801099	113	12769	10.6301458	173	29929	13.1529464
54	2916	7.3484692	114	12996	10.6770783	174	30276	13.1909060
55	3025	7.4161985	115	13225	10.7238053	175	30625	13.2287566
56	3136	7.4833148	116	13456	10.7703296	176	30976	13.2664992
57	3249	7.5498344	117	13689	10.8166538	177	31329	13.3041347
58	3364	7.6157731	118	13924	10.8627805	178	31684	13.3416641
59	3481	7.6811457	119	14161	10.9087121	179	32041	13.3790882
60	3600	7.7459667	120	14400	10.9544512	180	32400	13.4164079

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
181	32761	13.4536240	241	58081	15.5241747	301	90601	17.3493516
182	33124	13.4907376	242	58564	15.5563492	302	91204	17.3781472
183	33489	13.5277493	243	59049	15.5884573	303	91809	17.4068952
184	33856	13.5646600	244	59536	15.6204994	304	92416	17.4355958
185	34225	13.6014705	245	60025	15.6524758	305	93025	17.4642492
186	34596	13.6381817	246	60516	15.6843871	306	93636	17.4928557
187	34969	13.6747943	247	61009	15.7162336	307	94249	17.5214155
188	35344	13.7113092	248	61504	15.7480157	308	94864	17.5499288
189	35721	13.7477271	249	62001	15.7797338	309	95481	17.5783958
190	36100	13.7840488	250	62500	15.8113883	310	96100	17.6068169
191	36481	13.8202750	251	63001	15.8429795	311	96721	17.6351921
192	36864	13.8564065	252	63504	15.8745079	312	97344	17.6635217
193	37249	13.8924400	253	64009	15.9059737	313	97969	17.6918060
194	37636	13.9283883	254	64516	15.9373775	314	98596	17.7200451
195	38025	13.9642400	255	65025	15.9687194	315	99225	17.7482393
196	38416	14.0000000	256	65536	16.0000000	316	99856	17.7763888
197	38809	14.0356688	257	66049	16.0312195	317	100489	17.8044938
198	39204	14.0712473	258	66564	16.0623784	318	101124	17.8325545
199	39601	14.1067360	259	67081	16.0934769	319	101761	17.8605711
200	40000	14.1421356	260	67600	16.1245155	320	102400	17.8885438
201	40401	14.1774469	261	68121	16.1554944	321	103041	17.9164729
202	40804	14.2126704	262	68644	16.1864141	322	103684	17.9443584
203	41209	14.2478068	263	69169	16.2172747	323	104329	17.9722008
204	41616	14.2828569	264	69696	16.2480768	324	104976	18.0000000
205	42025	14.3178211	265	70225	16.2788206	325	105625	18.0277564
206	42436	14.3527001	266	70756	16.3095064	326	106276	18.0554701
207	42849	14.3874946	267	71289	16.3401346	327	106929	18.0831413
208	43264	14.4222051	268	71824	16.3707055	328	107584	18.1107703
209	43681	14.4568323	269	72361	16.4012195	329	108241	18.1383571
210	44100	14.4913767	270	72900	16.4316767	330	108900	18.1659021
211	44521	14.5258390	271	73441	16.4620776	331	109561	18.1934054
212	44944	14.5602198	272	73984	16.4924225	332	110224	18.2208672
213	45369	14.5945195	273	74529	16.5227116	333	110889	18.2482876
214	45796	14.6287388	274	75076	16.5529454	334	111556	18.2756669
215	46225	14.6628783	275	75625	16.5831240	335	112225	18.3030052
216	46656	14.6969385	276	76176	16.6132477	336	112896	18.3303028
217	47089	14.7309199	277	76729	16.6433170	337	113569	18.3575598
218	47524	14.7648231	278	77284	16.6783320	338	114244	18.3847763
219	47961	14.7986486	279	77841	16.7032931	339	114921	18.4119526
220	48400	14.8323970	280	78400	16.7332005	340	115600	18.4390889
221	48841	14.8660687	281	78961	16.7630546	341	116281	18.4661853
222	49284	14.8996644	282	79524	16.7928556	342	116964	18.4932420
223	49729	14.9331845	283	80089	16.8226038	343	117649	18.5202592
224	50176	14.9666295	284	80656	16.8522995	344	118336	18.5472370
225	50625	15.0000000	385	81225	16.8819430	345	119025	18.5741756
226	51076	15.0332964	286	81796	16.9115345	346	119716	18.6010752
227	51529	15.0665192	287	82369	16.9410743	347	120409	18.6279360
228	51984	15.0996689	288	82944	16.9705627	348	121104	18.6547581
229	52441	15.1327460	289	83521	17.0000000	349	121801	18.6815417
230	52900	15.1657509	290	84100	17.0293864	350	122500	18.7082869
231	53361	15.1986842	291	84681	17.0587221	351	123201	18.7349940
232	53824	15.2315462	292	85264	17.0880075	352	123904	18.7616630
233	54289	15.2643375	293	85849	17.1172428	353	124609	18.7882942
234	54756	15.2970585	294	86436	17.1464282	354	125316	18.8148877
235	55225	15.3297097	295	87025	17.1755640	355	126025	18.8414437
236	55696	15.3622915	296	87616	17.2046505	356	126736	18.8679623
237	56169	15.3948043	297	88209	17.2336879	357	127449	18.8944436
238	56644	15.4272486	298	88804	17.2626765	358	128164	18.9208879
239	57121	15.4596248	299	89401	17.2916165	359	128881	18.9472953
240	57600	15.4919334	300	90000	17.3205081	360	129600	18.9736660

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
361	130321	19.0000000	421	177241	20.5182845	481	231361	21.9317122
362	131044	19.0262976	422	178084	20.5426386	482	232324	21.9544984
363	131769	19.0525589	423	178929	20.5669638	483	233289	21.9772610
364	132496	19.0787840	424	179776	20.5912603	484	234256	22.0000000
365	133225	19.1049732	425	180625	20.6155281	485	235225	22.0227155
366	133956	19.1311265	426	181476	20.6397674	486	236196	22.0454077
367	134689	19.1572441	427	182329	20.6639783	487	237169	22.0680765
368	135424	19.1833261	428	183184	20.6881609	488	238144	22.0907220
369	136161	19.2093727	429	184041	20.7123152	489	239121	22.1133444
370	136900	19.2353841	430	184900	20.7364414	490	240100	22.1359436
371	137641	19.2613603	431	185761	20.7605395	491	241081	22.1585198
372	138384	19.2873015	432	186624	20.7846097	492	242064	22.1810730
373	139129	19.3132079	433	187489	20.8086520	493	243049	22.2036033
374	139876	19.3390796	434	188356	20.8326667	494	244036	22.2261108
375	140625	19.3649167	435	189225	20.8566536	495	245025	22.2485955
376	141376	19.3907194	436	190096	20.8806130	496	246016	22.2710575
377	142129	19.4164878	437	190969	20.9045450	497	247009	22.2934968
378	142884	19.4422221	438	191844	20.9284495	498	248004	22.3159136
379	143641	19.4679223	439	192721	20.9523268	499	249001	22.3383079
380	144400	19.4935887	440	193600	20.9761770	500	250000	22.3606798
381	145161	19.5192213	441	194481	21.0000000	501	251001	22.3830293
382	145924	19.5448203	442	195364	21.0237960	502	252004	22.4053565
383	146689	19.5703858	443	196249	21.0475652	503	253009	22.4276615
384	147456	19.5959179	444	197136	21.0713075	504	254016	22.4499443
385	148225	19.6214169	445	198025	21.0950231	505	255025	22.4722051
386	148996	19.6468827	446	198916	21.1187121	506	256036	22.4944438
387	149769	19.6723156	447	199809	21.1423745	507	257049	22.5166605
388	150544	19.6977156	448	200704	21.1660105	508	258064	22.5388553
389	151321	19.7230829	449	201601	21.1896201	509	259081	22.5610283
390	152100	19.7484177	450	202500	21.2132034	510	260100	22.5831796
391	152881	19.7737199	451	203401	21.2367606	511	261121	22.6053091
392	153664	19.7989899	452	204304	21.2602916	512	262144	22.6274170
393	154449	19.8242276	453	205209	21.2837967	513	263169	22.6495033
394	155236	19.8494332	454	206116	21.3072758	514	264196	22.6715681
395	156025	19.8746069	455	207025	21.3307290	515	265225	22.6936114
396	156816	19.8997487	456	207936	21.3541565	516	266256	22.7156334
397	157609	19.9248588	457	208849	21.3775583	517	267289	22.7376340
398	158404	19.9499373	458	209764	21.4009346	518	268324	22.7596134
399	159201	19.9749844	459	210681	21.4242853	519	269361	22.7815715
400	160000	20.0000000	460	211600	21.4476106	520	270400	22.8035085
401	160801	20.0249844	461	212521	21.4709106	521	271411	22.8254244
402	161604	20.0499377	462	213444	21.4941853	522	272484	22.8473193
403	162409	20.0748599	463	214369	21.5174348	523	273529	22.8691933
404	163216	20.0997512	464	215296	21.5406592	524	274576	22.8910463
405	164025	20.1246118	465	216225	21.5638587	525	275625	22.9128775
406	164836	20.1494417	466	217156	21.5870331	526	276676	22.9346899
407	165649	20.1742410	467	218089	21.6101828	527	277729	22.9564806
408	166464	20.1990099	468	219024	21.6333077	528	278784	22.9782506
409	167281	20.2237484	469	219961	21.6564078	529	279841	23.0000000
410	168100	20.2484567	470	220900	21.6794834	530	280900	23.0217289
411	168921	20.2731349	471	221841	21.7025344	531	281961	23.0434372
412	169744	20.2977831	472	222784	21.7255610	532	283024	23.0651252
413	170569	20.3224014	473	223729	21.7485632	533	284089	23.0867928
414	171396	20.3469899	474	224676	21.7715411	534	285156	23.1084400
415	172225	20.3715488	475	225625	21.7944947	535	286225	23.1300670
416	173056	20.3960781	476	226576	21.8174242	536	287296	23.1516738
417	173889	20.4205779	477	227529	21.8403297	537	288369	23.1732605
418	174724	20.4450483	478	228484	21.8632111	538	289444	23.1948270
419	175561	20.4694895	479	229441	21.8860686	539	290521	23.2163735
420	176400	20.4939015	480	230400	21.9089023	540	291600	23.2379001

No.	Square	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
541	292681	23.2594067	601	361201	24.5153013	661	436921	25.7099203
542	293764	23.2808935	602	362404	24.5356883	662	438244	25.7293607
543	294849	23.3023604	603	363609	24.5560583	663	439569	25.7487864
544	295936	23.3238076	604	364816	24·5764115	664	440896	25.7681975
545	297025	23.3452351	605	366025	24.5967478	665	442225	25.7875939
546	298116	23.3666429	606	367236	24.6170673	666	443556	25.8069758
547	299209	23.3880311	607	368449	24.6373700	667	444889	25.8263431
548	300304	23.4093998	608	369664	24.6576560	668	446224	25.8456960
549	301401	23.4307490	609	370881	24.6779254	669	447561	25.8650343
550	302500	23.4520788	610	372100	24.6981781	670	448900	25.8843582
551	303601	23.4733892	611	373321	24.7184142	671	450241	25.9036677
552	304704	23.4946802	612	374544	24.7386338	672	451584	25.9229628
553	305809	23.5159520	613	375769	24.7588368	673	452929	25.9422435
554	306916	23.5372046	614	376996	24.7790234	674	454276	25.9615100
555	308025	23.5584380	615	378225	24.7991935	675	455625	25.9807621
556	309136	23.5796522	616	379456	24.8193473	676	456976	26.0000000
557	310249	23.6008474	617	380689	24.8394847	677	458329	26.0192237
558	311364	23.6220236	618	381924	24.8596058	678	459684	26.0384331
559	312481	23.6431808	619	383161	24.8797106	679	461041	26.0576284
560	313600	23.6643191	620	384400	24.8997992	680	462400	26.0768096
561	314721	23.6854386	621	385641	24.9198716	681	463761	26.0959767
562	315844	23.7065392	622	386884	24.9399278	682	465124	26.1151297
563	316969	23.7276210	623	388129	24.9599679	683	466489	26.1342687
564	318096	23.7486842	624	389376	24.9799920	684	467856	26.1533937
565	319225	23.7697286	625	390625	25.0000000	685	469225	26.1725047
566	320356	23.7907545	626	391876	25.0199920	686	470596	26.1916017
567	321489	23.8117618	627	393129	25.0399681	687	471969	26.2106848
568	322624	23.8327506	628	394384	25.0599282	688	473344	26.2297541
569	323761	23.8537209	629	395641	25.0798724	689	474721	26.2488095
570	324900	23.8746728	630	396900	25.0998008	690	476100	26.2678511
571	326041	23.8956063	631	398161	25.1197134	691	477481	26.2868789
572	327184	23.9165215	632	399424	25.1396102	692	478864	26.3058929
573	328329	23.9374184	633	400689	25.1594913	693	480249	26.3248932
574	329476	23.9582971	634	401956	25.1793566	694	481636	26.3438797
575	330625	23.9791576	635	403225	25.1992063	695	483025	26.3628527
576	331776	24.0000000	636	404496	25.2190404	696	484416	26.3818119
577	332929	24.0208243	637	405769	25.2388589	697	485809	26.4007576
578	334084	24.0416306	638	407044	25.2586619	698	487204	26.4196896
579	335241	24.0624188	639	408321	25.2784493	699	488601	26.4386081
580	336400	24.0831891	640	409600	25.2982213	700	490000	26.4575131
581	337561	24.1039416	641	410881	25.3179778	701	491401	26.4764046
582	338724	24.1246762	642	412164	25.3377189	702	492804	26.4952826
583	339889	24.1453929	643	413449	25.3574447	703	494209	26.5141472
584	341056	24.1660919	644	414736	25.3771551	704	495616	26.5329983
585	342225	24.1867732	645	416025	25.3968502	705	497025	26.5518361
586	343396	24.2074369	646	417316	25.4165301	706	498436	26.5706605
587	344569	24.2280829	647	418609	25.4361947	707	499849	26.5894716
588	345744	24.2487113	648	419904	25.4558441	708	501264	26.6082694
589	346921	24.2693222	649	421201	25.4754784	709	502681	26.6270539
590	348100	24.2899156	650	422500	25.4950976	710	504100	26.6458252
591	349281	24.3104916	651	423801	25.5147016	711	505521	26.6645833
592	350464	24.3310501	652	425104	25.5342907	712	506944	26.6833281
593	351649	24.3515913	653	426409	25.5538647	713	508369	26.7020598
594	352836	24.3721152	654	427716	25.5734237	714	509796	26.7207784
595	354025	24.3926218	655	429025	25.5929678	715	511225	26.7394839
596	355216	24.4131112	656	430336	25.6124969	716	512656	26.7581763
597	356409	24.4335834	657	431649	25.6320112	717	514089	26.7768557
598	357604	24.4540385	658	432964	25.6515107	718	515524	26.7955220
599	358801	24.4744765	659	434281	25.6709953	719	516961	26.8141754
600	360000	24.4948974	660	435600	25.6904652	720	518400	26.8328157

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
721	519841	26.8514432	781	609961	27.9463772	841	707281	29.0000000
722	521284	26.8700577	782	611524	27.9642629	842	708964	29.0172363
723	522729	26.8886593	783	613089	27.9821372	843	710649	29.0344623
724	524176	26.9072481	784	614656	28.0000000	844	712336	29.0516781
725	525625	26.9258240	785	616225	28.0178515	845	714025	29.0688837
726	527076	26.9443872	786	617796	28.0356915	846	715716	29.0860791
727	528529	26.9629375	787	619369	28.0535203	847	717409	29.1032644
728	529984	26.9814751	788	620944	28.0713377	848	719104	29.1204396
729	531441	27.0000000	789	622521	28.0891438	849	720801	29.1376046
730	532900	27.0185122	790	624100	28.1069386	850	722500	29.1547595
731	534361	27.0370117	791	625681	28.1247222	851	724201	29.1719043
732	535824	27.0554985	792	627624	28.1424946	852	725904	29.1890390
733	537289	27.0739727	793	628849	28.1602557	853	727609	29.2061637
734	538756	27.0924344	794	630436	28.1780056	854	729316	29.2232784
735	540225	27.1108834	795	632025	28.1957444	855	731025	29.2403830
736	541696	27.1293199	796	633616	28.2134720	856	732736	29.2574777
737	543169	27.1477439	797	635209	28.2311884	857	734449	29.2745623
738	544644	27.1661554	798	636804	28.2488938	858	736164	29.2916370
739	546121	27.1845544	799	638401	28.2665881	859	737881	29.3087018
740	547600	27.2029410	800	640000	28.2842712	860	739600	29.3257566
741	549081	27.2213152	801	641601	28.3019434	861	741321	29.3428015
742	550564	27.2396769	802	643204	28.3196045	862	743044	29.3598365
743	552049	27.2580263	803	644809	28.3372546	863	744769	29.3768616
744	553536	27.2763634	804	646416	28.3548938	864	746496	29.3938769
745	555025	27.2946881	805	648025	28.3725219	865	748225	29.4108823
746	556516	27.3130006	806	649636	28.3901391	866	749956	29.4278779
747	558009	27.3313007	807	651249	28.4077454	867	751689	29.4448637
748	559504	27.3495887	808	652864	28.4253108	868	753424	29.4618397
749	561001	27.3678644	809	654481	28.4429253	869	755161	29.4788059
750	562500	27.3861279	810	656100	28.4604989	870	756900	29.4957624
751	564001	27,4043792	811	657721	28.4780617	871	758641	29.5127091
752	565504	27.4226184	812	659344	28.4956137	872	760384	29.5296461
753	567009	27.4408455	813	660969	28.5131549	873	762129	29.5465734
754	568516	27.4590604	814	662596	28.5306852	874	763876	29.5634910
755	570025	27.4772633	815	664225	28.5482048	875	765625	29.5803989
756	571536	27.4954542	816	665856	28.5657137	876	767376	29.5972972
757	573049	27.5136330	817	667489	28.5832119	877	769129	26.6141858
758	574564	27.5317998	818	669124	28.6006993	878	770884	29.6310648
759	576081	27.5499546	819	670761	28.6181760	879	772641	29.6479342
760	577600	27.5680975	820	672400	28.6356421	880	774400	29.6647939
761	579121	27.5862284	821	674041	28.6530976	881	776161	29.6816442
762	580644	27.6043475	822	675684	28.6705424	882	777924	29.6984848
763	582169	27.6224546	823	677329	28.6879766	883	779689	29.7153159
764	583696	27.6405499	824	678976	28.7054002	884	781456	29.7321375
765	585225	27.6586334	825	680625	28.7228132	885	783225	29.7489496
766	586756	27.6767050	826	682276	28.7402157	886	784996	29.7657521
767	588289	37.6947648	827	683929	28.7576077	887	786769	29.7825452
768	589824	27.7128129	828	685584	28.7749891	888	788544	29.7993289
769	591361	27.7308492	829	687241	28.7923601	889	790321	29.8161030
770	592900	27.7488739	830	688900	28.8097206	890	792100	29.8328678
771	594441	27.7668868	831	690561	28.8270706	891	793881	29.8496231
772	595984	27.7848880	832	692224	28.8444102	892	795664	29.8663690
773	597529	27.8028775	833	693889	28.8617394	893	797449	29.8831056
774	599076	27.8208555	834	695556	28.8790582	894	799236	29.8998328
775	600625	27.8388218	835	697225	28.8963666	895	801025	29.9165506
776	602176	27.8567766	836	698896	28.9136646	896	802816	29.9332591
777	603729	27.8747197	837	700569	28.9309523	897	804609	29.9499583
778	605284	27.8926514	838	702244	28.9482297	898	806404	29.9666481
779	606841	27.9105715	839	703921	28.9654967	899	808201	29.9833287
780	608400	27.9284801	840	705600	28.9827535	900	810000	30.0000000

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre root.
901	811801	30.0166621	961	923521	31.0000000	1021	1042441	31.9530906
902	813604	30.0333148	962	925444	31.0161248	1022	1044484	31.9687347
903	815409	30.0499584	963	927369	31.0322413	1023	1046529	31.9843712
904	817216	30.0665928	964	929296	31.0483494	1024	1048576	32.0000000
905	819025	30.0832179	965	931225	31.0644491	1025	1050625	32.0156212
906	820836	30.0998339	966	933156	31.0805405	1026	1052676	32.0312348
907	822649	30.1164407	967	935089	31.0966236	1027	1054729	32.0468407
908	824464	30.1330383	968	937024	31.1126984	1028	1056784	32.0624391
909	826281	30.1496269	969	938961	31.1287648	1029	1058841	32.0780298
910	828100	30.1662063	970	940900	31.1448230	1030	1060900	32.0936131
911	829921	30.1827765	971	942841	31.1608729	1031	1062961	32.1091877
912	831744	30.1993377	972	944784	31.1769145	1032	1065024	32.1247568
913	833569	30.2158899	973	946729	31.1929479	1033	1067089	32.1403173
914	835396	30.2324329	974	948676	31.2089731	1034	1069156	32.1558704
915	837225	30.2489669	975	950625	31.2249900	1035	1071225	32.1714159
916	839056	30.2654919	976	952576	31.2409987	1036	1073296	32.1869539
917	840889	30.2820079	977	954529	31.2569992	1037	1075369	32.2024844
918	842724	30.2985148	978	956484	31.2729915	1038	1077444	32.2180074
919	844561	30.3150128	979	958441	31.2889757	1039	1079521	32.2335229
920	846400	30.3315018	980	960400	31.3049517	1040	1081600	32.2490310
921	848241	30.3479818	981	962361	31.3209195	1041	1083681	32.2645316
922	850084	30.3644529	982	964324	31.3368792	1042	1085764	32.2800248
923	851929	30.3809151	983	966289	31.3528308	1043	1087849	32.2955105
924	853776	30.3973683	984	968256	31.3687743	1044	1089936	32.3109888
925	855625	30.4138127	985	970225	31.3847097	1045	1092025	32.3264598
926	857476	30.4302481	986	972196	31.4006369	1046	1094116	32.3419233
927	859329	30.4466747	987	974169	31.4165561	1047	1096209	32.3573794
928	861184	30.4630924	988	976144	31.4324673	1048	1098304	32.3728281
929	863041	30.4795013	989	978121	31.4483704	1049	1100401	32.3882695
930	864900	30.4959014	990	980100	31.4642654	1050	1102500	32.4037035
931	866761	30.5122926	991	982081	31.4801525	1051	1104601	32.4191301
932	868624	30.5286750	992	984064	31.4960315	1052	1106704	32.4345495
933	870489	30.5450487	993	986049	31.5119025	1053	1108899	32.4499615
934	872356	30.5614136	994	988036	31.5277655	1054	1110916	32.4653662
935	874225	30.5777697	995	990025	31.5436206	1055	1113025	32.4807635
936	876096	30.5941171	996	992016	31.5594677	1056	1115136	32.4961536
937	877969	30.6104557	997	994009	31.5753068	1057	1117249	32.5115364
938	879844	30.6267857	998	996004	31.5911380	1058	1119364	32.5269119
939	881721	30.6431069	999	1998001	31.6069613	1059	1121481	32.5422802
940	883600	30.6594194	1000	1000000	31.6227766	1060	1123600	32.5576412
941	885481	30.6757233	1001	1000201	31.6385840	1061	1125721	32.5729949
942	887364	30.6920185	1002	1004004	31.6543836	1062	1127844	32.5883415
943	889249	30.7083051	1003	1006009	31.6701752	1063	1129969	32.6036807
944	891136	30.7245830	1004	1008016	31.6859590	1064	1132096	32.6190129
945	893025	30.7408523	1005	1010025	31.7017349	1065	1134225	32.6343377
946	894916	30.7571130	1006	1010036	31.7175030	1066	1136356	32.6496554
947	896808	30.7733651	1007	1014049	31.7332633	1067	1138489	32.6649659
948	898704	30.7896086	1008	1016064	31.7490157	1068	1140624	32.6802693
949	900601	30.8058436	1009	1018081	31.7647603	1069	1142761	32.6955654
950	902500	30.8220700	1010	1020100	31.7804972	1070	1144900	32.7108544
951	904401	30.8382879	1011	1020121	31.7962262	1071	1147041	32·7261363
952	906304	30.8544972	1012	1024144	31.8119474	1072	1149184	32.7414111
953	908209	30.8706981	1013	1026169	31.8276609	1073	1151329	32.7566787
954	910116	30.8868904	1014	1028196	31.8433666	1074	1153476	32.7719392
955	912025	30.9030743	1015	1030225	31.8590646	1075	1155625	32.7871926
956	913936	30.9192477	1016	1032256	31.8747549	1076	1157776	32.8024398
957	915849	30.9354166	1017	1034289	31.8904374	1077	1159929	32.8176782
958	917764	30.9515751	1018	1036324	31.9061123	1078	1162084	32.8329103
959	919681	30.9677251	1019	1038361	31.9217794	1079	1164241	32.8481354
960	921600	30.9838668	1020	1040400	31.9374388	1080	1166400	32.8633535

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root
1081	1168561	32.8785644	1141	1301881	33.7786915	1201	1442401	34.6554469
1082	1170724	32.8937684	1142	1304164	33.7934905	1202	1444804	34.6698716
1083	1172889	32.9089653	1143	1306449	33·8082830	1203	1447209	34.6842904
1084	1175056	32.9241553	1144	1308736	33.8230691	1204	1449616	34.6987031
1085	1177225	32.9393382	1145	1311025	33.8378486	1205	1452025	34.7131099
1086	1179396	32.9545141	1146	1313316	33.8526218	1206	1454436	34.7275107
1087	1181569	32.9696830	1147	1315609	33.8673884	1207	1456849	34.7419055
1088	1183744	32.9848450	1148	1317904	33.8821487	1208	1459264	34.7562944
1089	1185921	33.0000000	1149	1320201	33.8969025	1209	1461681	34.7706773
1090	1188100	33.0151480	1150	1322500	33.9116499	1210	1464100	34.7850543
1091	1190281	33.0302891	1151	1324801	33.9263909	1211	1466521	34.7994253
1092	1192464	33.0454233	1152	1327104	33.9411255	1212	1468944	34.8137904
1093	1194649	33.0605505	1153	1329409	33.9558537	1213	1471369	34.8281495
1094	1196836	33.0756708	1154	1331716	33.9705755	1214	1473796	34 8425028
1095	1199025	33.0907842	1155	1334025	33.9852910	1215	1476225	34.8568501
1096	1201216	33.1058907	1156	1336336	34.0000000	1216	1478656	34.8711915
1097	1203409	33.1209903	1157	1338649	34.0147027	1217	1481089	34.8855271
1098	1205604	33.1360830	1158	1340964	34.0293990	1218	1483524	34.8998567
1099	1207801	33.1511689	1159	1343281	34.0440890	1219	1485961	34.9141805
1100	1210000	33.1662479	1160	1345600	34.0587727	1220	1488400	34,9284984
1101	1212201	33.1813200	1161	1347921	34.0734501	1221	1490841	34.9428984
1102	1214404	33.1963853	1162	1350244	34.0881211	1222	1493284	34.9428104
1103	1216609	33.2114438	1163	1352569	34.1027858	1223	1495729	34.9571166
1104	1218816	33.2266955	1164	1354896	34.1174442	1224	1498176	34.9714169
1105	1221025	33.2415403	1165	1357225	34.1320963	1225	1500625	34.9857114
1106	1223236	33.2565783	1166	1359556	34.1467422	1226	1503076	35.0000000
1107	1225449	33.2716095	1167	1361889	34.1613817	1227	1505529	35.0142828
1108	1227664	33.2866339	1168	1364224	34.1760150	1228	1507984	35.0285598
1109	1229881	33.3016516	1169	1366561	34.1906420	1229	1510441	35.0428309
1110	1232100	33.3166625	1170	1368900	34.2052627	1230	1512900	35.0570963
1111	1234321	33.3316666	1171	1371241	34.2198773	1231	1515361	35.0713558
1112	1236544	33.3466640	1172	1373584	34.2344855	1232	1517824	35.0856096
1113	1238769	33.3616546	1173	1375929	34.2490875	1233	1520289	35.0998575
1114	1240996	33.3766385	1174	1378276	34.2636834	1234	1522756	35.1140997
1115	1243225	33.3916157	1175	1380625	34.2782730	1235	1525225	35.1283361
1116	1245456	33.4065862	1176	1382976	34.2928564	1236	1527696	35.1425568
1117	1247689	33.4215499	1177	1385329	34.3074336	1237	1530169	35.1567917
1118	1249924	33.4365070	1178	1387684	34.3220046	1238	1532644	35.1710108
1119	1252161	33.4514573	1179	1390041	34.3365694	1239	1535121	35.1852242
1120	1254400	33.4664011	1180	1392400	34.3511281	1240	1537600	35.1994318
1121	1256641	33.4813381	1181	1394761	34.3656805	1241	1540081	35.2136337
1122	1258884	33.4962684	1182	1397124	34.3802268	1242	1542564	35.2278299
1123	1261129	33.5111921	1183	1399489	34.3947670	1243	1545049	35.2420204
1124	1263376	33.5261092	1184	1401856	34.4093011	1244	1547536	35.2562051
1125	1265625	33.5410196	1185	1404225	34.4238289	1245	1550025	35.2703842
1126	1267876	33.5559234	1186	1406596	34.4383507	1246	1552516	35.2845575
1127	1270129	33.5708206	1187	1408969	34.4528663	1247	1555009	35.2987252
1128	1272384	33.5857112	1188	1411344	34.4673759	1248	1557509	35.3128872
1129	1274641	33.6005952	1189	1413721	34.4818793	1249	1560001	35.3270435
1130	1276900	33.6154726	1190	1416100	34.4963766	1250	1562500	35.3411941
1131	1279161	33.6303434	1191	1418481	34.5108678	1251	1565001	35.3553391
1132	1281424	33.6452077	1192	1420864	34.5253530	1252	1567504	35.3694784
1133	1283689	33.6600653	1193	1423249	34.5398321	1253	1570009	35.3836120
1134	1285956	33.6749165	1194	1425636	34.5543051	1254	1572516	35.3977400
1135	1288225	33.6897610	1195	1428025	34.5687720	1255	1575025	35.4118624
1136	1290496	33.7045991	1196	1430416	34.5832329	1256	1577536	35.4259792
1137	1292769	33.7194306	1197	1432809	34.5976879	1257	1580049	35.4400903
1138	*1295044*	*33.7340556*	1198	1435204	34.6121366	1258	1582564	35.4541958
1139	*1297321*	*33.7490741*	1199	1437601	34.6265794	1259	1585081	35.4682957
1140	*1299600*	*33.7638860*	1200	1440000	34.6410162	1260	1587600	35.4823900

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
1261	1590121	35.5105618	1321	1745041	36.3455637	1381	1907161	37.1618084
1262	1592644	35.5246393	1322	1747684	36.3593179	1382	1909924	37.1752606
1263	1595166	35.5387113	1323	1759329	36.3730670	1383	1912689	37.1887079
1264	1597696	35.5527777	1324	1752976	36.3868108	1384	1915456	37.2021505
1265	1600225	35.5668385	1325	1755625	36.4005494	1385	1918225	37.2155881
1266	1602756	35.5808937	1326	1758276	36.4142829	1386	1920996	37.2290209
1267	1605289	35.5949434	1327	1760929	36.4280112	1387	1923769	37.2224489
1268	1607824	35.6089876	1328	1763584	36.4417343	1388	1926544	37.2558720
1269	1610361	35.6230262	1329	1766241	36.4554523	1389	1929321	37.2692903
1270	1612900	35.6370593	1330	1768900	36.4691650	1390	1932100	37.2827037
1271	1615441	35.6510869	1331	1771561	36.4828727	1391	1934881	37.2961124
1272	1617984	35.6651090	1332	1774224	36.4965752	1392	1937664	37.3095162
1273	1620529	35.6791255	1333	1776889	36.5102725	1393	1940449	37.3229152
1274	1623076	35.6931366	1334	1779556	36.5239647	1394	1943236	37.3363094
1275	1625625	35.7071421	1335	1782225	36.5376518	1395	1946025	37.3496988
1276	1628176	35.7211422	1336	1784896	36.5513388	1396	1948816	37.3630834
1277	1630729	35.7351367	1337	1787569	36.5650106	1397	1951609	37.3764632
1278	1633284	35.7491258	1338	1790244	36.5786823	1398	1954404	37.3898382
1279	1635841	35.7631095	1339	1792921	36.5923489	1399	1957201	37.4032084
1280	1638400	35.7770876	1340	1795600	36.6060104	1400	1960000	47.4165738
1281	1640961	35.7910603	1341	1798281	36.6196668	1401	1962801	37.4299345
1282	1643524	35.8050276	1342	1800964	36.6333181	1402	1965604	37.4432904
1283	1646089	35.8189894	1343	1803649	36.6469644	1403	1968409	37.4566416
1284	1648656	35.8329457	1344	1806336	36.6606056	1404	1971216	37.4699880
1285	1651225	35.8468966	1345	1809025	36.6742416	1405	1974025	37.4833296
1286	1653796	35.8608421	1346	1811716	36.6878726	1406	1976836	37.4966665
1287	1656369	35.8747822	1347	1814409	36.7014986	1407	1979649	37.5099987
1288	1658944	35.8887169	1348	1817104	36.7151195	1408	1982464	37.5233261
1289	1661521	35.9026461	1349	1819801	36.7287353	1409	1985281	37.5366487
1290	1664100	35.9165699	1350	1822500	36.7423461	1410	1938100	37.5499667
1291	1666681	35.9304884	1351	1825201	36.7559519	1411	1990921	37.5632799
1292	1669264	33.9444015	1352	1827904	36.7695526	1412	1993744	37.5765885
1293	1671849	35.9583092	1353	1830609	36.7831483	1413	1996569	37.5898922
1294	1674436	35.9722115	1354	1833316	36.7967390	1414	1999396	37.6031913
1295	1677025	35.9861084	1355	1836025	36.8103246	1415	2002225	37.6164857
1296	1679616	36.0000000	1356	1838736	36.8239053	1416	2005056	37.6297754
1297	1682209	36.0138862	1357	1841449	36.8374809	1417	2007889	37.6430604
1298	1684804	36.0277671	1358	1844164	36.8510515	1418	2010724	37.6563407
1299	1687401	36.0416426	1359	1846881	36.8646172	1419	2013561	37.6696164
1300	1690000	36.0555128	1360	1849600	36.8781778	1420	2016400	37.6828874
1301	1692601	36.0693776	1361	1852321	36.8917335	1421	2019241	37.6961536
1302	1695204	36.0832371	1362	1855044	36.9052842	1422	2022084	37.7094153
1303	1697809	36.0970913	1363	1857769	36.9188299	1423	2024929	37.7226722
1304	1700416	36.1109402	1364	1860496	36.9323706	1424	2027776	37.7359245
1305	1703025	36.1247837	1365	1863225	36.9459064	1425	2030625	37.7491722
1306	1705636	36.1386220	1366	1865956	36.9594372	1426	2033476	37.7624152
1307	1708249	36.1524550	1367	1868689	36.9729631	1427	2036329	37.7756535
1308	1710864	36.1662826	1368	1871424	36.9864840	1428	2039184	37.7888873
1309	1713481	36.1801050	1369	1874161	37.0000000	1429	2042041	37.8021163
1310	1716100	36.1939221	1370	1876900	37.0135110	1430	2044900	37.8153408
1311	1718721	36.2077340	1371	1879641	37.0270172	1431	2047761	37.8285606
1312	1721344	36.2215406	1372	1882384	37.0405184	1432	2050624	37.8417759
1313	1723969	36.2353419	1373	1885129	37.0540146	1433	2053489	37.8549864
1314	1726596	36.2491379	1374	1887876	37.0675060	1434	2056356	37.8681924
1315	1729225	36.2626287	1375	1890625	37.0899924	1435	2059225	37.8813938
1316	1731856	36.2767143	1376	1893376	37.0944740	1436	2062096	37.8945906
1317	1734489	36.2904946	1377	1896129	37.1079506	1437	2064969	37.9077828
1318	1737124	36.3042697	1378	1898884	37.1214224	1438	2067844	37.9209704
1319	1739761	36.3180396	1379	1901641	37.1348893	1439	2070721	37.9341535
1320	1742400	36.3318042	1380	1904400	37.1483512	1440	2073600	37.9473319

No.	Square.	Sqre. root.	No.	Square.	Sqre. root.	No.	Square.	Sqre. root.
1441	2076481	37.9605058	1495	2235025	38.6652299	1548	2396304	39.3446311
1442	2079364	37.9736751	1496	2238016	38.6781593	1549	2399401	39.3573373
1443	2082249	37.9868398	1497	2241009	38.6910843	1550	2402500	39.3700394
1444	2085136	38.0000000	1498	2244004	38.7040050	1551	2405601	39.3827373
1445	2088025	38.0131556	1499	2247001	38.7169214	1552	2408704	39.3954312
1446	2090916	38.0263067	1500	2250000	38.7298335	1553	2411809	39.4081210
1447	2093809	38.0394532	1501	2253001	38.7427412	1554	2414916	39.4208067
1448	2096704	38.0525952	1502	2256004	38.7556447	1555	2418025	39.4331883
1449	2099601	38.0657326	1503	2259009	38.7685439	1556	2421136	39.4461658
1450	2102500	38.0788655	1504	2262016	38.7814389	1557	2424249	39.4588393
1451	2105401	38.0919939	1505	2265025	38.7943294	1558	2427364	39.4715087
1452	2108304	38.1051178	1506	2268034	38.8032158	1559	2430481	39.4841740
1453	2111209	38.1182371	1507	2271049	38.8200978	1560	2433600	39.4968353
1454	2114116	38.1313519	1508	2274064	38.8329757	1561	2436721	39.5094925
1455	2117025	38.1444622	1509	2277081	38.8458491	1562	2439844	39.5221457
1456	2119936	38.1575681	1510	2280100	38.8587184	1563	2442969	39.5347948
1457	2122849	38.1706693	1511	2283121	38.8715834	1564	2446096	39.5474399
1458	2125764	38.1837662	1512	2286144	38.8844442	1565	2449225	39.5600809
1459	2128681	38.1968585	1513	2289169	38.8973006	1566	2452356	39.5727179
1460	2131600	38.2099463	1514	2292196	38.9101529	1567	2455489	39.5853508
1461	2134521	38.2230297	1515	2295225	38.9230009	1568	2458624	39.5979797
1462	2137444	38.2361085	1516	2298256	38.9358447	1569	2461761	39.6106046
1463	2140369	38.2491829	1517	2301289	38.9486841	1570	2464900	39.6232255
1464	2143296	38.2622529	1518	2304334	38.9615194	1571	2468041	39.6358424
1465	3146225	38.2753184	1519	2307361	38.9743505	1572	2471184	39.6484552
1466	2149156	38.2883794	1520	2310400	38.9871774	1573	2474319	39.6610640
1467	2152089	38.3014360	1521	2313441	39.0000000	1574	2477475	39.6736688
1468	2155024	38.3144881	1522	2316484	39.0128184	1575	2480625	39.6862696
1469	2157961	38.3275358	1523	2319529	39.0256326	1576	2483776	39.6988665
1470	2160900	38.3405790	1524	2322576	39.0384426	1577	2486929	39.7114593
1471	2163841	38.3536178	1525	2325625	39.0512483	1578	2490084	39.7240481
1472	2166784	38.3666522	1526	2328676	39.0640499	1679	2493241	39.7366329
1473	2169729	38.3796821	1527	2331729	39.0768473	1180	2496400	39.7492138
1474	2172676	38.3927076	1528	2334784	39.0896406	1581	2499561	39.7617907
1475	2175625	38.4057287	1529	2337841	39.1024296	1582	2502724	39.7743636
1476	2178576	38.4187454	1530	2340900	39.1152144	1583	2505889	39.7869325
1477	2181529	38.4317577	1531	2343961	39.1279951	1584	2509056	39.7994976
1478	2184484	38.4447656	1532	2347024	39.1407716	1585	2512225	39.8120585
1479	2187441	38.4577691	1533	2350089	39.1535439	1586	2515396	39.8241155
1480	2190400	38.4707681	1534	2353156	39.1663120	1587	2518569	39.8376646
1481	2193361	38.4837627	1535	2356225	39.1790760	1588	2521744	39.8497177
1482	2196324	38,4967530	1536	2359296	39.1918359	1589	2524921	39.8622628
1483	2199289	38.5097390	1537	2362369	39.2045915	1590	2528100	39.8748040
1484	2202256	38.5227206	1538	2365444	39.2173431	1591	2531281	39.8873413
1485	2205225	38.5356977	1539	2368521	39.2300905	1592	2534464	39.8998747
1486	2208196	38.5486705	1540	2371600	39.2428337	1593	2537649	39.9124041
1487	2211169	38.5616389	1541	2374681	39.2555728	1594	2540836	39.9249295
1488	2214144	38.5746030	1542	2377764	39.2683078	1595	2544025	39.9374511
1489	2217121	38.5875627	1543	2380849	39.2810387	1596	2547216	39.9499687
1490	2220100	38.6005181	1544	2383936	39.2937654	1597	2550409	39.9624824
1491	2223081	38.6134691	1545	2387025	39.3064880	1598	2553604	39.9749922
1492	2226004	38.6264158	1546	2390116	39.3192065	1599	2556801	39.9874980
1493	2229049	38.6393582	1547	2393209	39.3319208	1600	2560000	40.0000000
1494	2232036	38.6522962						

TABLE II. a.

AREAS OF CIRCLES, FROM 1/64 TO 150.

[*Advancing by an Eighth.*]

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
1/64	.00019	4.	12.5664	10.	78.54	16.	201.062	22.	380.134
1/32	.00077	.⅛	13.364	.⅛	80.5157	.⅛	204.216	.⅛	384.465
1/16	.00307	.¼	14.1862	.¼	82.5161	.¼	207.394	.¼	388.822
1/8	.01227	.⅜	15.0331	.⅜	84.5409	.⅜	210.597	.⅜	393.203
3/16	.02761	.½	15.9043	.½	86.59	.½	213.825	.½	397.608
1/4	.04909	.⅝	16.8001	.⅝	88.6643	.⅝	217.073	.⅝	402.038
5/16	.0767	.¾	17.7205	.¾	90.7628	.¾	220.353	.¾	406.493
3/8	.11045	.⅞	18.6655	.⅞	92.8858	.⅞	223.654	.⅞	440.972
7/16	.15033	5.	19.635	11.	95.0334	17.	226.981	23.	416.477
1/2	.19635	.⅛	20.629	.⅛	97.2055	.⅛	230.33	.⅛	420.004
9/16	.2185	.¼	21.6475	.¼	99.4022	.¼	233.705	.¼	424.557
5/8	.30679	.⅜	22.6907	.⅜	101.6234	.⅜	237.104	.⅜	429.135
11/16	.37122	.½	23.7583	.½	103.8691	.½	240.528	.½	433.731
3/4	.44178	.⅝	24.8505	.⅝	106.1394	.⅝	243.977	.⅝	438.363
13/16	.51848	.¾	25.9672	.¾	108.4343	.¾	247.45	.¾	443.014
7/8	.60132	.⅞	27.1085	.⅞	110.7536	.⅞	250.947	.⅞	447.699
15/16	.69029	6.	28.2744	12.	113.098	18.	254.467	24.	452.39
		.⅛	29.4647	.⅛	115.466	.⅛	258.016	.⅛	457.115
		.¼	30.6796	.¼	117.859	.¼	261.587	.¼	461.864
		.⅜	31.9192	.⅜	120.276	.⅜	265.182	.⅜	466.638
		.½	33.1831	.½	122.718	.½	268.803	.½	471.436
		.⅝	34.4717	.⅝	125.184	.⅝	272.447	.⅝	476.259
		.¾	35.7847	.¾	127.676	.¾	276.117	.¾	481.106
		.⅞	37.1224	.⅞	130.192	.⅞	279.811	.⅞	485.978
1.	.7854	7.	38.4846	13.	132.733	19.	283.529	25.	490.875
.⅛	.99402	.⅛	39.8713	.⅛	135.297	.⅛	287.272	.⅛	495.796
.¼	1.2271	.¼	41,2825	.¼	137.886	.¼	291.039	.¼	500.741
.⅜	1.4848	.⅜	42.7184	.⅜	140.5	.⅜	294.831	.⅜	505.711
.½	1.7671	.½	44.1787	.½	143.139	.½	298.648	.½	510.706
.⅝	2.0739	.⅝	45.6636	.⅝	145.802	.⅝	302.489	.⅝	515.725
.¾	2.4052	.¾	47.173	.¾	148.489	.¾	306.355	.¾	520.769
.⅞	2.7611	.⅞	48.707	.⅞	151.201	.⅞	310.245	.⅞	525.837
2.	3.1416	8.	50.2656	14.	153.938	20.	314.16	26.	530.93
.⅛	3.5465	.⅛	51.8486	.⅛	156.699	.⅛	318.099	.⅛	536.047
.¼	3.976	.¼	53.4562	.¼	159.485	.¼	322.063	.¼	541.189
.⅜	4.4302	.⅜	55.0885	.⅜	162.295	.⅜	326.051	.⅜	546.356
.½	4.9087	.½	56.7451	.½	165.13	.½	330.064	.½	551.547
.⅝	5.4119	.⅝	58.4264	.⅝	167.989	.⅝	334.101	.⅝	556.762
.¾	5.9395	.¾	60.1321	.¾	170.873	.¾	338.163	.¾	562.002
.⅞	6.4918	.⅞	61.8625	.⅞	173.782	.⅞	342.25	.⅞	567.267
3.	7.0686	9.	63.6174	15.	176.715	21.	346.361	27.	572.557
.⅛	7.6699	.⅛	65.3968	.⅛	179.672	.⅛	350.497	.⅛	577.87
.¼	8.2957	.¼	67.2007	.¼	182.654	.¼	354.657	.¼	583.208
.⅜	8.9462	.⅜	69.0293	.⅜	185.661	.⅜	358.841	.⅜	588.571
.½	9.6211	.½	70.8823	.½	188.692	.½	363.051	.½	593.958
.⅝	10.3206	.⅝	72.7599	.⅝	191.748	.⅝	367.284	.⅝	599.376
.¾	11.0446	.¾	74.662	.¾	194.828	.¾	371.543	.¾	604.807
.⅞	11.7932	.⅞	76.5887	.⅞	197.933	.⅞	375.826	.⅞	610.268

TABLE.—(*Continued.*)

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
28.	615.754	35.	962.115	42.	1385.44	49.	1885.74	56.	2463.01
.⅛	621.263	.⅛	968.999	.⅛	1393.7	.⅛	1895.37	.⅛	2474.02
.¼	626.798	.¼	975.908	.¼	1401.98	.¼	1905.03	.¼	2485.05
.⅜	632.357	.⅜	982.842	.⅜	1410.29	.⅜	1914.7	.⅜	2496.11
.½	637.941	.½	989.8	.½	1418.63	.½	1924.42	.½	2507.19
.⅝	643.549	.⅝	996.783	.⅝	1426.98	.⅝	1934.15	.⅝	2418.3
.¾	649.182	.¾	1003.79	.¾	1435.36	.¾	1943.91	.¾	2529.43
.⅞	654.839	.⅞	1010.822	.⅞	1443.77	.⅞	1953.69	.⅞	2540.54
29.	660.521	36.	1017.878	43.	1452.21	50.	1963.5	57.	2551.76
.⅛	666.227	.⅛	1024.959	.⅛	1460.65	.⅛	1973.33	.⅛	2562.97
.¼	671.958	.¼	1032.065	.¼	1469.13	.¼	1983.18	.¼	2574.2
.⅜	677.714	.⅜	1039.195	.⅜	1477.63	.⅜	1993.05	.⅜	2585.45
.½	683.494	.½	1046.349	.½	1486.17	.½	2002.97	.½	2596.73
.⅝	689.298	.⅝	1053.528	.⅝	1494.72	,⅝	2012.89	.⅝	2608.03
.¾	695.128	.¾	1060.732	.¾	1503.3	.¾	2022.85	.¾	2619.36
.⅞	700.981	.⅞	1067.96	.⅞	1511.9	.⅞	2032.82	.⅞	2630.71
30.	706.86	37.	1075.213	44.	1520.53	51.	2042.82	58.	2642.09
.⅛	712.762	.⅛	1082.49	.⅛	1529.18	.⅛	2052.85	.⅛	2653.49
.¼	718.69	.¼	1089.792	.¼	1537.86	.¼	2062.9	.¾	2664.91
.⅜	724.641	.⅜	1097.118	.⅜	1546.55	.⅜	2072.98	.⅜	2676.36
.½	730.618	.½	1104.469	.½	1555.28	.½	2083.08	.½	2687.84
.⅝	736.619	.⅝	1111.844	.⅝	1564.03	.⅝	2093.2	.⅝	2699.33
.¾	742.644	.¾	1119.244	.¾	1572.81	.¾	2103.35	.¾	2710.86
.⅞	748.694	.⅞	1126.668	.⅞	1581.61	.⅞	2113.52	.⅞	2722.4
31.	754.769	38.	1134.118	45.	1590.43	52.	2123.72	59.	2733.98
.⅛	760.868	.⅛	1141.591	.⅛	1599.28	.⅛	2133.94	.⅛	2745.57
.¼	766.992	.¼	1149.089	.¼	1608.15	.¼	2144.19	.¼	2757.2
.⅜	773.14	.⅜	1156.612	.⅜	1617.04	.⅜	2154.46	.⅜	2768.84
.½	779.313	.½	1164.159	.½	1625.97	.½	2164.76	.½	2780.51
.⅝	785.51	.⅝	1171.731	.⅝	1634.92	.⅝	2175.08	.⅝	2792.21
.¾	791.732	.¾	1179.327	.¾	1643.89	.¾	2185.42	.¾	2803.93
.⅞	797.978	.⅞	1186.948	.⅞	1652.88	.⅞	2195.79	.⅞	2815.67
32.	804.25	39.	1194.593	46.	1661.91	53.	2206.19	60.	2827.44
.⅛	810.545	.⅛	1202.263	.⅛	1670.95	.⅛	2216.61	.⅛	2839.23
.¼	816.865	.¼	1209.958	.¼	1680.01	.¼	2227.05	.¼	2851.05
.⅜	823.209	.⅜	1217.677	.⅜	1689.1	.⅜	2237.52	.⅜	2862.89
.½	829.578	.½	1225.42	.½	1698.23	.½	2248.01	.½	2874.76
.⅝	835.972	.⅝	1233.188	.⅝	1707.37	.⅝	2258.53	.⅝	2886.65
.¾	842.390	.¾	1240.981	.¾	1716.54	.¾	2269.07	.¾	2898.57
.⅞	848.833	.⅞	1248.798	.⅞	1725.73	.⅞	2279.64	.⅞	2910.51
33.	855.301	40.	1256.64	47.	1734.95	54.	2290.23	61.	2922.47
.⅛	861.792	.⅛	1264.5	.⅛	1744.18	.⅛	2300.84	.⅛	2934.46
.¼	868.309	.¼	1272.39	.¼	1753.45	.¼	2311.48	.¼	2946.48
.⅜	874.85	.⅜	1280.31	.⅜	1762.73	.⅜	2322.14	.⅜	2958.52
.½	881.415	.½	1288.25	.½	1772.05	.½	2332.83	.½	2970.58
.⅝	888.005	.⅝	1296.21	.⅝	1781.39	.⅝	2343.55	.⅝	2982.67
.¾	894.62	.¾	1304.2	.¾	1790.76	.¾	2354.28	.¾	2994,78
.⅞	901.259	.⅞	1312.21	.⅞	1800.14	.⅞	2365.05	.⅞	3006.92
34.	907.922	41.	1320.26	48.	1809.56	55.	2375.83	62.	3019.08
.⅛	914.61	.⅛	1328.32	.⅛	1818.99	.⅛	2386.65	.⅛	3031.26
.¼	921.323	.¼	1336.4	.¼	1828.46	.¾	2397.48	.¼	3043.47
.⅜	928.06	.⅜	1344.51	.⅜	1837.93	.⅜	2408.34	.⅜	3055.71
.½	934.822	.½	1352.65	.½	1847.45	.½	2419.22	.½	3067.97
.⅝	941.609	.⅝	1360.81	.⅝	1856.99	.⅝	2430.18	.⅝	3080.25
.⅞	948.419	.¾	1369.	.¾	1866.55	.¾	2441.07	.¾	3092.56
.¾	955.255	.⅞	1377.21	.⅞	1876.13	.⅞	2452.03	.⅞	3104.89

TABLE II. b.

CIRCUMFERENCES OF CIRCLES, FROM 1/64 TO 150.

[*Advancing by an Eighth.*]

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
1/64	.04909	4.	12.5664	10.	31.416	16.	50.2656	22.	69.1152
1/32	.09817	.1/8	12.9591	.1/8	31.8087	.1/8	50.6583	.1/8	69.5079
1/16	.19635	.1/4	13.3518	.1/4	32.2014	.1/4	51.051	.1/4	69.9066
1/8	.3927	.3/8	13.7445	.3/8	32.5941	.3/8	51.4437	.3/8	70.2933
3/16	.589	.1/2	14.1372	.1/2	32.9868	.1/2	51.8364	.1/2	70.686
1/4	.7854	.5/8	14.5299	.5/8	33.3795	.5/8	52.2291	.5/8	71.0787
5/16	.98175	.3/4	14.9226	.3/4	33.7722	.3/4	52.6218	.3/4	71.4714
3/8	1.1781	.7/8	15.3153	.7/8	34.1649	.7/8	53.0145	.7/8	71.8641
7/16	1.37445	5.	15.708	11.	34.5576	17.	53.4072	23.	72.2568
1/2	1.5708	.1/8	16.1007	.1/8	34.9503	.1/8	53.7999	.1/8	72.6495
9/16	1.76715	.1/4	16.4934	.1/4	35.343	.1/4	54.1926	.1/4	73.0422
5/8	1.9635	.3/8	16.8861	.3/8	35.7353	.3/8	54.5853	.3/8	73.4349
11/16	2.15985	.1/2	17.2788	.1/2	36.1284	.1/2	54.978	.1/2	73.8276
3/4	2.3562	.5/8	17.6715	.5/8	36.5211	.5/8	55.3707	.5/8	74.2203
13/16	2.55255	.3/4	18.0642	.3/4	36.9138	.3/4	55.7634	.3/4	74.613
7/8	2.7489	.7/8	18.4569	.7/8	37.3065	.7/8	56.1561	.7/8	75.0057
15/16	2.94525	6.	18.8496	12.	37.6992	18.	56.5488	24.	75.3984
		.1/8	19.2423	.1/8	38.0919	.1/8	56.9415	.1/8	75.7911
		.1/4	19.635	.1/4	38.4846	.1/4	57.3342	.1/4	76.1838
		.3/8	20.0277	.3/8	38.8773	.3/8	57.7269	.3/8	76.5765
		.1/2	20.4204	.1/2	39.27	.1/2	58.1196	.1/2	76.9692
		.5/8	20.8131	.5/8	39.6627	.5/8	58.5123	.5/8	77.3619
		.3/4	21.2058	.3/4	40.0554	.3/4	58.905	.3/4	77.7546
		.7/8	21.5985	.7/8	40.4481	.7/8	59.2977	.7/8	78.1473
1.	3.1416	7.	21.9912	13.	40.8408	19.	59.6904	25.	78.54
.1/8	3.5343	.1/8	22.3839	.1/8	41.2335	.1/8	60.0831	.1/8	78.9327
.1/4	3.927	.1/4	22.7766	.1/4	41.6262	.1/4	60.4758	.1/4	79.3254
.3/8	4.3197	.3/8	23.1693	.3/8	42.0189	.3/8	60.8685	.3/8	79.7181
.1/2	4.7124	.1/2	23.562	.1/2	42.4116	.1/2	61.2612	.1/2	80.1102
.5/8	5.1051	.5/8	23.9547	.5/8	42.8043	.5/8	61.6539	.5/8	80.5035
.3/4	5.4978	.3/4	24.3474	.3/4	43.197	.3/4	62.0466	.3/4	80.8962
.7/8	5.8905	.7/8	24.7401	.7/8	43.5897	.7/8	62.4393	.7/8	81.2889
2.	6.2832	8.	25.1328	14.	43.9824	20.	62.832	26.	81.6816
.1/8	6.6759	.1/8	25.5255	.1/8	44.3751	.1/8	63.2247	.1/8	82.0743
.1/4	7.0686	.1/4	25.9182	.1/4	44.7678	.1/4	63.6174	.1/4	82.467
.3/8	7.4613	.3/8	26.3109	.3/8	45.1605	.3/8	94.0101	.3/8	82.8597
.1/2	7.854	.1/2	26.7036	.1/2	45.5532	.1/2	64.4028	.1/2	83.2524
.5/8	8.2467	.5/8	27.0963	.5/8	45.9459	.5/8	64.7955	.5/8	83.6451
.3/4	8.6394	.3/4	27.489	.3/4	46.3386	.3/4	65.1882	.3/4	84.0378
.7/8	9.0321	.7/8	27.8817	.7/8	46.7313	.7/8	65.5809	.7/8	84.4305
3.	9.4248	9.	28.2744	15.	47.124	21.	65.9736	27.	84.8232
.1/8	9.8175	.1/8	28.6671	.1/8	47.5167	.1/8	66.3663	.1/8	85.2159
.1/4	10.2102	.1/4	29.0598	.1/4	47.9094	.1/4	66.759	.1/4	85.6086
.3/8	10.6029	.3/8	29.4525	.3/8	48.3021	.3/8	67.1517	.3/8	86.0013
.1/2	10.9956	.1/2	29.8452	.1/2	48.6948	.1/2	67.5444	.1/2	86.394
.5/8	11.3883	.5/8	30.2379	.5/8	49.0875	.5/8	67.9371	.5/8	86.7867
.3/4	11.781	.3/4	30.6306	.3/4	49.4802	.3/4	68.3298	.3/4	87.1794
.7/8	12.1737	.7/8	31.6233	.7/8	49.8729	.7/8	68.7225	.7/8	87.5721

TABLE.—(*Continued.*)

Diam.	Circum.	Diam	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
28.	87.9648	35.	109.956	42.	131.947	49.	153.938	56.	175.93
.⅛	88.3575	.⅛	110.349	.⅛	132.34	.⅛	154.331	.⅛	176.322
.¼	88.7502	.¼	110.741	.¼	132.733	.¼	154.724	.¼	176.715
.⅜	89.1429	.⅜	111.134	.⅜	133.125	.⅜	155.117	.⅜	177.108
.½	89.5356	.½	111.527	.½	133.518	.½	155.509	.½	177.5
.⅝	89.9283	.⅝	111.919	.⅝	133.911	.⅝	155.902	.⅝	177.893
.¾	90.321	.¾	112.312	.¾	134.303	.¾	156.295	.¾	178.286
.⅞	90.7137	.⅞	112.705	.⅞	134.696	.⅞	156.687	.⅞	178.679
29.	91.1064	36.	113.098	43.	135.089	50.	157.08	57.	179.071
.⅛	91.4991	.⅛	113.49	.⅛	135.481	.⅛	157.473	.⅛	179.464
.¼	91.8918	.¼	113.883	.¼	135.874	.¼	157.865	.¼	179.857
.⅜	92.2845	.⅜	114.276	.⅜	136.267	.⅜	158.258	.⅜	180.249
.½	92.6772	.½	114.668	.½	136.66	.½	158.651	.½	180.642
.⅝	93.0699	.⅝	115.061	.⅝	137.052	.⅝	159.044	.⅝	181.035
.¾	93.4626	.¾	115.454	.¾	137.445	.¾	159.436	.¾	181.427
.⅞	93.8553	.⅞	115.846	.⅞	137.838	.⅞	159.823	.⅞	181.82
30.	94.248	37.	116.239	44.	138.23	51.	160.222	58.	182.213
.⅛	94.6407	.⅛	116.632	.⅛	138.623	.⅛	160.614	.⅛	182.606
.¼	95.0334	.¼	117.025	.¼	139.016	.¼	161.007	.¼	182.998
.⅜	95.4261	.⅜	117.417	.⅜	139.408	.⅜	161.4	.⅜	183.391
.½	95.8188	.½	117.81	.½	139.801	.½	161.792	.½	183.784
.⅝	96.2115	.⅝	118.203	.⅝	140.194	.⅝	162.185	.⅝	184.176
.¾	96.6042	.¾	118.595	.¾	140.587	.¾	162.578	.¾	184.569
.⅞	96.9969	.⅞	118.988	.⅞	140.979	.⅞	162.971	.⅞	184.962
31.	97.3896	38.	119.381	45.	141.372	52.	163.363	59.	185.354
.⅛	97.7823	.⅛	119.774	.⅛	141.765	.⅛	163.756	.⅛	185.747
.¼	98.175	.¼	120.166	.¼	142.157	.¼	164.149	.¼	186.14
.⅜	98.5677	.⅜	120.559	.⅜	142.55	.⅜	164.541	.⅜	186.533
.½	98.9604	.½	120.952	.½	142.943	.½	164.934	.½	186.925
.⅝	99.3531	.⅝	121.344	.⅝	143.336	.⅝	165.327	.⅝	187.318
.¾	99.7458	.¾	121.737	.¾	143.728	.¾	165.719	.¾	187.711
.⅞	100.1385	.⅞	122.13	.⅞	144.121	.⅞	166.112	.⅞	188.103
32.	100.5312	39.	122.522	46.	144.514	53.	166.505	60.	188.496
.⅛	100.9239	.⅛	122.915	.⅛	144.906	.⅛	166.898	.⅛	188.889
.¼	101.3166	.¼	123.308	.¼	145.299	.¼	167.29	.¼	189.281
.⅜	101.7093	.⅜	123.701	.⅜	145.692	.⅜	167.683	.⅜	189.674
.½	102.102	.½	124.093	.½	146.084	.½	168.076	.½	190.067
.⅝	102.4947	.⅝	124.486	.⅝	146.477	.⅝	168.468	.⅝	190.46
.¾	102.8874	.¾	124.879	.¾	146.87	.¾	168.861	.¾	190.852
.⅞	103.2801	.⅞	125.271	.⅞	147.263	.⅞	169.254	.⅞	191.245
33.	103.673	40.	125.664	47.	147.655	54.	169.646	61.	191.638
.⅛	104.066	.⅛	126.057	.⅛	148.018	.⅛	170.039	.⅛	192.03
.¼	104.458	.¼	126.449	.¼	148.441	.¼	170.432	.¼	192.423
.⅜	104.851	.⅜	126.842	.⅜	148.833	.⅜	170.825	.⅜	192.816
.½	105.244	.½	127.235	.½	149.226	.½	171.217	.½	193.208
.⅝	105.636	.⅝	127.627	.⅝	149.619	.⅝	171.61	.⅝	193.601
.¾	106.029	.¾	128.02	.¾	150.011	.¾	172.003	.¾	193.994
.⅞	106.422	.⅞	128.413	.⅞	150.404	.⅞	172.396	.⅞	194.387
34.	106.814	41.	128.806	48.	150.797	55.	172.788	62.	194.779
.⅛	107.207	.⅛	129.198	.⅛	151.19	.⅛	173.181	.⅛	195.172
.¼	107.6	.¼	129.591	.¼	151.582	.¼	173.573	.¼	195.565
.⅜	107.993	.⅜	129.984	.⅜	151.975	.⅜	173.966	.⅜	195.957
.½	108.385	.½	130.376	.½	152.368	.½	174.359	.½	196.35
.⅝	108.778	.⅝	130.769	.⅝	152.76	.⅝	174.752	.⅝	196.743
.¾	109.171	.¾	131.162	.¾	153.153	.¾	175.144	.¾	197.135
.⅞	109.563	.⅞	131.554	.⅞	153.546	.⅞	175.537	.⅞	197.528

TABLE—(*Continued*).

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
63.	3117.25	70.	3848.46	77.	4656.64	84.	5541.78	91.	6503.9
.⅛	3129.63	.⅛	3862.23	.⅛	4671.77	.⅛	5558.29	.⅛	6521.78
.¼	3142.04	.¼	3876.	.¼	4686.92	.¼	5574.82	.¼	6539.68
.⅜	3154.47	.⅜	3889.8	.⅜	4702.1	.⅜	5591.37	.⅜	6557.61
.½	3166.93	.½	3903.63	.½	4717.31	.½	5607.95	.½	6575.56
.⅝	3179.41	.⅝	3917.49	.⅝	4732.54	.⅝	5624.56	.⅝	6593.54
.¾	3191.91	.¾	3931.37	.¾	4747.79	.¾	5641.18	.¾	6611.55
.⅞	3204.44	.⅞	3945.27	.⅞	4763.07	.⅞	5657.84	.⅞	6629.57
64.	3217.	71.	3959.2	78.	4778.37	85.	5674.51	92.	6647.63
.⅛	3229.58	.⅛	3973.15	.⅛	4793.7	.⅛	5691.22	.⅛	6665.7
.¼	3242.18	.¼	3987.13	.¼	4809.05	.¼	5707.94	.¼	6683.8
.⅜	3254.81	.⅜	4001.13	.⅜	4824.43	.⅜	5724.69	.⅜	6701.93
.½	3267.46	.½	4015.16	.½	4839.83	.½	5741.47	.½	6720.08
.⅝	3280.18	.⅝	4029.21	.⅝	4855.26	.⅝	5758.27	.⅝	6738.25
.¾	3292.84	.¾	4043.29	.¾	4870.71	.¾	5775.1	.¾	6756.45
.⅞	3305.56	.⅞	4057.39	.⅞	4886.18	.⅞	5791.94	.⅞	6774.68
65.	2318.31	72.	4071.51	79.	4901.68	86.	5808.82	93.	6792.92
.⅛	3331.09	.⅛	4085.66	.⅛	4917.21	.⅛	5825.72	.⅛	6811.2
.¼	3343.89	.¼	4099.83	.¼	4932.75	.¼	5842.64	.¼	6829.49
.⅜	3356.71	.⅜	4114,04	.⅜	4948.33	.⅜	5859.59	.⅜	6847.82
.½	3369.56	.½	4128.26	.½	4963.92	.½	5876.56	.½	6866.16
.⅝	3382.43	.⅝	4142.51	.⅝	4979.55	.⅝	5893.55	.⅝	6884.53
.¾	3395.33	.¾	4156.78	.¾	4995.19	.¾	5910.58	.¾	6902.93
.⅞	3408.26	.⅞	4171,08	.⅞	5010.87	.⅞	5927.62	.⅞	6921.35
66.	3421.2	73.	4185.4	80.	5026.56	87.	5944.69	94.	6939.79
.⅛	3434.17	.⅛	4199,74	.⅛	5042.28	.⅛	5961.79	.⅛	6958.26
.¼	3447.17	.¼	4214.11	.¼	5058.02	.⅜	5978.9	.¼	6976.76
.⅜	3460.19	.⅜	4228.51	.⅜	5073.79	.¼	5996.05	.⅜	6995.28
.½	3473.24	.½	4242.93	.½	5089.59	.½	6013.22	.½	7013.82
.⅝	3486.3	.⅝	4257.37	.⅝	5105.41	.⅝	6030.41	.⅝	7032.39
.¾	3499.4	.¾	4271.84	.¾	5121.25	.¾	6047.63	.¾	7050.98
.⅞	3512.52	.⅞	4286.33	.⅞	5137.12	.⅞	6064.87	.⅞	7069.59
67.	3525.66	74.	4300.85	81.	5153.01	88.	6082.14	95.	7088.24
.⅛	3538.83	.⅛	4315.39	.⅛	5168.93	.⅛	6099.43	.⅛	7106.9
.¼	3552.02	.¼	4329.96	.¼	5184.87	.¼	6116.74	.¼	7125.59
.⅜	3565.24	.⅜	4344.55	.⅜	5200.83	.⅜	6134.08	.⅜	7144.31
.½	3578.48	.½	4359.17	.½	5216.82	.½	6151.45	.½	7163.04
.⅝	3591.74	.⅝	4373.81	.⅝	5232.84	.⅝	6168.84	.⅝	7181.81
.¾	3605.03	.¾	4388.47	.¾	5248.88	.¾	6186.25	.¾	7200.6
.⅞	3618.35	.⅞	4403.16	.⅞	5264.94	.⅞	6203.69	.⅞	7219.41
68.	3631.69	75:	4417.87	82.	5281.03	89.	6221.15	96.	7238.25
.⅛	3645.05	.⅛	4432.16	.⅛	5297.14	.⅛	6238.64	.⅛	7257.11
.¼	3658.44	.¼	4447.37	.¼	5313,28	.¼	6256.15	.¼	7275.99
.⅜	3671.85	.⅜	4462.16	.⅜	5329.44	.⅜	6273.69	.⅜	7294.91
.½	3685.29	.½	4476.98	.½	5345.63	.½	6291.25	.½	7313.84
.⅝	3698.76	.⅝	4491.81	.⅝	5361.84	.⅝	6308,84	.⅝	7332.8
.¾	3712.24	.¾	4506.67	.¾	5378.08	.¾	6326.44	.¾	7351.79
.⅞	3625.75	.⅞	4521.56	.⅞	5394.34	.⅞	6344.08	.⅞	7370.79
69.	3739.29	76.	4536.47	83.	5410.62	90.	6361.74	97.	7389.83
.⅛	3752.85	.⅛	4551.4	.⅛	5426.93	.⅛	6379.42	.⅛	7408.89
.¼	3766.43	.¼	4566.36	.¼	5443.26	.¾	6397.13	.¼	7427.97
.⅜	3780.04	.⅜	4581.35	.⅜	5459.62	.⅜	6414.86	.⅜	7447.08
.½	3793.68	.½	4596.36	.½	5476.01	.½	6432.62	.½	7466.21
.⅝	3807.34	.⅝	4611.39	.⅝	5492.41	.⅝	6450.4	.⅝	7485.36
.¾	3821.02	.¾	4626.45	.¾	5508.84	.¾	6468.21	.¾	7504.55
.⅞	3834.73	.⅞	4641.53	.⅞	5525.3	.⅞	6486.04	.⅞	7523.75

TABLE—*(Continued).*—[*Advancing by a Quarter and a half.*]

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
98.	7542.98	105.	8659.03	114.	10207.06	123.	11882.32	139.	15174.71
.⅛	7562.24	.¼	8700.32	.¼	10251.88	.¼	11930.67	.½	15284.08
.¼	7581.61	.½	8741.7	.½	10296.79	.½	11979.2	140.	15393.84
.⅜	7600.82	.¾	8783.18	.¾	10341.8	.¾	12027.66	.½	15503.98
.½	7620.15	106.	8824.75	115.	10386.91	124.	12076.31	141.	15614.53
.⅝	7639.5	.¼	8866.43	.¼	10432.12	.¼	12125.05	.½	15725.47
.¾	7658.88	.½	8908.2	.½	10477.43	.½	12173.9	142.	15836.8
.⅞	7678.28	.¾	8950.07	.¾	10522.84	.¾	12222.84	.½	15948.52
99.	7697.71	107.	8992.04	116.	10568.34	125.	12271.87	143.	16060.64
.⅛	7717.16	.¼	9034.11	.¼	10613.94	.½	12370.25	.½	16173.15
.¼	7736.63	.½	9076.28	.½	10659.64	126.	12469.01	144.	16286.05
.⅜	7756.13	.¾	9118.53	.¾	10705.44	.½	12568.17	.½	16399.34
.½	7775.66	108.	9160.91	117.	10751.34	127.	12667.72	145.	16513.03
.⅝	7795.2	.¼	9203.37	.¼	10797.34	.½	12767.66	.½	16627.11
.¾	7814.78	.½	9245.92	.½	10843.43	128.	12867.99	146.	16741.59
.⅞	7834.38	.¾	9288.58	.¾	10889.62	.½	12968.71	.½	16856.44
100.	7854.	109.	9331.34	118.	10935.9	129.	13069.84	147.	16971.71
.¼	7993.32	.¼	9374.19	.¼	10982.3	.½	13171.35	.½	17087.36
.½	7932.74	.½	9417.14	.½	11028.78	130.	13273.26	148.	17203.4
.¾	7972.21	.¾	9460.19	.¾	11075.37	.½	13375.55	.½	17319.83
101.	8011.87	110.	9503.34	119.	11122.06	131.	13478.25	149.	17436.67
.¼	8051.58	.¼	9546.69	.¼	11168.83	.½	13581.33	.½	17553.89
.½	8091.39	.½	9589.93	.½	11215.71	132.	13684.81	150.	17671.5
.¾	8131.3	.¾	9633.37	.¾	11262.69	.½	13788.67	.½	17789.51
102.	8171.3	111.	9676.91	120.	11309.76	133.	13992.94		
.¼	8211.41	.¼	9720.73	.¼	11356.93	.½	13997.54		
.½	8251.61	.½	9764.29	.½	11404.2	134.	14102.64		
.¾	8291.91	.¾	9808.12	.¾	11451.57	.½	14208.07		
103.	8332.31	112.	9852.06	121.	11499.04	135.	14313.91		
.¼	8372.81	.¼	9896.09	.¼	11546.61	.½	14420.14		
.½	8413.4	.½	9940.22	.½	11594.27	136.	14526.76		
.¾	8454.09	.¾	9984.45	.¾	11642.03	.½	14633.76		
104.	8494.89	113.	10028.77	122.	11689.89	137.	14741.17		
.¼	8535.78	.¼	10073.2	.¼	11737.85	.½	14848.96		
.½	8576.77	.½	10117.72	.½	11785.91	138.	14957.16		
.¾	8617.85	.¾	10162.34	.¾	11834.06	.½	15065.73		

To Compute the Area of a Diameter greater than any in the preceding Table.

RULE.—Divide the dimension by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in the table.

Take the tabular area for the diameter, multiply it by the square of the divisor, and the product will give the area required.

EXEMPLE.—What is the area for a diameter of 1050 ?

1050 ÷ 7 = 150; tab. area, 150 = 17671.5, which × 7^2 = 865903.5, *area required.*

To Compute the Area of an Integer and a Fraction not given in the Table.

RULE.—Double, treble, or quadruple the dimension given, until the fraction is increased to a whole number, or to one of those in the table, as ⅛, ¼, etc., provided it is practicable to do so.

Take the area for this diameter; and if it is double of that for wich the area is required, take one fourth of it; if treble, take one 9 th. of it and if quadruple, take one sixteenth of it, etc., etc.

EXAMPLE.—Required the area for a circle of 2. $\frac{3}{16}$ inches.

2. $\frac{3}{16}$ × 2 = 4⅜, area for which = 15.0331, which ÷ 4 = 3,758 *ins.*

TABLE.—(*Continued.*)

Diam.	Circum.	Diam	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
63.	197.921	70.	219.912	77.	241.903	84.	263.894	91.	285.886
.1/8	198.314	.1/8	220.305	.1/8	242.296	.1/8	264.287	.1/8	286.278
.1/4	198.706	.1/4	223.697	.1/4	242.689	.1/4	264.68	.1/4	286.671
.3/8	199.099	.3/8	221.09	.3/8	243.081	.3/8	265.073	.3/8	287.064
.1/2	199.492	.1/2	221.483	.1/2	243.474	.1/2	265.465	.1/2	287.456
.5/8	199.884	.5/8	221.876	.5/8	243.867	.5/8	265.858	.5/8	287.849
.3/4	200.277	.3/4	222.268	.3/4	244.259	.3/4	266.251	.3/4	288.242
.7/8	200.67	.7/8	222.661	.7/8	244.652	.7/8	266.643	.7/8	288.634
64.	201.062	71.	223.054	78.	245.045	85.	267.036	92.	289.027
.1/8	201.455	.1/8	223.446	.1/8	245.438	.1/8	267.429	.1/8	289.42
.1/4	201.848	.1/4	223.839	.1/4	245.83	.1/4	267.821	.1/4	289.813
.3/8	202.241	.3/8	224.232	.3/8	246.223	.3/8	268.214	.3/8	290.205
.1/2	202.633	.1/2	224.624	.1/2	246.616	.1/2	268.607	.1/2	290.598
.5/8	203.026	.5/8	225.017	.5/8	247.008	.5/8	268.999	.5/8	290.991
.3/4	203.419	.3/4	225.41	.3/4	247.401	.3/4	269.392	.3/4	291.383
.7/8	203.811	.7/8	225.803	.7/8	247.794	.7/8	269.785	.7/8	291.776
65.	204.204	72.	226.195	79.	248.186	86.	270.178	93.	292.169
.1/8	204.597	.1/8	226.588	.1/8	248.579	.1/8	270.57	.1/8	292.562
.1/4	204.989	.1/4	226.981	.1/4	248.972	.1/4	270.963	.1/4	292.954
.3/8	205.382	.3/8	227.373	.3/8	249.365	.3/8	271.356	.3/8	293.347
.1/2	205.775	.1/2	227.766	.1/2	249.757	.1/2	271.748	.1/2	293.74
.5/8	206.168	.5/8	228.159	.5/8	250.15	.5/8	272.141	.5/8	294.132
.3/4	206.56	.3/4	228.551	.3/4	250.543	.3/4	272.534	.3/4	294.525
.7/8	206.953	.7/8	228.944	.7/8	250.935	.7/8	272.926	.7/8	294.918
66.	207.346	73.	229.337	80.	251.328	87.	273.319	94.	295.31
.1/8	207.738	.1/8	229.73	.1/8	251.721	.1/8	273.712	.1/8	295.703
.1/4	208.131	.1/4	230.122	.1/4	252.113	.1/4	274.105	.1/4	296.096
.3/8	208.524	.3/8	230.515	.3/8	252.506	.3/8	274.497	.3/8	296.489
.1/2	208.916	.1/2	230.908	.1/2	252.899	.1/2	274.89	.1/2	296.881
.5/8	209.309	.5/8	231.3	.5/8	253.292	.5/8	275.283	.5/8	297.274
.3/4	209.702	.3/4	231.693	.3/4	253.684	.3/4	275.675	.3/4	297.667
.7/8	210.095	.7/8	232.086	.7/8	254.077	.7/8	276.068	.7/8	298.059
67.	210.487	74.	232.478	81.	254.47	88.	276.461	95.	298.452
.1/8	210.88	.1/8	232.871	.1/8	254.862	.1/8	276.853	.1/8	298.845
.1/4	211.273	.1/4	233.264	.1/4	255.255	.1/4	277.246	.1/4	299.237
.3/8	211.665	.3/8	233.657	.3/8	255.648	.3/8	277.629	.3/8	299.63
.1/2	212.058	.1/2	234.049	.1/2	256.04	.1/2	278.032	.1/2	300.023
.5/8	212.451	.5/8	234.442	.5/8	256.433	.5/8	278.424	.5/8	300.416
.3/4	212.843	.3/4	234.835	.3/4	256.826	.3/4	278.817	.3/4	300.808
.7/8	213.236	.7/8	235.227	.7/8	257.219	.7/8	279.21	.7/8	301.201
68.	213.629	75.	235.62	82.	257.611	89.	279.602	96.	301.594
.1/8	214.022	.1/8	236.013	.1/8	258.004	.1/8	279.995	.1/8	301.986
.1/4	214.414	.1/4	236.405	.1/4	258.397	.1/4	280.388	.1/4	302.379
.3/8	214.807	.3/8	236.798	.3/8	258.789	.3/8	280.781	.3/8	302.772
.1/2	215.2	.1/2	237.191	.1/2	259.182	.1/2	281.173	.1/2	303.164
.5/8	215.592	.5/8	237.584	.5/8	259.575	.5/8	281.566	.5/8	303.557
.3/4	215.985	.3/4	237.976	.3/4	259.967	.3/4	281.959	.3/4	303.95
.7/8	216.378	.7/8	238.369	.7/8	260.36	.7/8	282.351	.7/8	304.343
69.	216.77	76.	238.762	83.	260.753	90.	282.744	97.	304.735
.1/8	217.163	.1/8	239.154	.1/8	261.146	.1/8	283.137	.1/8	305.128
.1/4	217.556	.1/4	239.547	.1/4	261.538	.1/4	283.529	.1/4	305.521
.3/8	217.948	.3/8	239.94	.3/8	261.931	.3/8	283.922	.3/8	305.913
.1/2	218.341	.1/2	240.332	.1/2	262.324	.1/2	284.315	.1/2	306.306
.5/8	218.734	.5/8	240.725	.5/8	262.716	.5/8	284.708	.5/8	306.699
.3/4	219.127	.3/4	241.118	.3/4	263.109	.3/4	285.1	.3/4	307.091
.7/8	219.519	.7/8	241.511	.7/8	263.502	.7/8	285.493	.7/8	307.484

TABLE—*(Continued).*

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
98.	307.877	105.	329.868	114.	358.142	123.	386.417	139.	436.682
.⅛	308.27	.¼	330.653	.¼	358.928	.¼	387.202	.½	438.253
.¼	308.662	.½	331.439	.½	359.713	.½	387.988	140.	439.824
.⅜	309.055	.¾	332.224	.¾	360.499	.¾	388.773	.½	441.395
.½	309.448	106.	333.01	115.	361.284	124.	389.558	141.	442.966
.⅝	309.84	.¼	333.795	.¼	362.069	.¼	390.344	.½	444.536
.¾	310.233	.½	334.58	.½	362.855	.½	391.129	142.	446.107
.⅞	310.626	.¾	335.366	.¾	363.64	.¾	391.915	.½	447.678
99.	311.018	107.	336.151	116.	364.426	125.	392.7	143.	449.249
.⅛	311.411	.¼	336.937	.¼	365.211	.½	394.271	.½	450.82
.¼	311.804	.½	337.722	.½	365.996	126.	395.842	144.	452.39
.⅜	312.196	.¾	338.507	.¾	366.782	.½	397.412	.½	453.961
.½	312.589	108.	339.293	117.	367.567	127.	398.983	145.	455.532
.⅝	312.982	.¼	340.078	.¼	368.353	.½	400.554	.½	457.103
.¾	313.375	.½	340.864	.½	369.138	128.	402.125	146.	458.674
.⅞	313.767	.¾	341.649	.¾	369.923	.½	403.696	.½	460.244
100.	314.16	109.	342.434	118.	370.709	129.	405.266	147.	461.815
.¼	314.945	.¼	343.22	.¼	371.494	.½	406.837	.½	463.386
.½	315.731	.½	344.005	.½	372.28	130.	408.408	148.	464.957
.¾	316.516	.¾	344.791	.¾	373.065	.½	409.979	.½	466.528
101.	317.302	110.	345.576	119.	373.85	131.	411.55	149.	468.098
.¼	318.087	.¼	346.361	.¼	374.636	.½	413.12	.½	469.669
.½	318.872	.½	347.147	.½	375.421	132.	414.691	150.	471.24
.¾	319.658	.¾	347.932	.¾	376.207	.½	416.262	.½	472.811
102.	320.443	111.	348.718	120.	376.992	133.	417.833		
.¼	321.229	.¼	349.503	.¼	377.777	.½	419.404		
.½	322.014	.½	350.288	.½	378.563	134.	420.974		
.¾	322.799	.¾	350.074	.¾	379.348	.½	422.545		
103.	323.585	112.	351.859	121.	380.134	135.	424.116		
.¼	324.37	.½	352.645	.¼	380.919	.½	425.687		
.½	325.156	.¾	353.43	.½	381.704	136.	427.258		
.¾	325.941	.¼	354.215	.¾	382.49	.½	428.828		
104.	326.726	113.	355.001	122.	383.275	137.	430.399		
.¼	327.512	.¼	355.786	.¼	384.061	.½	431.97		
.½	328.297	.½	356.572	.½	384.846	138.	433.541		
.¾	329.083	.¾	357.357	.¾	385.631	.½	435.112		

To Compute the Circum. of a Diameter greater than any in the preceding Table.

Rule.—Divide the dimention by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in the table.

Take the tabular circumference for this dimention, multiply it by 2, 3, 4, 5, etc., according as it was divided, and the product will give the circumference required.

Example.—What is the circumference for a diameter of 1050?

1050 ÷ 7 = 150; tab. circum., 150 = 471,239, which × 7 = 3299.073, *circum. required.*

To Cumpute the Circumference for an Integer and Fraction not given in the Table.

Rule.—Double, treble, or quadruple the dimention given, until the fraction is increased to a whole number or to one of those in the table, as ⅛, ¼, etc., provided it is practical to do so.

Take the circumference for this diameter; and if it is double of that for which the circumference is required, take one half of it; if treble, take one third of it; and if quadruple, one fourth of it.

Example.—Required the circumference of 2.21875 inches.

2.21875 × 2 = 4.4375 = $4\frac{7}{16}$, which × 2 = 8.⅞; tab. circum. = 27.8817, which ÷ 4 = 6.9704 *ins.*

To Compute the Circum. of a Diameter in Feet and Inches etc., by the preceding Table.

Rule.—Reduce the dimention to inches or eighths, as the case may be, and take the circumference in that term from the table for that number.

Divide this number by 8 if it is in eighths, and by 12 if in inches, and the quotient will give the area in feet.

Example.—Required the circumference of a circle of 1 foot 6⅜ inches.

ot 6⅜ ins = 18⅜ ins. = 147 eighths. Circum. of 147 = 461.815, which ÷ 8 = 57.727 *inches;* and y 12 = 4.81 *feet.*

TABLE III.

AREAS AND CIRCUMFERENCES OF CIRCLES, FROM $\frac{1}{10}$ TO 100.

[*Advancing by Tenths.*]

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
			5.	19.635	15.708	10.	78.54	31.416
.1	.007854	.31416	.1	20.4282	16.0221	.1	80.1186	31.7301
.2	.031416	.62832	.2	21.2372	16.3363	.2	81.713	32.0443
.3	.070686	.94248	.3	22.0618	16.6504	.3	83.323	32.358
.4	.12566	1.2566	.4	22.9022	16.9646	.4	84.9488	32.6726
.5	.19635	1.5708	.5	23.7583	17.2788	.5	86.5903	32.9868
.6	.28274	1.885	.6	24.6301	17.5929	.6	88.2475	33.3009
.7	.38485	2.1991	.7	25.5176	17.9071	.7	89.9204	33.6151
.8	.50266	2.5133	.8	26.4208	18.2212	.8	91.609	33.9292
.9	.63617	2.8274	.9	27.3397	18.5354	.9	93.3133	34.2434
.1	.7854	3.1416	6.	28.2744	18.8496	11.	95.0334	34.5576
.1	.9503	3.4557	.1	29.2247	19.1637	.1	96.7691	34.8717
.2	1.1309	3.7699	.2	30.1907	19.4779	.2	98.5205	35.1859
.3	1.3273	4.084	.3	31.1725	19.792	.3	100.2877	35.501
.4	1.5393	4.3982	.4	32.1699	20 1062	.4	102.0705	35.8142
.5	1.7671	4.7124	.5	33.1831	20.4204	.5	103.8691	36.1284
.6	2.0106	5.0265	.6	34.212	20.7345	.6	105.6834	36.4425
.7	2 2698	5.3407	.7	35.2566	21.0487	.7	107.5134	36.7567
.8	2.5446	5.6548	.8	36.3168	21.3628	.8	109.359	37.0708
.9	2.8352	5.969	.9	37.3928	21.677	.9	111.2204	37.384
.2	3.1416	6.2832	7.	38.4846	21.9912	12.	113.0976	37.6992
.1	3.4636	6.5973	.1	39.592	22.3053	.1	114.9904	38.0133
.2	3.8013	6.9115	.2	40.7151	22.6195	.2	116.8989	38.3275
.3	4.1547	7.2256	.3	41.8539	22.9336	.3	118.8231	38.6416
.4	4.5239	7.5398	.4	43.0085	23.2478	.4	120.7631	38.9558
.5	4.9087	7.854	.5	44.1787	23.562	.5	122.7187	39.27
.6	5.3093	8.1681	.6	45.3647	23.8761	.6	124.6901	39.5841
.7	5.7255	8.4823	.7	46.5663	24.1903	.7	126.6771	39.8983
.8	6.1575	8.7964	.8	47.7837	24.5044	.8	128.6799	40.2124
.9	6.6052	9.1106	.9	49.0168	24.8186	.9	130.6984	40.5266
.3	7.0686	9.4248	8.	50.2656	25.1328	13.	132.7326	40.8408
.1	7.5476	9.7389	.1	51.53	25.4469	.1	134.7824	41.1549
.2	8.0424	10.0531	.2	52.8102	25.7611	.2	136.848	41.4691
.3	8.553	10.3672	.3	54.1062	26.0752	.3	138.9294	41.7832
.4	9.0792	10.6814	.4	55.4178	26.3894	.4	141.0264	42.0974
.5	9.6211	10.9956	.5	56.7451	26.7036	.5	143.1391	42.4116
.6	10.1787	11.3097	.6	58.0881	27.0177	.6	145.2675	42.7257
.7	10.7521	11.6239	.7	59.4469	27.3319	.7	147.4117	43.0399
.8	11.3411	11.938	.8	60.8213	27.646	.8	149.5715	43.354
.9	11.9459	12.2522	.9	62.2115	27.9602	.9	151.7471	43.6682
.4	12.5664	12.5664	9.	63.6174	28.2744	14.	153.9384	43.9824
.1	13.2025	12.8805	.1	65.0389	28.5885	.1	156.1453	44.2965
.2	13.8544	13.1947	.2	66.4762	28.9027	.2	158.368	44.6107
.3	14.522	13.5088	.3	67.9292	29.2168	.3	160.6064	44.9248
.4	15.2053	13.823	.4	69.3979	29.531	.4	162.8605	45.239
.5	15.9043	14.1372	.5	70.8823	29.8452	.5	165.1303	45.5532
.6	16.619	14.4513	.6	72.3824	30.1593	.6	167.4158	45.8673
.7	17.3494	14.7655	.7	73.3982	30.4735	.7	169.717	46.1815
.8	18.0956	15.0796	.8	75.4298	30.7876	.8	172.034	46.4956
.9	18.8574	15.3938	.9	76.977	31.1018	.9	174.3666	46.8098

TABLE.—(*Continued.*)

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
15.	176.715	47.124	.6	333.2923	64.7161	.2	539.1299	82.3099
.1	179.079	47.4381	.7	336.536	65.0311	.3	543.2533	82.624
.2	181.4588	47.7523	.8	339.7954	65.3452	.4	547.3923	82.9382
.3	183.8542	48.0664	.9	343.0705	65.6594	.5	551.5471	83.2524
.4	186.2654	48.3806	21.	346.3614	65.9736	.6	555.7176	83.5665
.5	188.6923	48.6918	.1	349.6679	66.2877	.7	559.9038	83.8807
.6	191.1349	49.0089	.2	352.9901	66.6019	.8	564.1056	84.1918
.7	193.5932	49.3231	.3	356.3281	66.916	.9	568.3232	84.509
.8	196.0672	49.6372	.4	359.6817	67.2302	27.	572.5566	84.8232
.9	198.5569	49.9514	.5	363.0511	67.5144	.1	576.8056	85.1373
16.	201.0624	50.2656	.6	366.4362	67.8585	.2	581.0703	85.4515
.1	203.5835	50.5797	.7	369.837	68.1727	.3	585.3503	85.7656
.2	206.1203	50.8939	.8	373.2534	68.4868	.4	589.6469	86.0798
.3	208.6729	51.208	.9	376.6856	68.801	.5	593.9587	86.394
.4	211.2411	51.5224	22.	380.1336	69.1152	.6	598.2863	86.7081
.5	213.8251	51.8364	.1	383.5972	69.4293	.7	602.6295	87.0223
.6	216.4248	52.1505	.2	387.0765	69.7435	.8	606.9885	87.3364
.7	219.0402	52.4647	.3	390.5751	70.0576	.9	611.3632	87.6506
.8	221.6712	52.7788	.4	394.0823	70.3718	28.	615.7536	87.9648
.9	224.318	53.093	.5	397.6087	70.686	.1	620.1596	88.2789
17.	226.9806	53.4072	.6	401.1509	71.0001	.2	624.5814	88.5931
.1	229.6588	53.7213	.7	404.7087	71.3143	.3	629.019	88.9072
.2	232.3527	54.0355	.8	408.2823	71.6284	.4	633.4722	89.2214
.3	235.0623	54.3496	.9	411.8716	71.9426	.5	637.9411	89.5356
.4	237.7877	54.6038	23.	415.4766	72.2568	.6	642.4257	89.8497
.5	240.5287	54.978	.1	419.0972	72.5709	.7	646.9261	90.1639
.6	243.2855	55.2921	.2	422.7336	72.8851	.8	651.4421	90.478
.7	246.0579	55.6063	.3	426.3858	73.1992	.9	655.9739	90.7922
.8	248.8461	55.9204	.4	430.0536	73.5134	29.	660.5214	91.1064
.9	251.65	56.2346	.5	433.7371	73.8276	.1	665.0845	91.4205
18.	254.4696	56.5488	.6	437.4363	74.1417	.2	669.6634	91.7347
.1	257.3048	56.8629	.7	441.1511	74.4559	.3	674.258	92.0488
.2	260.1558	57.1771	.8	444.8819	74.768	.4	678.8683	92.363
.3	263.0226	57.4912	.9	448.6283	75.0882	.5	683.4943	92.3772
.4	265.905	57.8054	24.	452.3904	75.3984	.6	688.136	92.9913
.5	268.8031	58.1196	.1	456.1681	75.7125	.7	692.7934	93.3055
.6	271.7169	58.4337	.2	459.9616	76.0267	.8	697.4666	93.6196
.7	274.6465	58.7479	.3	463.7708	76.3408	.9	702.1554	93.9338
.8	277.5917	59.062	.4	467.5957	76.6523	30.	706.86	94.248
.9	280.5527	59.3762	.5	471.4363	76.9692	.1	711.5802	94.5621
19.	283.5294	59.6904	.6	475.2926	77.2833	.2	716.3162	94.8763
.1	286.5217	60.0045	.7	479.1646	77.5975	.3	721.0678	95.1904
.2	289.5298	60.3187	.8	483.0524	77.9116	.4	725.8352	95.5046
.3	292.5536	60.6328	.9	486.9558	78.2258	.5	730.6183	95.8188
.4	295.5931	60.947	25.	490.875	78.54	.6	735.4171	96.1329
.5	298.6483	61.2612	.1	494.8098	78.8541	.7	740.2316	96.1471
.6	301.7192	61.5753	.2	498.7604	78.1683	.8	745.0618	96.7612
.7	301.806	61.8895	.3	502.7266	79.4824	.9	749.9077	97.0754
.8	307.9082	62.2036	.4	506.7086	79.7966	31.	754.7694	97.3896
.9	311.0252	62.5178	.5	510.7063	80.1108	.1	759.6467	97.7037
20.	314.16	62.832	.6	514.7196	80.4248	.2	764.5397	98.0179
.1	317.3094	63.1461	.7	518.7488	80.7391	.3	769.4485	98.332
.2	320.4746	63.4603	.8	522.7936	81.0532	.4	774.3729	98.6452
.3	323.6554	63.7744	.9	526.8541	81.3674	.5	779.3131	98.9604
.4	326.852	64.0886	26.	530.9304	81.6816	.6	781.2689	99.2745
.5	*330.0643*	64.4028	.1	555.0223	81.9976	.7	789.2106	99.5887

TABLE.—(*Continued.*)

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
.8	794.2278	99.9028	.4	1098.5862	117.4958	43.	1452.2046	135.0888
.9	799.2308	100.217	.5	1104.4687	117.81	1.	1458.9668	135.4029
32.	804.2496	100.5312	.6	1110.3671	118.1241	.2	1465.7448	135.7171
.1	809.284	100.8453	.7	1116.2811	118.4383	.3	1472.5385	136.0332
.2	814.3341	101.1595	.8	1122.2109	118.7524	.4	1479.348	136.3454
.3	819.3999	101.4736	.9	1128.1564	119.0666	.5	1486.1731	136.6596
.4	824.4815	101.7478	38.	1134.1176	119.3808	.6	1493.0139	136.9737
.5	829.5787	102.102	.1	1140.0946	119.6949	.7	1499.8705	137.2879
.6	834.6917	102.4161	.2	1146.087	120.0091	.8	1506.7427	137.602
.7	839.8203	102.7303	.3	1152.0954	120.3232	.9	1513.6287	137.9162
.8	844.9647	103.0444	.4	1158.1194	120.6374	44.	1520.5344	138.2304
.9	850.1248	103.3586	.5	1164.1591	120.9516	.1	1527.4537	138.5445
33.	855.3006	103.6728	.6	1170.2145	121.2657	.2	1534.3888	138.8587
.1	860.492	103.9869	.7	1176.2857	121.5799	.3	1541.3396	139.1728
.2	865.6992	104.3011	.8	1182.3725	121.894	.4	1548.3061	139,487
.3	870.9222	104.6151	.9	1188.4651	122.2082	.5	1555.2883	139.8012
.4	876.1608	104.9294	39.	1194.5434	122.5224	.6	1562.2862	140.1153
.5	881.4151	105.2436	.1	1200.7273	122.8365	.7	1569.2998	140.4295
.6	886.6851	105.5577	.2	1206.877	123.1507	.8	1576.3292	140.7436
.7	891.9709	105.8719	.3	1213.0424	123.4648	.9	1583.3742	141.0578
.8	897.2723	106.186	.4	1219.2243	123.779	45.	1590.435	141.372
.9	902.5895	106.5002	.5	1225.4203	124.0932	.1	1597.5114	141.6861
34.	907.9224	106.8144	.6	1231.6328	124.4073	.2	1604.6036	142.0003
.1	913.2709	107.1285	.7	1237.861	124.7215	.3	1611.7114	142.3144
.2	918.6352	107.4272	.8	1244.121	125.0356	.4	1613.835	142.6286
.3	924.0115	107.7568	.9	1250.3646	125.3498	.5	1625.9743	142.9428
.4	929.4109	108.071	40.	1256.64	125.664	.6	1633.1293	143.2569
.5	934.8223	108.3852	.1	1262.931	125.9781	.7	1640.302	143.5711
.6	940.2494	108.6993	.2	1269.2388	126.2923	.8	1647.4846	143.8852
.7	945.6922	109.0352	.3	1275.5602	126.6064	.9	1654.6885	144.1994
.8	951.1508	109.3076	.4	1281.8984	126.9206	46.	1661.9064	144.5136
.9	956.625	109.6418	.5	1288.2523	127.2348	.1	1669.1399	144.8277
35.	962.115	109.956	.6	1294.6219	127.5489	.2	1676.3891	145.1419
.1	967.6206	110.2701	.7	1301.0071	127.8631	.3	1683.6541	145.456
.2	973.142	110.5843	.8	1307.4082	128.1772	.4	1690.9347	145.7702
.3	978.679	110.8984	.9	1313.8249	128.4914	.5	1698.2311	146.0844
.4	984.2318	111.2126	41.	1320.2574	128.8056	.6	1705.5432	146.3985
.5	989.8003	111.5268	.1	1326.7055	129.1197	.7	1712.871	146.7127
.6	995.3845	111.8409	.2	1333.1693	129.4323	.8	1720.2144	147.0268
.7	1000.9843	112.1551	.3	1339.6489	129.748	.9	1727.5736	147.341
.8	1006.6	112.4692	.4	1346.1441	130.0622	47.	1734.9486	147.6552
.9	1012.2313	112.7834	.5	1352.6551	130.3764	.1	1742.3392	147.9693
36.	1017.8784	113.0976	.6	1359.1818	130.6905	.2	1749.7455	148.2835
.1	1023.5411	113.4117	.7	1365.7242	131.0047	.3	1757.1675	148.5976
.2	1029.2195	113.7259	.8	1372.2822	131.3188	.4	1764.6045	148.9118
.3	1034.9131	114.04	.9	1378.856	131.632	.5	1772.0587	149.226
.4	1040.6235	114.3542	42.	1385.4456	131.9472	.6	1779.5279	149.5361
.5	1046.3491	114.6684	.1	1392.0508	132.2613	.7	1787.0127	149.8543
.6	1052.0904	114.9825	.2	1398.6717	132.5755	.8	1794.5133	150.1684
.7	1057.8474	115.2967	.3	1405.3083	132.8896	.9	1802.0296	150.4826
.8	1063.62	115.6108	.4	1411.9607	133.2038	48.	1809.5616	150.7968
.9	1069.4084	115.925	.5	1418.6287	133.518	.1	1817.1092	151.1109
37.	1075.2126	116.2392	.6	1425.3125	133.8321	.2	1824.6726	151.4251
.1	1081.0324	116.5533	.7	1432.0119	134.1463	.3	1832.2518	151.7392
.2	1086.8679	116.8675	.8	1438.7271	134.4604	.4	1839.8466	152.0534
.3	1092.7191	117.1816	.9	1445.458	134.7746	.5	1847.4576	152.3676

TABLE—*(Continued)*.

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
.6	1855.0833	152.6817	.2	2307.2224	170.2747	.8	2808.6218	187.8676
.7	1862.7253	152.9959	.3	2315.744	170.5883	.9	2818.023	188.1818
.8	1870.3829	153.31	.4	2324.2813	170.903	60.	2827.44	188.496
.9	1878.0563	153.6242	.5	2332.8343	171.2172	.1	2836.8726	188.8101
49.	1885.7454	153.9384	.6	2341.403	171.5343	.2	2846.321	189.1243
.1	1893.4501	154.2525	.7	2349.9874	171.8455	.3	2855.785	189.4384
.2	1901.1706	154.5667	.8	2358.5876	172.1596	.4	2865.2648	189.7526
.3	1908.9068	154.8808	.9	2367.2034	172.4738	.5	2874.7603	190.0668
.4	1916.6587	155.195	55.	2375.835	172.788	.6	2884.2615	190.3809
.5	1924.4263	155.5092	.1	2384.4822	173.1021	.7	2893.7984	190.6951
.6	1932.2096	155.8233	.2	2393.1452	173.4163	.8	2903.341	191.0092
.7	1940.0086	156.1375	.3	2401.8238	173.7304	.9	2912.8993	191.3234
.8	1947.8234	156.4516	.4	2410.5182	174.0446	61.	2922.4734	191.6376
.9	1955.6538	156.7558	.5	2419.2283	174.3588	.1	2932.0631	191.9517
50.	1963.5	157.08	.6	2427.9541	174.6729	.2	2941.6685	192.2659
.1	1971.3618	157.3941	.7	2436.6956	174.9771	.3	2951.2897	192.58
.2	1979.2394	157.7083	.8	2445.4528	175.3092	.4	2960.9265	193.8942
.3	1987.1326	158.0224	.9	2454.2257	175.6154	.5	2970.5791	193.2084
.4	1995.0416	158.3366	56.	2463.0144	175.9296	.6	2980.2474	193.5225
.5	2902.9663	158.6508	.1	2471.8187	176.2437	.7	2989.9314	193.8367
.6	2010.9067	158.9649	.2	2480.6387	176.5579	.8	2999.63	194.1508
.7	2018.8628	159.2791	.3	2489.4745	176.872	.9	3009.3464	194.465
.8	2026.8346	159.5932	.4	2498.3259	177.1862	62.	3019.0776	194.7792
.9	2034.877	159.9074	.5	2507.1931	177.5004	.1	3028.8244	195.0933
51.	2042.8254	160.2216	.6	2516.076	177.8145	.2	3038.5869	195.4075
.1	2050.8443	160.5357	.7	2524.9736	178.1287	.3	3048.3651	195.7216
.2	2058.8784	160.8499	.8	2533.8888	178.4428	.4	3058.1591	196.0358
.3	2066.9293	161.164	.9	2542.8188	178.757	.5	3067.9687	196.35
.4	2074.9953	161.4782	57.	2551.7646	179.0712	.6	3077.7941	196.6641
.5	2083.0771	161.7924	.1	2560.726	179.3853	.7	3087.6341	196.9783
.6	2091.1746	162.1065	.2	2569.7031	179.6995	.8	3097.4919	197.2924
.7	2099.2878	162.4207	.3	2578.6959	180.0136	.9	3107.3644	197.6066
.8	2107.4166	162.7348	.4	2587.7045	180.3278	63.	3117.2526	197.9208
.9	2115.5612	163.049	.5	2596.7287	180.642	.1	3127.1564	198.2349
52.	2123.7216	163.3632	.6	2605.7687	180.9561	.2	3137.0758	198.5491
.1	2131.8976	163.6773	.7	2614.8243	181.2803	.3	3147.0114	198.8632
.2	2140.0893	163.9935	.8	2623.8957	181.5844	.4	3156.9664	199.1774
.3	2148.2967	164.3056	.9	2632.9828	181.8986	.5	3166.9291	199.4916
.4	2156.5199	164.6198	58.	2642.0856	182.2128	.6	3176.9115	199.8057
.5	2164.7587	164.934	.1	2651.2046	182.5269	.7	3186.9097	200.1199
.6	2173.0133	165.2481	.2	2660.3382	182.8411	.8	3196.9235	200.434
.7	2181.2835	165.5623	.3	2669.4882	183.1552	.9	3206.9531	200.7482
.8	2189.5695	165.8764	.4	2678.6538	183.4694	64.	3216.9984	201.0624
.9	2197.8712	166.1906	.5	2687.8351	183.7836	.1	3227.0593	201.3765
53.	2206.1886	166.5048	.6	2697.0321	184.0977	.2	3237.136	201.6907
.1	2214.5216	166.8189	.7	2706.2449	184.4119	.3	3247.2284	202.0048
.2	2222.8704	167.1331	.8	2715.4733	184.726	.4	3257.3365	202.319
.3	2231.235	167.4472	.9	2724.7175	185.0402	.5	3267.4603	202.6332
.4	2239.6152	167.7614	59.	2733.9774	185.3544	.6	3277.5998	202.9473
.5	2248.0111	168.0756	.1	2743.2529	185.6685	.7	3287.755	203.2615
.6	2256.4227	168.3897	.2	2752.5442	185.9827	.8	3297.926	203.5756
.7	2264.8701	168.7049	.3	2761.8512	186.2696	.9	3308.1126	203.8898
.8	2273.2931	169.018	.4	2771.1739	186.611	65.	3318.315	204.204
.9	2281.7519	169.3322	.5	2780.5123	186.9252	.1	3328.534	204.5181
54.	2290.2264	169.6464	.6	2789.8664	187.2393	.2	3338.7668	204.8323
.1	2298.7165	169.9605	.7	2799.2362	187.5535	.3	3349.0162	205.1464

TABLE.—(*Continued.*)

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
.4	3359.2814	205.4606	71.	3959.2014	223.0536	.6	4608.3816	240.6465
.5	3369.5623	205.7748	.1	3970.3619	223.3677	.7	4620.4218	240.9607
.6	3379.8589	206.0889	.2	3981.5381	223.6819	.8	4632.4776	241.2748
.7	3390.1712	206.4031	.3	3992.7301	223.996	.9	4644.5492	241.5987
.8	3400.4992	206.7172	.4	4003.9373	224.3102	77.	4656.6366	241.9032
.9	3410.8429	207.0314	.5	4015.1611	224.6244	.1	4668.7396	242.2173
66.	3421.2024	207.3456	.6	4026.4002	224.9385	.2	4680.8583	242.5315
.1	3431.5775	207.6597	.7	4037.655	225.2527	.3	4692.9927	242.8456
.2	3441.9633	207.9739	.8	4048.9254	225.5668	.4	4705.1429	243.1598
.3	3452.3749	208.288	.9	4060.2116	225.881	.5	4717.3087	243.474
.4	3462.7971	208.6022	72.	4071.5136	226.1952	.6	4729.4903	243.7881
.5	3473.2351	208.9164	.1	4082.8332	226.5093	.7	4741.6875	244.1023
.6	3483.6888	209.2305	.2	4094.1645	226.8235	.8	4753.9605	244.4164
.7	3494.164	209.5446	.3	4105.5125	227.1376	.9	4766.1292	244.7306
.8	3504.6432	209.8588	.4	4116.8793	227.4518	78.	4778.3736	245.0448
.9	3515.143	210.173	.5	4128.2587	227.766	.1	4790.6336	245.3589
67.	3525.6606	210.4872	.6	4139.6524	228.0801	.2	4802.9094	245.6731
.1	3536.1928	210.8013	.7	4151.0667	228.3943	.3	4815.201	245.9872
.2	3546.7404	211.1155	.8	4162.4943	228.7084	.4	4827.5082	246.3014
.3	3557.3043	211.4296	.9	4173.9376	229.0226	.5	4839.8311	246.6156
.4	3567.8837	211.7438	73.	4185.3966	229.3368	.6	4852.1697	246.9297
.5	3578.4787	212.058	.1	4196.8712	229.6509	.7	4864:5241	247.2439
.6	3589.0895	212.3721	.2	4208.3614	229.9651	.8	4876.8973	247.548
.7	3599.7159	212.6863	.3	4219.8678	230.2792	.9	4889.2799	247.8722
.8	3610.3581	213.0004	.4	4231.3896	230.5934	79.	4901.6814	248.1864
.9	3621.016	213.3146	.5	4242.9271	230.9076	.1	4914.0985	248.5005
68.	3631.6896	213.6288	.6	4254.4803	231.2217	.2	4926.5314	248.8147
.1	3642.3788	213.9429	.7	4266.0493	231.5359	.3	4938.982	249.1288
.2	3653.0838	214.2571	.8	4277.6339	231.85	.4	4951.4443	249.443
.3	3663.804	214.5712	.9	4289.2343	232.1642	.5	4963.9243	249.7572
.4	3674.541	214.8454	74.	4300.8504	232.4784	.6	4976.484	250.0713
.5	3685.2931	215.1996	.1	4312.4821	232.7925	.7	4988.9314	250.3855
.6	3696.006	215.5137	.2	4324.1296	233.1067	.8	5001.4586	250.6996
.7	3706.8445	215.8279	.3	4335.7928	233.4208	.9	5014.0014	251.0138
.8	3717.6437	216.142	.4	4347.4717	234.735	80.	5026.56	251.3280
.9	3728.4587	216.4562	.5	4359.1663	234.0492	.1	5039.1342	251.6421
69.	3739.2894	216.7704	.6	4370.8766	234.3633	.2	5051.7242	251.9563
.1	3750.1357	217.0845	.7	4382.6026	234.6775	.3	5064.3258	252.2704
.2	3760.9978	217.3987	.8	4394.3448	234.9916	.4	5076.9552	252.5846
.3	3771.8756	217.7128	.9	4406.1018	235.3058	.5	5089.5883	252.8988
.4	3782.7691	218.027	75.	4417.875	235.62	.6	5102.2411	253.2129
.5	3793.6783	218.3412	.1	4429.6638	235.9341	.7	5114.9096	253.5271
.6	3804.6032	218.6553	.2	4441.4684	236.2483	.8	5127.5938	253.8412
.7	3815.5438	218.9695	.3	4453.2886	236.5624	.9	5140.2937	254.1554
.8	3826.5002	219.2836	.4	4465.1246	236.8766	81.	5153.0094	254.4696
.9	3847.4722	219.5978	.5	4476.9763	237.1908	.1	5165.7407	254.7837
70.	3848.46	219.912	.6	4488.8437	237.5049	.2	5178.4877	255.0979
.1	3859.4952	220.2261	.7	4500.7268	237.8191	.3	5191.2505	255.412
.2	3870.4826	220.5403	.8	4512.6256	238.1332	.4	5204.0285	255.7262
.3	3881.5174	220.8544	.9	4524.5401	238.4474	.5	5216.8231	256.0404
.4	3892.568	221.1686	76.	4536.4704	238.7616	.6	5229.633	256.3545
.5	3903.6343	221.4828	.1	4548.4163	239.0757	.7	5242.4586	256.6687
.6	3914.7163	221.7969	.2	4560.3787	239.3899	.8	5255.2998	256.9828
.7	3925.814	222.1111	.3	4572.3553	239.704	.8	5268.1568	257.297
.8	3936.9274	222.4252	.4	4584.3583	240.0182	82.	5281.0296	257.6112
.9	3948.0565	222.7394	.5	4596.3571	240.3324	.1	5293.918	257.9253

TABLE.—(*Continued.*)

Diam	Area.	Circum.	Diam.	Area.	Circum.	Diam·	Area.	Circum.
.2	5306.8221	258.2395	.8	6054.5149	275.8324	.4	6851.4840	293.4254
.3	5319.7439	258.5536	.9	6068.3224	275.1466	.5	6866.1631	293.7396
.4	5332.6775	258.8646	88.	6082.1376	276.4608	.6	6880.8579	294.0537
.5	5345.6287	259.182	.1	6095.9684	276.7749	.7	6895.5685	294.3679
.6	5358.5957	259.4961	.2	6109.815	277.0891	.8	6910.2947	294.682
.7	5371.5983	259.8103	.3	6123.6774	277.4032	.9	6925.0367	294.9962
.8	5384.5762	260.1244	.4	6137.5554	277.7174	94.	6939.7944	295.3104
.9	5397.5908	260.4386	.5	6151.4491	278.0316	.1	6954.5677	295.6245
83.	5410.6206	260.7528	.6	6165.3585	278.3457	.2	6969.3568	295.9387
.1	5423.666	261.0669	.7	6179.2837	278.6599	.3	6984.1614	296.2436
.2	5436.7272	261.3811	.8	6193.2245	278.975	.4	6998.9821	296.567
.3	5449.8042	261.6952	.9	6207.1811	279.2882	.5	7013.8183	296.8812
.4	5462.8968	262.0094	89.	6221.1534	279.6024	.6	7028.6702	297.1953
.5	5476.0051	262.3236	.1	6235.1413	279.9165	.7	7043.5025	297.5095
.6	5489.1291	262.6376	.2	6249.145	280.2307	.8	7058.418	297.8236
.7	5502.2689	262.9519	.3	6263.1644	280.5448	.9	7073.3202	298.1378
.8	5515.4243	263.264	.4	6277.1995	280.859	95.	7088.235	298.452
.9	5528.5958	263.5802	.5	6291.2035	281.1732	.1	7103.1654	298.7661
84.	5541.7824	263.8944	.6	6305.3168	281.4873	.2	7118.1116	299.0723
.1	5554.9847	264.2085	.7	6319.399	281.8825	.3	7133.0734	299.3944
.2	5568.2032	264.5227	.8	6333.497	282.1156	.4	7148.051	299.7086
.3	5581.4372	264.8368	.9	6347.6813	282.4298	.5	7163.0443	300.0228
.4	5594.6869	265.151	90.	6361.74	282.744	.6	7178.0533	300.3369
.5	5607.9523	265.4652	.1	6375.885	283.0581	.7	7193.078	300.6511
.6	5621.2334	265.7793	.2	6390.0458	283.3723	.8	7208.1184	300.9652
.7	5634.5682	266.0935	.3	6404.2222	283.6864	.9	7223.1745	301.2794
.8	5647.8428	266.4076	.4	6418.4144	284.0006	96.	7238.2464	301.5936
.9	5661.171	266.7218	.5	6432.6223	284.3148	.1	7253.3339	301.9077
85.	5674.515	267.036	.6	6446.8474	384.6289	.2	7268.4371	302.2219
.1	5687.8746	267.3501	.7	6461.0852	284.9431	.3	7283.5561	302.536
.2	5701.25	267.6643	.8	6475.3402	285.2572	.4	7298.6907	302.8502
.3	5714.641	267.9784	.9	6489.6109	285.5714	.5	7313.8411	303.1644
.4	5728.0478	268.2926	91.	6503.8674	285.8856	.6	7329.0072	303.4785
.5	5741.4703	268.6068	.1	6518.1995	286.1997	.7	7344.189	303.7927
.6	5754.9085	268.9209	.2	6532.5173	286.5139	.8	7359.3864	304.1068
.7	5768.3624	269.2351	.3	6546.8909	286.829	.9	7374.5996	304.421
.8	5781.832	269.5492	.4	6561.2081	287.1422	97.	7389.8286	304.7352
.9	5795.3173	269.8634	.5	6575.5651	287.4564	.1	7405.0732	305.0493
86.	5808.8184	270.1776	.6	6589.9458	287.7705	.2	7420.3335	305.3635
.1	5822.3351	270.4917	.7	6604.3222	288.0847	.3	7435.6095	305.6776
.2	5835.8675	270.8059	.8	6618.7542	288.3988	.4	7450.9013	305.9918
.3	5849.4157	271.12	.9	6633.182	288.713	.5	7466.2087	306.306
.4	5862.9795	271.4342	92.	6647.6356	289.0272	.6	7481.5319	306.6201
.5	5876.5591	271.7484	.1	6662.0848	289.3413	.7	7496.8707	306.9363
.6	5890.1541	272.0665	.2	6676.5597	289.6555	.8	7512.2253	307.2484
.7	5903.7654	272.3767	.3	6691.0161	289.9696	.9	7527.5956	307.5626
.8	5917.392	272.6908	.4	6705.5567	290.2838	98.	7542.9816	307.8768
.9	5931.0344	273.005	.5	6720.0787	290.598	.1	7558.3832	308.1909
87.	5944.6926	273.3192	.6	6734.6165	290.9121	.2	7573.8006	308.5051
.1	5958.3644	273.6333	.7	6749.1699	291.2263	.3	7589.2338	308.8192
.2	5972.0559	273.9875	.8	6763.7391	291.5404	.4	7604.6826	309.1334
.3	5985.7691	274.2616	.9	6778.324	291.8546	.5	7620.1471	309.4476
.4	5999.4821	274.5758	93.	6792.9246	292.1688	.6	7635.6273	309.7617
.5	6013.2187	274.89	.1	6807.5408	292.4829	.7	7651.1933	310.0759
.6	6026.9711	275.2041	.2	6822.173	292.7971	.8	7666.9349	310.396
.7	6040.7391	275.5183	.3	6836.8206	293.1112	.9	7682.1623	310.7042

TABLE.—*(Continued.)*

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
99.	7697.7054	311.0184	.4	7760.0347	312.275	.8	7822.6154	313.5116
.1	7713.2641	311.3325	.5	7775.6563	312.5892	.9	7838.2998	313.8458
.2	7728.8336	311.6467	.6	7791.2936	312.9033	100.	7854.	314.16
.3	7744.4288	311.9608	.7	7806.9466	313.2175			

To Compute the Area or Circumference of a Diameter greater than any in the preceding Table.

See Rules, pages 176 and 181.

Or, *If the Diameter exceeds* 100 *and is less than* 1001.

Remove the decimal point, and take out the area or circumference as for a Whole Number by removing the decimal point, if for the area, two places to the right; and if for the circumference, one place.

ILLUSTRATION.—The area of 96.7 is 7344.189; hence for 967 it is 734418.9; and the circumference of 96.7 is 303.7927, and for 967 it is 3037.927.

TABLE IIII.

AREAS AND CIRCUMFERENCES OF CIRCLES.

FROM 1 TO 50 FEET.

[*Advancing by an Inch.*]

Diam.	Area.	Circum.		Diam.	Area.	Circum.		Diam.	Area.	Circum.	
	Feet.	Feet.	Ins.		Feet.	Feet.	Ins.		Feet.	Feet.	Ins.
1 *ft.*	.7854	3	1⅝	3 *ft.*	7.0686	9	5	5 *ft.*	19.635	15	8⅛
1	.9217	3	4⅝	1	7.4666	9	8¼	1	20.2947	15	11⅝
2	1.069	3	8	2	7.8757	9	11⅜	2	20.9656	16	2¾
3	1.2271	3	11	3	8.2957	10	2½	3	21.6475	16	5¾
4	1.3962	4	2⅛	4	8.7265	10	5⅝	4	22.34	16	9
5	1.5761	4	5⅜	5	9.1683	10	8¾	5	23.0437	17	⅛
6	1.7671	4	8½	6	9.6211	10	11⅞	6	23.7583	17	3¼
7	1.9689	4	11⅝	7	10.0846	11	3	7	24.4835	17	6⅜
8	2.1816	5	2¾	8	10.5591	11	6⅛	8	25.2199	17	9⅝
9	2.4052	5	5⅞	9	11.0446	11	9⅜	9	25.9672	18	¾
10	2.6398	5	9	10	11.5409	12	½	10	26.7251	18	3⅞
11	2.8852	6	2¼	11	12.0481	12	3⅝	11	27.4943	18	7⅛
2 *ft.*	3.1416	6	3⅜	4 *ft.*	12.5664	12	6¾	6 *ft.*	28.2744	18	10⅛
1	3.4087	6	6½	1	13.0952	12	9⅞	1	29.0649	19	1¼
2	3.6869	6	9⅝	2	13.6353	13	1	2	29.8668	19	4⅜
3	3.976	7	¾	3	14.1862	13	4⅛	3	30.6796	19	7½
4	4.276	7	3⅞	4	14.7479	13	7¼	4	31.5029	19	10⅝
5	4.5869	7	7	5	15.3206	13	10½	5	32.3376	20	1⅞
6	4.9087	7	10¼	6	15.9043	14	1⅝	6	33.1831	20	4⅞
7	5.2413	8	1⅜	7	16.4986	14	4⅝	7	34.0391	20	8⅛
8	5.585	8	4½	8	17.1041	14	7⅞	8	34.9065	20	11½
9	5.9395	8	7⅝	9	17.7205	14	11	9	35.7847	21	2⅜
10	6.3049	8	10¾	10	18.3476	15	2⅛	10	36.6735	21	5½
11	6.6813	9	1⅞	11	18.9858	15	5¼	11	37.5736	21	8¾

TABLE.—(*Continued.*)

Diam.	Area.	Circum.		Diam.	Area.	Circum.		Diam	Area.	Circum.	
	Feet.	Feet.	Ins.		Feet.	Feet.	Ins.		Feet.	Feet.	Ins.
7 *ft*	38.4846	21	11⅞	7	105.3794	36	4½	2	205.2726	50	9⅝
1	39.406	22	3	8	106.9013	36	7¾	3	207.3946	51	½
2	40.3388	22	6⅛	9	108.4342	36	10⅞	4	209.5264	51	3¾
3	41.2825	22	9¼	10	109.9772	37	2¾	5	211.6703	51	6½
4	42.2367	23	⅜	11	111.5319	37	5¼	6	213.8251	51	10
5	43.2022	23	2⅛	12 *ft*.	113.0976	37	8¾	7	215.9896	52	1⅛
6	44.1787	23	6¾	1	114.6732	37	11½	8	218.1662	52	4¼
7	45.1656	23	9⅞	2	116.2607	38	2⅝	9	220.3537	52	7⅜
8	46.1638	24	1⅛	3	117.859	38	5¾	10	222,551	52	10½
9	47.173	24	4⅛	4	119.4674	38	8⅞	11	224.7603	53	1⅝
10	48.1926	24	7¼	5	121.0876	39		17 *ft*.	226.9806	53	4⅞
11	49.2236	24	10⅜	6	122.7187	39	3¼	1	229.2105	53	8
8 *ft*.	50.2656	25	1½	7	124.3598	39	6⅜	2	231.4525	53	11⅛
1	51.3178	25	4⅝	8	126.0127	39	9½	3	233.7055	54	2⅛
2	52.3816	25	7⅞	9	127.6765	40	⅝	4	235.9682	54	5⅜
3	53.4562	25	11	10	129.3504	40	3¾	5	238.243	54	8½
4	54.5412	26	2⅛	11	131.036	40	6⅞	6	240.5287	54	11⅝
5	55.6377	26	5¼	13 *ft*.	132.7326	40	10	7	242.8241	55	2⅞
6	56.7451	26	8⅜	1	134.4391	41	1⅛	8	245.1316	55	6
7	57.8628	26	11½	2	136.1574	41	4⅜	9	247.45	55	9⅛
8	58.992	27	2¾	3	137.8867	41	7½	10	249.7781	56	¼
9	60.1321	27	5¾	4	139.626	41	10⅝	11	252.1184	56	3½
10	61.2826	27	9	5	141.3771	42	1⅝	18 *ft*.	254.4696	56	6½
11	62.4445	28	⅛	6	143.1391	42	4⅞	1	256.8303	56	9⅝
9 *ft*.	63.6174	28	3¼	7	144.9111	42	8	2	259.2033	57	⅞
1	64.8006	28	6⅜	8	146.6949	42	11⅛	3	261.5872	57	4
2	65.9951	28	9½	9	148.4896	43	2¼	4	263.9807	57	7⅛
3	67.2007	29	⅝	10	150.2943	43	5½	5	266.3864	57	10¼
4	68.4166	29	3¾	11	152.1109	43	8⅝	6	268.8031	58	1⅜
5	69.644	29	7	14 *ft*.	153.9384	43	11¾	7	271.2293	58	4½
6	70.8823	29	10⅛	1	155.7758	44	2⅞	8	273.6678	58	7⅝
7	72.1309	30	1¼	2	157.625	44	6	9	276.1171	58	10¾
8	73.391	30	4⅜	3	159.4852	44	9⅛	10	278.5761	58	2
9	74.662	30	7½	4	161.3553	44	¼	11	281.0472	59	5⅛
10	75.9433	30	11⅝	5	163.2373	45	3½	19 *ft*.	283.5294	59	8¼
11	77.2362	31	1¾	6	165.1303	45	6⅝	1	286.021	59	11½
10 *ft*.	78.54	31	5	7	167.0331	45	9¾	2	288.5249	60	2½
1	79.854	31	8⅛	8	168.9479	46	⅞	3	291.0397	60	5⅝
2	81.1795	31	11¼	9	170.8735	46	4	4	293.5641	60	8¾
3	82.516	32	2⅜	10	172.8091	46	7⅛	5	296.1107	60	11⅞
4	83.8627	32	5½	11	174.7565	46	11¼	6	298.6483	60	3⅛
5	85.2211	32	8⅝	15 *ft*.	176.715	47	1½	7	301.2054	61	6¼
6	86.5903	32	11¾	1	178.6832	47	4⅝	8	303.7747	61	9½
7	87.9697	33	2⅞	2	180.6634	47	7¾	9	306.355	61	½
8	89.3608	33	6⅛	3	182.6545	47	10⅞	10	308.9448	61	3⅝
9	90.7627	33	9¼	4	184.6555	48	2½	11	311.5469	62	6¾
10	92.1749	34	⅜	5	186.6684	48	5⅛	20 *ft*.	314.16	62	9⅞
11	93.5986	34	3½	6	188.6923	48	8¼	1	316.7824	62	1⅛
11 *ft*.	95.0334	34	6⅝	7	190.726	48	11⅜	2	319.4173	63	4¼
1	96.4783	34	9¾	8	192.7716	49	2⅝	3	322.063	63	7⅜
2	97.9347	35	⅞	9	194.8282	49	5¾	4	324.7182	63	11½
3	99.4021	35	4⅛	10	196.8946	49	8⅞	5	327.3858	63	1⅝
4	100.8797	35	7¼	11	198.973	50		6	330.0643	64	4¾
5	102.3689	35	10⅝	16 *ft*.	201.0624	50	3⅛	7	332.7522	64	7⅞
6	103.8691	36	1½	1	203.1615	50	6¼	8	335.4525	64	11½

TABLE.—*(Continued.)*

Diam.	Area.	Circum.		Diam.	Area.	Circum.		Diam.	Area.	Circum.	
	Feet.	Feet.	Ins.		Feet.	Feet.	Ins.		Feet.	Feet.	Ins.
9	338.1637	65	2¼	4	504.051	79	7⅛	11	702.9377	93	11⅞
10	340.8844	65	5⅜	5	507.3732	79	11⅛	30 *ft.*	706.86	94	2⅞
11	343.6174	65	8¼	6	510.7063	80	1¼	1	710.7909	94	6
21 *ft.*	346.3614	65	11⅝	7	514.0484	80	4⅜	2	714.735	94	9¼
1	349.1147	66	2¾	8	517.4034	80	7⅝	3	718.69	95	⅜
2	351.8804	66	5⅞	9	520.7692	80	10¾	4	722.6537	95	3½
3	354.6571	66	9	10	524.1441	81	1⅞	5	726.6305	95	6⅝
4	357.4432	66	⅛	11	527.5318	81	5	6	730.6183	95	9¾
5	360.2417	67	3⅜	26 *ft.*	530.9304	81	8⅛	7	734.6147	96	⅞
6	363.0511	67	6½	1	534.3379	81	11¼	8	738.6242	96	4
7	365.8698	67	9⅝	2	537.7583	82	2⅜	9	742.6447	96	7¼
8	368.7011	68	¾	3	541.1896	82	5¼	10	746.6738	96	10⅜
9	371.5432	68	3⅞	4	544.6299	82	8⅝	11	750.7161	97	1½
10	374.3947	68	7	5	548.083	82	11⅞	31 *ft.*	754.7694	97	4⅝
11	377.2587	68	10¼	6	551.5471	83	3	1	758.8311	97	7¾
22 *ft.*	380.1336	69	1⅜	7	555.0201	83	6⅛	2	762.9062	97	10⅞
1	383.0177	69	4½	8	558.5059	83	9¼	3	766.9921	98	2
2	385.9144	69	7⅝	9	562.0027	84	⅜	4	771.0866	98	5⅛
3	388.822	69	10¾	10	565.5084	84	3½	5	775.1914	98	8⅜
4	391.7389	70	1⅞	11	569.027	84	6⅝	6	779.3131	98	11½
5	394.6683	70	5	27 *ft.*	572.5566	84	9⅞	7	783.4403	99	2⅝
6	397.6087	70	8¼	1	576.0949	85	1	8	787.5808	99	5¾
7	400.5583	70	11⅛	2	579.6463	85	4¼	9	791.7322	99	8⅞
8	403.5204	71	2½	3	583.2085	85	8⅛	10	795.8922	100	
6	406.4935	71	5⅝	4	586.7796	85	11⅜	11	800.0654	100	3⅛
10	409.4759	71	8¾	5	590.3637	86	1½	32 *ft.*	804.2496	100	6⅜
11	412.4707	71	11⅞	6	593.9587	86	4⅝	1	808.4422	100	9½
23 *ft.*	415.4766	72	3	7	597.5625	86	7⅞	2	812.6481	101	⅝
1	418.4915	72	6⅛	8	601.1793	86	11	3	816.865	101	3¾
2	421.5192	72	9⅜	9	604.807	87	2⅛	4	821.0904	101	6⅞
3	424.5577	73	½	10	608.4436	87	5¼	5	825.3291	101	10
4	427.6055	73	3⅝	11	612.0931	87	8¾	6	829.5787	102	1⅛
5	430.6658	73	6¾	28 *ft.*	615.7536	87	11½	7	833.8368	102	4⅜
6	433.7371	73	9⅞	1	619.4228	88	2⅝	8	838.1082	102	7½
7	436.8175	74	1	2	623.105	88	5¾	9	842.3095	102	10⅝
8	439.9106	74	4⅛	3	626.7982	88	9	10	846.6813	103	1¾
9	443.0146	74	7¼	4	630.5002	89	⅛	11	850.9855	103	4⅞
10	446.1278	74	10⅝	5	634.2152	89	3¼	33 *ft.*	855.3006	103	8
11	449.2536	75	1⅝	6	637.9411	89	6⅜	1	859.624	103	11⅛
24 *ft.*	452.3904	75	4¾	7	641.6758	89	9½	2	863.9609	104	2¼
1	455.5362	75	7⅞	8	645.4235	90	⅝	3	868.3087	104	5⅜
2	458.6948	75	11	9	649.1821	90	3¾	4	872.6649	104	8⅝
3	461.8642	76	2⅛	10	652.9495	90	6⅞	5	877.0346	104	11¾
4	465.0428	76	5¼	11	656.73	90	11⅛	6	881.4151	105	2⅞
5	468.2341	76	8½	29 *ft.*	660.5214	91	1¼	7	885.804	105	6
6	471.4363	76	11⅝	1	664.3214	91	4⅜	8	890.2064	105	9⅛
7	474.6476	77	2¾	2	668.1346	91	7½	9	894.6196	106	¼
8	477.8716	77	5⅞	3	671.9587	91	10⅝	10	899.0413	106	3⅜
9	481.1065	77	9	4	675.7915	92	1¾	11	903.4763	106	6⅝
10	484.3506	78	⅛	5	679.6375	92	4⅞	34 *ft.*	907.9224	106	9¾
11	487.6073	78	3¼	6	683.4943	92	8⅛	1	912.3767	107	0⅞
25 *ft*	490.875	78	6½	7	687.3598	92	11⅛	2	916.8445	107	4
1	494.1516	78	9½	8	691.2385	93	2⅜	3	921.3232	107	7⅛
2	497.4411	79	¾	9	695.128	93	5½	4	925.8103	107	10¼
3	500.7415	79	3⅞	10	699.0263	93	8⅝	5	930.3108	108	1⅜

TABLE—*(Continued)*.

Diam.	Area.	Circum.		Diam.	Area.	Circum.		Diam.	Area.	Circum.	
	Feet.	Feet.	Ins.		Feet.	Feet.	Ins.		Feet.	Feet.	Ins.
6	934.8223	108	4⅝	1	1199.7195	122	9½	8	1497.5821	137	2⅛
7	939.3421	108	7¾	2	1204.8244	123	½	9	1503.3046	137	5¼
8	943.8753	108	10⅞	3	1209.9577	123	3⅝	10	1509.0348	137	8⅜
9	948.4195	109	2	4	1215.099	123	6¾	11	1514.7791	137	11⅝
10	952.972	109	5⅛	5	1220.2542	123	9⅞	44 *ft*	1520.5344	138	2¾
11	957.538	109	8¼	6	1225.4203	124	1⅛	1	1526.2971	138	5⅞
35 *ft*	962.115	109	11⅜	7	1230.5943	124	4¼	2	1532.0742	138	9
1	966.7701	110	2⅝	8	1235.7822	124	7⅜	3	1537.8622	139	⅛
2	971.2989	110	5¾	9	1240.981	124	10½	4	1543.6578	139	3¼
3	975.9085	110	8⅞	10	1246.1878	125	1⅝	5	1549.4776	139	6⅜
4	980.5264	111		11	1251.4084	125	4¾	6	1555.2883	139	9⅝
5	985.1579	111	3⅛	40 *ft*.	1256.64	125	7⅞	7	1561.1165	140	¾
6	989.8003	111	6¼	1	1261.8794	125	11	8	1566.9591	140	3⅞
7	994.4509	111	9⅜	2	1267.1327	126	2¼	9	1572.8125	140	7½
8	999.1151	112	½	3	1272.397	126	5⅜	10	1578.6735	141	10⅛
9	1003.7902	112	3¾	4	1277.6692	126	8½	11	1584.5488	141	1¼
10	1008.4736	112	6⅞	5	1282.9553	126	11⅝	45 *ft*.	1590.435	141	4⅜
11	1013.1705	112	10	6	1288.2523	127	2¾	1	1596.3286	141	7½
36*ft*	1017.8784	113	1⅛	7	1293.5572	127	5⅞	2	1602.2366	141	10¾
1	1022.5944	113	4¼	8	1298.876	127	9	3	1608.1555	142	1⅞
2	1027.324	113	7⅜	9	1304.2057	128	¼	4	1614.0819	142	5
3	1032.0646	113	10⅝	10	1305.5433	128	3⅜	5	1620.0226	142	8⅛
4	1036.8134	114	1¾	11	1314.8949	128	6½	6	1625.9743	142	11¼
5	1041.5758	114	4⅞	41 *ft*.	1320.2574	128	9⅝	7	1631.9334	143	2⅜
6	1046.3491	114	8	1	1325.6276	129	¾	8	1637.9068	143	5½
7	1051.1306	114	11⅛	2	1331.0119	129	3⅞	9	1643.8912	143	8¾
8	1055.9257	115	2¼	3	1336.4071	129	7	10	1649.8831	143	11⅞
9	1060.7317	115	5⅜	4	1341.8101	129	10⅛	11	1655.8892	144	3
10	1065.5459	115	9¼	5	1347.2271	130	1⅜	46 *ft*.	1661.9064	144	6⅛
11	1070.3738	115	11⅝	6	1352.6551	130	4½	1	1667.9308	144	9¼
37 *ft*.	1075.2126	116	2⅞	7	1358.0908	130	7⅝	2	1673.9698	145	⅜
1	1080.0594	116	6	8	1363.5406	130	10¾	3	1680.0196	145	3½
2	1084.9201	116	9⅛	9	1369.0012	131	1⅞	4	1686.0769	145	6⅝
3	1089.7915	117	¼	10	1374.4697	131	5	5	1692.1485	145	9⅞
4	1094.6711	117	3½	11	1379.9521	131	8⅛	6	1698.2311	146	1⅛
5	1099.5644	117	6½	42 *ft*.	1385.4456	131	11⅜	7	1704.321	146	4⅛
6	1104.4687	117	9⅝	1	1390.2467	132	2½	8	1710.4254	146	7¼
7	1109.381	118	¾	2	1396.4619	132	5⅝	9	1716.5407	146	10⅜
8	1114.3071	118	4	3	1401.988	132	8¾	10	1722.6634	147	1½
9	1119.244	118	7⅛	4	1407.5219	132	11⅞	11	1728.8005	147	4⅝
10	1124.1891	118	10¼	5	1413.0698	133	3	47 *ft*.	1734.9486	147	7¾
11	1129.1478	119	1⅜	6	1418.6287	133	6⅛	1	1741.1039	147	11
38 *ft*	1134.1176	119	4½	7	1424.1952	133	9¼	2	1747.2738	148	2⅛
1	1139.0953	119	7⅝	8	1429.7759	134	½	3	1753.4545	148	5¼
2	1144.0868	119	10¾	9	1435.3675	134	3⅝	4	1759.6426	148	8⅜
3	1149.0892	120	2	10	1440.9668	134	6¾	5	1765.8452	148	11½
4	1154.0997	120	5⅛	11	1446.5802	134	9⅞	6	1772.0587	149	2⅝
5	1159.1239	120	8⅜	43 *ft*.	1452.2046	135	1	7	1778.2795	149	5⅞
6	1164.1591	120	11⅜	1	1457.8365	135	4⅛	8	1784.5148	149	8⅞
7	1169.2023	121	2½	2	1463.4827	135	7¼	9	1790.761	150	⅛
8	1174.2592	121	5⅝	3	1469.1397	135	10½	10	1797.0145	150	3¼
9	1179.3271	121	8¾	4	1474.8044	136	1⅝	11	1803.2826	150	6⅜
10	1184.403	121	11⅞	5	1480.4833	136	4¾	48 *ft*.	1809.5616	150	9½
11	1189.4927	122	3⅛	6	1486.1731	136	7⅞	1	1815.8477	151	⅝
39 *ft*.	1194.5934	122	6¼	7	1491.8705	136	11	2	1822.1485	151	3¾

TABLE.—*(Continued.)*

Diam.	Area.	Circum.	Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet. Ins.		Feet.	Feet. Ins.		Feet.	Feet. Ins.
3	1828.4602	151 6⅞	11	1879.3355	153 8⅛	7	1930.9188	155 9¼
4	1834.7791	151 10⅛	49 *ft.*	1885.7454	153 11¼	8	1937.3159	156 ⅛
5	1841.1727	152 1¼	1	1892.1724	154 2⅜	9	1943.914	156 3½
6	1847.4571	152 4⅜	2	1898.5041	154 5½	10	1950.4392	156 6⅝
7	1853.8087	152 7½	3	1905.0367	154 8⅝	11	1956.9691	156 9¾
8	1860.175	152 10⅝	4	1911.4965	154 11⅞	50 *ft.*	1963.5	157 ⅞
9	1866.5521	153 1¾	5	1917.9609	155 2⅞			
10	1872.9365	153 3⅞	6	1924.4263	155 6			

TABLE V.

TABLE OF THE SIDES OF SQUARES-EQUAL IN AREA TO A CIRCLE OF ANY DIAMETER.

FROM 1 TO 100.

Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.
1.	.8862	8.	7.0898	15.	13.2934	22.	19.497	29.	25.7006
.¼	1.1078	.¼	7.3114	.¼	13.515	.½	19.7185	.¼	25.9221
.½	1.3293	.½	7.5329	.½	13.7365	.½	19.9401	.½	26.1437
.¾	1.5509	.¾	7.7545	.¾	13.9581	.¾	20.1617	.¾	26.3653
2.	1.7724	.	7.976	16.	14.1796	23.	20.3832	30.	26.5868
.¼	1.994	.¼	8.1976	.¼	14.4012	.¼	20.6048	.¼	26.8084
.½	2.2156	.½	8.4192	.½	14.6227	.½	20.8263	.½	27.0299
.¾	2.4371	.¾	8.6407	.¾	14.8443	.¾	21.0479	.¾	27.2515
3.	2.6587	10.	8.8623	17.	15.0659	24.	21.2694	31.	27.473
.¼	2.8802	.¼	9.0838	.¼	15.2874	.¼	21.491	.¼	27.6946
.½	3.1018	.½	9.3054	.½	15.509	.½	21.7126	.½	27.9161
.¾	3.3233	.¾	9.5269	.¾	15.7305	.¾	21.9341	.¾	28.1377
4.	3.5149	11.	9.7485	18.	15.9521	25.	22.1557	32.	28.3593
.¼	3.7665	.¼	9.97	.¼	16.1736	.¼	22.3772	.¼	28.5808
.½	3.988	.½	10.1916	.½	16.3952	.½	22.5988	.½	28.8024
.¾	4.2096	.¾	10.4132	.¾	16.6168	.¾	22.8203	.¾	29.0239
5.	4.4311	12.	10.6347	19.	16.8383	26.	23.0419	33.	29.2455
.¼	4.6527	.¼	10.8563	.¼	17.0599	.¼	23.2634	.¼	29.467
.½	4.8742	.½	11.0778	.½	17.2814	.½	23.485	.½	29.6886
.¾	5.0958	.¾	11.2994	.¾	17.503	.¾	23.7066	.¾	29.9102
6.	5.3174	13.	11.5209	20.	17.7245	27	23.9281	34.	30.1317
.¼	5.5389	.¼	11.7425	.¼	17.9461	.¼	24.1497	.¼	30.3533
.½	5.7605	.½	11.9641	.½	18.1677	.½	24.3712	.½	30.5748
.¾	5.982	.¾	12.1856	.¾	18.3892	.¾	24.5928	.¾	30.7964
7.	6.2036	14.	12.4072	21.	18.6108	28.	24.8144	35.	31.0179
.¼	6.4251	.¼	12.6287	.¼	18.8323	.¼	25.0359	.¼	31.2395
.½	6.6467	.½	12.8503	.½	19.0539	.½	25.2575	.½	31.4611
.¾	6.8683	.¾	13.0718	.¾	19.2754	.¾	25.459	.¾	31.6826

TABLE—(*Continued*).

Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.
36.	31.9042	49.	43.4251	62.	54.9461	75.	66.467	88.	77.988
.¼	32.1257	.¼	43.6467	.¼	55.1676	.¼	66.6886	.¼	78.2095
.½	32.3473	.½	43.8682	.½	55.3892	.½	66.9104	.½	78.4316
.¾	32.5688	.¾	44.0898	.¾	55.6107	.¾	67.1317	.¾	78.6526
37.	32.7904	50.	44.3113	63.	55.8323	76.	67.3532	89.	78.8742
.¼	33.0112	.¼	44.5329	.¼	56.0538	.¼	67.5748	.¼	79.0957
.½	33.2335	.½	44.7545	.½	56.2754	.½	67.7964	.½	79.3173
.¾	33.4551	.¾	44.976	.¾	56.497	.¾	68.0179	.¾	79.5389
38.	33.6766	51.	45.1976	64.	56.7185	77.	68.2395	90.	79.7604
.¼	33.8982	.¼	45.4191	.¼	56.9401	.¼	68.461	.¼	79.982
.½	34.1197	.½	45.6407	.½	57.1616	.½	68.6826	.½	80.2035
.¾	34.3413	.¾	45.8622	.¾	57.3832	.¾	68.9041	.¾	80.4251
39.	34.5628	52.	46.0838	65.	57.6047	78.	69.1257	91.	80.6467
.¼	34.7884	.¼	46.3054	.¼	57.8263	.¼	69.3473	.¼	80.8682
.½	35.006	.½	46.5269	.½	58.0479	.½	69.5688	.½	81.0898
.¾	35.2275	.¾	46.7485	.¾	58.2694	.¾	69.7904	.¾	81.3113
40.	35.4491	53.	46.97	66.	58.491	79.	70.0119	92.	81.5329
.¼	35.6706	.¼	47.1916	.¼	58.7125	.¼	70.2335	.¼	81.7544
.½	35.8922	.½	47.4131	.½	58.9341	.½	70.455	.½	81.976
.¾	36.1137	.¾	47.6347	.¾	59.1556	.¾	70.6766	.¾	82.1975
41.	36.3353	54.	47.8562	67.	59.3772	80.	70.8981	93.	82.4191
.¼	36.5569	.¼	48.0778	.¼	59.5988	.¼	71.1197	.¼	82.6407
.½	36.7784	.½	48.2994	.½	59.8203	.½	71.3413	.½	82.8622
.¾	37.	.¾	48.5209	.¾	60.0419	.¾	71.5628	.¾	83.0838
42.	37.2215	55.	48.7425	68.	60.2634	81.	71.7844	94.	83.3053
.¼	37.4431	.¼	48.964	.¼	60.485	.¼	72.0059	.¼	83.5269
.½	37.6646	.½	49.1856	.½	60.7065	.½	72.2275	.½	83.7484
.¾	37.8862	.¾	49.4071	.¾	60.9281	.¾	72.4491	.¾	83.970
43.	38.1078	56.	49.6287	69.	61.1497	82.	72.6706	95.	84.1916
.¼	38.3293	.¼	49.8503	.¼	61.3712	.¼	72.8921	.¼	84.4131
.½	38.5509	.½	50.0718	.½	61.5928	.½	73.1137	.½	84.6347
.¾	38.7724	.¾	50.2934	.¾	61.8143	.¾	73.3353	.¾	84.8562
44.	38.994	57.	50.5149	70.	62.0359	83.	73.5568	96.	85.0778
.¼	39.2155	.¼	50.7365	.¼	62.2574	.¼	73.7784	.¼	85.2993
.½	39.4371	.½	50.958	.½	62.479	.½	73.9999	.½	85.5209
.¾	39.6587	.¾	51.1796	.¾	62.7006	.¾	74.2215	.¾	85.7425
45.	39.8802	58.	51.4012	71.	62.9221	84.	74.4431	97.	85.9646
.¼	40.1018	.¼	51.6227	.¼	63.1437	.¼	74.6647	.¼	86.185
.½	40.3233	.½	51.8443	.½	63.3652	.½	74.8862	.½	86.4071
.¾	40.5449	.¾	52.0658	.¾	63.5868	.¾	75.1077	.¾	86.6289
46.	40.7664	59.	52.2874	72.	63.8083	85.	75.3293	98.	86.8502
.¼	40.988	.¼	52.5089	.¼	64.0299	.¼	75.5508	.¼	87.0718
.½	41.2096	.½	52.7305	.½	64.2514	.½	75.7724	.½	87.2933
.¾	41.4311	.¾	52.9521	.¾	64.4730	.¾	75.9934	.¾	87.5419
47.	41.6527	60.	53.1736	73.	64.6946	86.	76.2155	99.	87.7364
.¼	41.8742	.¼	53.3952	.¼	64.9161	.¼	76.4371	.¼	87.958
.½	42.0958	.½	53.6167	.½	65.1377	.½	76.6586	.½	88.1796
.¾	42.3173	.¾	53.8383	.¾	65.3592	.¾	76.8802	.¾	88.4011
48.	42.5839	61.	54.0598	74.	65.5808	87.	77.1017	100.	88.6227
.¼	42.7604	.¼	54.2814	.¼	65.8023	.¼	77.3233	.¼	88.8442
.½	42.982	.½	54.503	.½	66.0239	.½	77.5449	.½	89.0658
.¾	43.2036	.¾	54.7245	.¾	66.2455	.¾	77.7664	.¾	89.2874

TABLE VI.

TABLE OF THE LENGTHS OF CIRCULAR ARCS.

The Diameter of a Circle assumed to be Unity, and divided into 1000 *equal Parts.*

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.1	1.02645	.148	1.05743	.196	1.09949	.244	1.15186	.292	1.21381
.101	1.02698	.149	1.05819	.197	1.10048	.245	1.15308	.293	1.2152
.102	1.02752	.15	1.05896	.198	1.10147	.246	1.15429	.294	1.21658
.103	1.02806	.151	1.05973	.199	1.10247	.247	1.15549	.295	1.21794
.104	1.0286	.152	1.06051	.2	1.10348	.248	1.1567	.296	1.21926
.105	1.02914	.153	1.0613	.201	1.10447	.249	1.15791	.297	1.22061
.106	1.0297	.154	1.06209	.202	1.10548	.25	1.15912	.298	1.22203
.107	1.03026	.155	1.06288	.203	1.1065	.251	1.16033	.299	1.22347
.108	1.03082	.156	1.06368	.204	1.10752	.252	1.16157	.3	1.22495
.109	1.03139	.157	1.06449	.205	1.10855	.253	1.16279	.301	1.22635
.110	1.03196	.158	1.0653	.206	1.10958	.254	1.16402	.302	1.22776
.111	1.03254	.159	1.06611	.207	1.11062	.255	1.16526	.303	1.22918
.112	1.03312	.16	1.06693	.208	1.11165	.256	1.16649	.304	1.23061
.113	1.03371	.161	1.06775	.209	1.11269	.257	1.16774	.305	1.23205
.114	1.0343	.162	1.06858	.21	1.11374	.258	1.16899	.306	1.23349
.115	1.0349	.163	1.06941	.211	1.11479	.259	1.17024	.307	1.23494
.116	1.03551	.164	1.07025	.212	1.11584	.26	1.1715	.308	1.23636
.117	1.03611	.165	1.07109	.213	1.11692	.261	1.17275	.309	1.2378
.118	1.03672	.166	1.07194	.214	1.11796	.262	1.17401	.31	1.23921
.119	1.03734	.167	1.07279	.215	1.11904	.263	1.17527	.311	1.2407
.12	1.03797	.168	1.07365	.216	1.12011	.264	1.17655	.312	1.24216
.121	1.0386	.169	1.07451	.217	1.12118	.265	1.17784	.313	1.2436
.122	1.03923	.17	1.07537	.218	1.12225	.266	1.17912	.314	1.24506
.123	1.03987	.171	1.07624	.219	1.12334	.267	1.1804	.315	1.24654
.124	1.04051	.172	1.07711	.22	1.12445	.268	1.18162	.316	1.24801
.125	1.04116	.173	1.07799	.221	1.12556	.269	1.18294	.317	1.24946
.126	1.04181	.174	1.07888	.222	1.12663	.27	1.18428	.318	1.25095
.127	1.04247	.175	1.07977	.223	1.12774	.271	1.18557	.319	1.25243
.128	1.04313	.176	1.08066	.224	1.12885	.272	1.18688	.32	1.25391
.129	1.0438	.177	1.08156	.225	1.12997	.273	1.18819	.321	1.25539
.13	1.04447	.178	1.08246	.226	1.13108	.274	1.18969	.322	1.25686
.131	1.04515	.179	1.08337	.227	1.13219	.275	1.19082	.323	1.25836
.132	1.04584	.18	1.08428	.228	1.13331	.276	1.19214	.324	1.25987
.133	1.04652	.181	1.08519	.229	1.13444	.277	1.19345	.325	1.26137
.134	1.04722	.182	1.08611	.23	1.13557	.278	1.19477	.326	1.26286
.135	1.04792	.183	1.08704	.231	1.13671	.279	1.1961	.327	1.26437
.136	1.04862	.184	1.08797	.232	1.13786	.28	1.19743	.328	1.26588
.137	1.04932	.185	1.0889	.233	1.13903	.281	1.19887	.329	1.2674
.138	1.05003	.186	1.08984	.234	1.1402	.282	1.20011	.33	1.26892
.139	1.05075	.187	1.09079	.235	1.14136	.283	1.20146	.331	1.27044
.14	1.05147	.188	1.09174	.236	1.14247	.284	1.20282	.332	1.27196
.141	1.0522	.189	1.09269	.237	1.14363	.285	1.20419	.333	1.27349
.142	1.05293	.19	1.09365	.238	1.1448	.286	1.20558	.334	1.27502
.143	1.05367	.191	1.09461	.239	1.14597	.287	1.20696	.335	1·27656
.144	1.05441	.192	1.09557	.24	1.14714	.288	1.20828	.336	1.2781
.145	1.05516	.193	1.09654	.241	1.14831	.889	1.20967	.337	1.27964
.146	1.05591	.194	1.09752	.242	1.14949	.29	1.21202	.338	1.28118
.147	1.05667	.195	1.0985	.243	1.15067	.291	1.21239	.339	1.28273

TABLE.—(*Continued.*)

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.34	1.28428	.373	1.3373	.406	1.39372	.439	1.45327	.472	1.51571
.341	1.28583	.374	1.33896	.407	1.39548	.44	1.45512	.473	1.51764
.342	1.28739	.375	1.34063	.408	1.39724	.441	1.45697	.474	1.51958
.343	1.28895	.376	1.34229	.409	1.399	.442	1.45883	.475	1.52152
.344	1.29052	.377	1.34396	.41	1.40077	.443	1.46069	.476	1.52346
.345	1.29209	.378	1.34563	.411	1.40254	.444	1.46255	.477	1.52541
.346	1.29366	.379	1.34731	.412	1.40432	.445	1.46441	.478	1.52736
.347	1.29523	.38	1.34899	.413	1.406	.446	1.46628	.479	1.52931
.348	1.29681	.381	1.35068	.414	1.40788	.447	1.46815	.48	1.53126
.349	1.29839	.382	1.35237	.415	1.40966	.448	1.47002	.481	1.53322
.35	1.29997	.383	1.35406	.416	1.41145	.449	1.47189	.482	1.53518
.351	1.30156	.384	1.35575	.417	1.41324	.45	1.47377	.483	1.53714
.352	1.30315	.385	1.35744	.418	1.41503	.451	1.47565	.484	1.5391
.353	1.30474	.386	1.35914	.419	1.41682	.452	1.47753	.485	1.54106
.354	1.30634	.387	1.36084	.42	1.41861	.453	1.47942	.486	1.54302
.355	1.30794	.388	1.36254	.421	1.42041	.454	1.48131	.487	1.54499
.356	1.30954	.389	1.36425	.422	1.42222	.455	1.4832	.488	1.54696
.357	1.31115	.39	1.36596	.423	1.42402	.456	1.48509	.489	1.54893
.358	1.31276	.391	1.36767	.424	1.42583	.457	1.48699	.49	1.5509
.359	1.31437	.392	1.36939	.425	1.42764	.458	1.48889	.491	1.55288
.36	1.31599	.393	1.37111	.426	1.42942	.459	1.49079	.492	1.55486
.361	1.31761	.394	1.37283	.427	1.43127	.46	1.49269	.493	1.55685
.362	1.31923	.395	1.37455	.428	1.43309	.461	1.4946	.494	1.55854
.363	1.32086	.396	1.37628	.429	1.43491	.462	1.49651	.495	1.56083
.364	1.32249	.397	1.37801	.43	1.43673	.463	1.49842	.496	1.56282
.365	1.32413	.398	1.37974	.431	1.43856	.464	1.50033	.497	1.56481
.366	1.32577	.399	1.38148	.432	1.44039	.465	1.50224	.498	1.5668
.367	1.32741	.4	1.38322	.433	1.44222	.466	1.50416	.499	1.56879
.368	1.32905	.401	1.38496	.434	1.44405	.467	1.50608	.5	1.57079
.369	1.33069	.402	1.38671	.435	1.44589	.468	1.508		
.37	1.33234	.403	1.38846	.436	1.44773	.469	1.50992		
.371	1.33399	.404	1.39021	.437	1.44957	.47	1.51185		
.372	1.33564	.405	1.39196	.438	1.45142	.471	1.51378		

To Ascertain the Length of an Arc of a Circle by the preceding Table.

Rule.—Divide the height by the base, find the quotient in the column of heights, and take the length of that height from the next righthand column. Multiply the length thus obtained by the base of the arc, and the product will give the length of the arc.

Example.—What is the length of an arc of a circle, the base or span of it being 100 feet, and the height 25 feet?

$25 \div 100 = .25$; and .25 per table, $= 1.15912$, *the length of the base, which, being multiplied by* $100 = 115.912$ *feet.*

Note.—When, in the division of a height by the base, the quotient has a remainder after the third place of decimals, and great accuracy is required.

Take the length for the first three figures, subtract it from the next following length; multiply the remainder by the said fractional remainder, add the product to the first length, and the sum will be the length for the whole quotient.

Example.—What is the length of an arc of a circle, the base of which in 35 feet, and the height or versed sine 8 feet?

$8 \div 35 = .2285714$; the tabular length for $.228 = 1.13331$, and for $.229 = 1.13444$, the difference between which is .00113. Then $.5714 \times .00113 = .000645682$.

Hence $.228 = 1.13331$,

and $.0005714 = .000645682$

1.133955682, the sum by which the base of the arc is to be multiplied; and $1.133955682 \times 35 = 39.68845$ *feet.*

TABLE VII.

TABLE OF THE LENGTHS OF SEMI-ELLIPTIC ARCS.

The Transverse Diameter of an Ellipse assumed to be Unity, and divided into 1000 *equal Parts.*

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.1	1.04162	.148	1.09119	.196	1.14531	.244	1.2038	.292	1.26601
.101	1.04262	.149	1.09228	.197	1.14646	.245	1.20506	.293	1.26734
.102	1.04362	.15	1.0933	.198	1.14762	.246	1.20632	.294	1.26867
.103	1.04462	.151	1.09443	.199	1.14888	.247	1.20758	.295	1.27
.104	1.04562	.152	1.09558	.2	1.15014	.248	1.20884	.296	1.27133
.105	1.04662	.153	1.09669	.201	1.15131	.249	1.2101	.297	1.27267
.106	1.04762	.154	1.0978	.202	1.15248	.25	1.21136	.298	1.27401
.107	1.04862	.155	1.09891	.203	1.15366	.251	1.21263	.299	1.27535
.108	1.04962	.156	1.10002	.204	1.15484	.252	1.2139	.3	1.27669
.109	1.05063	.157	1.10113	.205	1.15602	.253	1.21517	.301	1.27803
.11	1.05164	.158	1.10224	.206	1.1572	.254	1.21644	.302	1.27937
.111	1.05265	.159	1.10335	.207	1.15838	.255	1.21772	.303	1.28071
.112	1.05366	.16	1.10447	.208	1.15957	.256	1.219	.304	1.28205
.113	1.05467	.161	1.1056	.209	1.16076	.257	1.22028	.305	1.28339
.114	1.05568	.162	1.10672	.21	1.16196	.258	1.22156	.306	1.28474
.115	1.05669	.163	1.10784	.211	1.16316	.259	1.22284	.307	1.28609
.116	1.0577	.164	1.10896	.212	1.16436	.26	1.22412	.308	1.28744
.117	1.05872	.165	1.11008	.213	1.16557	.261	1.22541	.309	1.28879
.118	1.05974	.166	1.1112	.214	1.16678	.262	1.2267	.31	1.29014
.119	1.06076	.167	1.11232	.215	1.16799	.263	1.22799	.311	1.29149
.12	1.06178	.168	1.11344	.216	1.1692	.264	1.22928	.312	1.29285
.121	1.0628	.169	1.11456	.217	1.17041	.265	1.23057	.313	1.29421
.122	1.06382	.17	1.11569	.218	1.17163	.266	1.23186	.314	1.29557
.123	1.06484	.171	1.11682	.219	1.17285	.267	1.23315	.315	1.29603
.124	1.06586	.172	1.11795	.22	1.17407	.268	1.23445	.316	1.29829
.125	1.06689	.173	1.11908	.221	1.17529	.269	1.23575	.317	1.29965
.126	1.06792	.174	1.12021	.222	1.17651	.27	1.23705	.318	1.30102
.127	1.06895	.175	1.12134	.223	1.17774	.271	1.23835	.319	1.30239
.128	1.06998	.176	1.12247	.224	1.17897	.272	1.23966	.32	1.30376
.129	1.07001	.177	1.1236	.225	1.1802	.273	1.24097	.321	1.30513
.13	1.07204	.178	1.12473	.226	1.18143	.274	1.24228	.322	1.3065
.131	1.07308	.179	1.12586	.227	1.18266	.275	1.24359	.323	1.30787
.132	1.07412	.18	1.12699	.228	1.1839	.276	1.2448	.324	1.30924
.133	1.07516	.181	1.12813	.229	1.18514	.277	1.24612	.325	1.31061
.134	1.07621	.182	1.12927	.23	1.18638	.278	1.24744	.326	1.31198
.135	1.07726	.183	1.13041	.231	1.18762	.279	1.24876	.327	1.31335
.136	1.07831	.184	1.13155	.232	1.18886	.28	1.2501	.328	1.31472
.137	1.07937	.185	1.13269	.233	1.1901	.281	1.25142	.329	1.3161
.138	1.08043	.186	1.13383	.234	1.19134	.282	1.25274	.33	1.31748
.139	1.08149	.187	1.13497	.235	1.19258	.283	1.25406	.331	1.31886
.14	1.08255	.188	1.13611	.236	1.19382	.284	1.25538	.332	1.32024
.141	1.08362	.189	1.13726	.237	1.19506	.285	1.2567	.333	1.32162
.142	1.08469	.19	1.13841	.238	1.1963	.286	1.25803	.334	1.323
.143	1.08576	.191	1.13956	.239	1.19755	.287	1.25936	.335	1.32438
.144	1.08684	.192	1.14071	.24	1.1988	.288	1.26069	.336	1.32576
.145	1.08792	.193	1.14186	.241	1.20005	.289	1.26202	.337	1.32715
.146	1.08901	.194	1.14301	.242	1.2013	.29	1.26335	.338	1.32854
.147	1.0901	.195	1.14416	.243	1.20255	.291	1.26468	.339	1.32993

TABLE.—*(Continued.)*

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.34	1.33132	.396	1.41211	.452	1.49618	.508	1.58319	.564	1.67087
.341	1.33272	.397	1.41357	.453	1.49771	.509	1.58474	.565	1.67245
.342	1.33412	.398	1.41504	.454	1.49924	.51	1.58629	.566	1.67403
.343	1.33552	.399	1.41651	.455	1.50077	.511	1.58784	.567	1.67561
.344	1.33692	.4	1.41798	.456	1.5023	.512	1.5894	.568	1.67719
.345	1.33833	.401	1.41945	.457	1.50383	.513	1.59096	.569	1.67877
.346	1.33974	.402	1.42092	.458	1.50536	.514	1.59252	.57	1.68036
.347	1.34115	.403	1.42239	.459	1.50689	.515	1.59408	.571	1.68195
.348	1.34256	.404	1.42386	.46	1.50842	.516	1.59564	.572	1.68354
.349	1.34397	.405	1.42533	.461	1.50996	.517	1.5972	.573	1.68513
.35	1.34539	.406	1.42681	.462	1.5115	.518	1.59876	.574	1.68672
.351	1.34681	.407	1.42829	.463	1.51304	.519	1.60032	.575	1.68831
.352	1.34823	.408	1.42977	.464	1.51458	.52	1.60188	.576	1.6899
.353	1.34965	.409	1.43125	.465	1.51612	.521	1.60344	.577	1.69149
.354	1.35108	.41	1.43273	.466	1.51766	.522	1.605	.578	1.69308
.355	1.35251	.411	1.43421	.467	1.5192	.523	1.60656	.579	1.69467
.356	1.35394	.412	1.43569	.468	1.52074	.524	1.60812	.58	1.69626
.357	1.35537	.413	1.43718	.469	1.52229	.525	1.60968	.581	1.69785
.358	1.3568	.414	1.43867	.47	1.52384	.526	1.61124	.582	1.69945
.359	1.35823	.415	1.44016	.471	1.52539	.527	1.6128	.583	1.70105
.36	1.35967	.416	1.44165	.472	1.52691	.528	1.61436	.584	1.70264
.361	1.36111	.417	1.44314	.473	1.52849	.529	1.61592	.585	1.70424
.362	1.36255	.418	1.44463	.474	1.53004	.53	1.61748	.586	1.70584
.363	1.36399	.419	1.44613	.475	1.53159	.531	1.61904	.587	1.70745
.364	1.36543	.42	1.44763	.476	1.53314	.532	1.6206	.588	1.70905
.365	1.36688	.421	1.44913	.477	1.53469	.533	1.62216	.589	1.71065
.366	1.36833	.422	1.45064	.478	1.53625	.534	1.62372	.59	1.71225
.367	1.36978	.423	1.45214	.479	1.53781	.535	1.62528	.591	1.71286
.368	1.37123	.424	1.45364	.48	1.53937	.536	1.62684	.592	1.71546
.369	1.37268	.425	1.45515	.481	1.54093	.537	1.6284	.593	1.71707
.37	1.37414	.426	1.45665	.482	1.54249	.538	1.62996	.594	1.71868
.371	1.37662	.427	1.45815	.483	1.54405	.539	1.63152	.595	1.72029
.372	1.37708	.428	1.45966	.484	1.54561	.54	1.63309	.596	1.7219
.373	1.37854	.429	1.46167	.485	1.54718	.541	1.63465	.597	1.7235
.374	1.38	.43	1.46268	.486	1.54875	.542	1.63623	.598	1.72511
.375	1.38146	.431	1.46419	.487	1.55032	.543	1.6378	.599	1.72672
.376	1.38292	.432	1.4657	.488	1.55189	.544	1.63937	.6	1.72833
.377	1.38439	.433	1.46721	.489	1.55346	.545	1.64094	.601	1.72994
.378	1.38585	.434	1.46872	.49	1.55503	.546	1.64251	.602	1.73155
.379	1.38732	.435	1.47023	.491	1.5566	.547	1.64408	.603	1.73316
.38	1.38879	.436	1.47174	.492	1.55817	.548	1.64565	.604	1.73477
.381	1.39024	.437	1.47326	.493	1.55974	.549	1.64722	.605	1.73638
.382	1.39169	.438	1.47478	.494	1.56131	.55	1.64879	.606	1.73799
.383	1.39314	.439	1.4763	.495	1.56289	.551	1.65036	.607	1.7396
.384	1.39459	.44	1.47782	.496	1.56447	.552	1.65193	.608	1.74121
.385	1.39605	.441	1.47934	.497	1.56605	.553	1.6535	.609	1.74283
.386	1.39751	.442	1.48086	.498	1.56763	.554	1.65507	.61	1.74444
.387	1.39897	.443	1.48238	.499	1.56921	.555	1.65665	.611	1.74605
.388	1.40043	.444	1.48391	.5	1.57089	.556	1.65823	.612	1.74767
.389	1.40189	.445	1.48544	.501	1.57234	.557	1.65981	.613	1.74929
.39	1.40335	.446	1.48697	.502	1.57389	.558	1.66139	.614	1.75091
.391	1.40481	.447	1.4885	.503	1.57544	.559	1.66297	.615	1.75252
.392	1.40627	.448	1.49003	.504	1.57699	.56	1.66455	.616	1.75414
.393	1.40773	.449	1.49157	.505	1.57854	.561	1.66613	.617	1.75576
.394	1.40919	.45	1.49311	.506	1.58009	.562	1.66771	.618	1.75738
.395	1.41065	.451	1.49465	.507	1.58164	.563	1.66929	.619	1.759

TABLE.—*(Continued.)*

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.62	1.76062	.676	1.85215	.732	1.94552	.788	2.04117	.844	2.13976
.621	1.76224	.677	1.85379	.733	1.94721	.789	2.0429	.845	2.14155
.622	1.76386	.678	1.85544	.734	1.9489	.79	2.04462	.846	2.14334
.623	1.76548	.679	1.85709	.735	1.95059	.791	2.04635	.847	2.14513
.624	1.7671	.68	1.85874	.736	1.95228	.792	2.04809	.848	2.14692
.625	1.76872	.681	1.86039	.737	1.95397	.793	2.04983	.849	2.14871
.626	1.77034	.682	1.86205	.738	1.95566	.794	2.05157	.85	2.1505
.627	1.77197	.683	1.8637	.739	1.95735	.795	2.05331	.851	2.15229
.628	1.77359	.684	1.86535	.74	1.95994	.796	2.05505	.852	2.15409
.629	1.77521	.685	1.867	.741	1.96074	.797	2.05679	.853	2.15589
.63	1.77684	.686	1.86866	.742	1.96244	.798	2.05853	.854	2.1577
.631	1.77847	.687	1.87031	.743	1.96414	.799	2.06027	.855	2.1595
.632	1.78009	.688	1.87196	.744	1.96583	.8	2.06202	.856	2.1613
.633	1.78172	.689	1.87362	.745	1.96753	.801	2.06377	.857	2.16309
.634	1.78335	.69	1.87527	.746	1.96923	.802	2.06552	.858	2.16489
.635	1.78498	.691	1.87693	.747	1.97093	.803	2.06727	.859	2.16668
.636	1.7866	.692	1.87859	.748	1.97262	.804	2.06901	.86	2.16848
.637	1.78823	.693	1.88024	.749	1.97432	.805	2.07076	.861	2.17028
.638	1.78986	.694	1.8819	.75	1.97602	.806	2.07251	.862	2.17209
.639	1.79149	.695	1.88356	.751	1.97772	.807	2.07427	.863	2.17389
.64	1.79312	.696	1.88522	.752	1.97943	.808	2.07602	.864	2.1757
.641	1.79475	.697	1.88688	.753	1.98113	.809	2.07777	.865	2.17751
.642	1.79638	.698	1.88854	.754	1.98283	.81	2.07953	.866	2.17932
.643	1.79801	.699	1.8902	.755	1.98453	.811	2.08128	.867	2.18113
.644	1.79964	.7	1.89186	.756	1.98623	.812	2.08304	.868	2.18294
.645	1.80127	.701	1.89352	.757	1.98794	.813	2.0848	.869	2.18475
.646	1.8029	.702	1.89519	.758	1.98964	.814	2.08656	.87	2.18656
.647	1.80454	.703	1.89685	.759	1.99134	.815	2.08832	.871	2.18837
.648	1.80617	.704	1.89851	.76	1.99305	.816	2.09008	.872	2.19018
.649	1.8078	.705	1.90017	.761	1.99476	.817	2.09198	.873	2.192
.65	1.80943	.706	1.90184	.762	1.99647	.818	2.0936	.874	2.19382
.651	1.81107	.707	1.9035	.763	1.99818	.819	2.09536	.875	2.19564
.652	1.81271	.708	1.90517	.764	1.99989	.82	2.09712	.876	2.19746
.653	1.81435	.709	1.90684	.765	2.0016	.821	2.09888	.877	2.19928
.654	1.81599	.71	1.90852	.766	2.00331	.822	2.10065	.878	2.2011
.655	1.81763	.711	1.91019	.767	2.00502	.823	2.10242	.879	2.20292
.656	1.81928	.712	1.91187	.768	2.00673	.824	2.10419	.88	2.20474
.657	1.82091	.713	1.91355	.769	2.00844	.825	2.10596	.881	2.20656
.658	1.82255	.714	1.91523	.77	2.01016	.826	2.10773	.882	2.20839
.659	1.82419	.715	1.91691	.771	2.01187	.827	2.1095	.883	2.21022
.66	1.82583	.716	1.91859	.772	2.01359	.828	2.11127	.884	2.21205
.661	1.82747	.717	1.92027	.773	2.01531	.829	2.11304	.885	2.21388
.662	1.82911	.718	1.92195	.774	2.01702	.83	2.11481	.886	2.21571
.663	1.83075	.719	1.92363	.775	2.01874	.831	2.11659	.887	2.21754
.664	1.8324	.72	1.92531	.776	2.02045	.832	2.11837	.888	2.21937
.665	1.83404	.721	1.927	.777	2.02217	.833	2.12015	.889	2.2212
.666	1.83568	.722	1.92868	.778	2.02389	.834	2.12193	.89	2.22303
.667	1.83733	.723	1.93036	.779	2.02561	.835	2.12371	.891	2.22486
.668	1.83897	.724	1.93204	.78	2.02733	.836	2.12549	.892	2.2267
.669	1.84061	.725	1.93373	.781	2.02907	.837	2.12727	.893	2.22854
.67	1.84226	.726	1.93541	.782	2.0308	.838	2.12905	.894	2.23038
.671	1.84391	.727	1.9371	.783	2.03252	.839	2.13083	.895	2.23222
.672	1.84556	.728	1.93878	.784	2.03425	.84	2.13261	.896	2.23406
.673	1.8472	.729	1.94046	.785	2.03598	.841	2.13439	.897	2.2359
.674	1.84885	.73	1.94215	.786	2.03771	.842	2.13618	.898	2.23774
.675	1.8505	.731	1.94383	.787	2.03944	.843	2.13797	.899	2.23958

TABLE.—(*Continued.*)

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.9	2.24142	.921	2.27987	.942	2.31852	.963	2.3581	.984	2.39823
.901	2.24325	.922	2.2817	.943	2.32038	.964	2.36	.985	2.40016
.902	2.24508	.923	2.28354	.944	2.32224	.965	2.36191	.986	2.40208
.903	2.24691	.924	2.28537	.945	2.32411	.966	2.36381	.987	2.404
.904	2.24874	.925	2.2872	.946	2.32598	.967	2.36571	.988	2.40592
.905	2.25057	.926	2.28903	.947	2.32785	.968	2.36762	.989	2.40784
.906	2.2524	.927	2.29086	.948	2.32972	.969	2.36952	.99	2.40976
.907	2.25423	.928	2.2927	.949	2.3316	.97	2.37143	.991	2.41169
.908	2.25606	.929	2.29453	.95	2.33348	.971	2.37334	.992	2.41362
.909	2.25789	.93	2.29636	.951	2.33537	.972	2.37525	.993	2.41556
.91	2.25972	.931	2.2982	.952	2.33726	.973	2.37716	.994	2.41749
.911	2.26155	.932	2.30004	.953	2.33915	.974	2.37908	.995	2.41943
.912	2.26338	.933	2.30188	.954	2.34104	.975	2.381	.996	2.42136
.913	2.26521	.934	2.30373	.955	2.34293	.976	2.38291	.997	2.42329
.914	2.26704	.935	2.30557	.956	2.34483	.977	2.38482	.998	2.42522
.915	2.26888	.936	2.30741	.957	2.34673	.978	2.38673	.999	2.42715
.916	2.27071	.937	2.30926	.958	2.34862	.979	2.38864	1.	2.42908
.917	2.27254	.938	2.31111	.959	2.35051	.98	2.39055		
.918	2.27437	.939	2.31295	.96	2.35241	.981	2.39247		
.919	2.2762	.94	2.31479	.961	2.35431	.982	2.39439		
.92	2.27803	.941	2.31666	.962	2.35621	.983	2.39631		

To Ascertain the Length of a Semi-Elliptic Arc (right Semi-Ellipse) by the preceding Table.

Rule.—Divide the height by the base, find the quotient in the column of heights, and take the length of that height from the next righthand column. Multiply the length thus obtained by the base of the arc, and the product will be the length of the arc.

Example.—What is the length of the arc of a semi-ellipse, the base being 70 feet, and the height 30.10 feet.

$30.10 \div 70 = .43$; and .43 per table, $= 1.46268$.

Then $1.46268 \times 70 = 102.3876$ *feet.*

When the Curve is not that of a Right Semi-Ellipse, the Height being half of the Transverse Diameter.

Rule.—Divide half the base by twice the height, then proceed as in the preceding example; multiply the tabular length by twice the height, and the product will be the length required

Example.—What is the length of the arc of a semi-ellipse, the height being 35 feet, and the base 60 feet?

$60 \div 2 = 30$, and $30 \div \overline{35 \times 2} = .428$, the tabular length of which is 1.45966.

Then $1.45966 \times \overline{35 \times 2} = 102.1762$ *feet.*

Note.—If in the division of a height by the base there is a remainder, proceed in the manner given for the Lengths of Circular Arcs, page 32.

TABLE VIII.

TABLE OF THE AREAS OF THE SEGMENTS OF A CIRCLE.

The Diameter of a Circle assumed to be Unity, and divided into 1000 *equal Parts.*

Versed Sine.	Seg Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg Area.	Versed Sine.	Seg Area.
.001	.00004	.048	.01382	.095	.0379	.142	.06822	.189	.10312
.002	.00012	.049	.01425	.096	.03849	.143	.06892	.19	.1039
.003	.00022	.05	.01468	.097	.03908	.144	.06962	.191	.10468
.004	.00034	.051	.01512	.098	.03968	.145	.07033	.192	.10547
.005	.00047	.052	.01556	.099	.04027	.146	.07103	.193	.10626
.006	.00062	.053	.01601	.1	.04087	.147	.07174	.194	.10705
.007	.00078	.054	.01646	.101	.04148	.148	.07245	.195	.10784
.008	.00095	.055	.01691	.102	.04208	.149	.07316	.196	.10864
.009	.00113	.056	.01737	.103	.04269	.15	.07387	.197	.10943
.01	.00133	.057	.01783	.104	.0431	.151	.07459	.198	.11023
.011	.00153	.058	.0183	.105	.04391	.152	.07531	.199	.11102
.012	.00175	.059	.01877	.106	.04452	.153	.07603	.2	.11182
.013	.00197	.06	.01924	.107	.04514	.154	.07675	.201	.11262
.014	.0022	.061	.01972	.108	.04575	.155	.07747	.202	.11343
.015	.00244	.062	.0202	.109	.04638	.156	.0782	.203	.11423
.016	.00268	.063	.02068	.11	.047	.157	.07892	.204	.11503
.017	.00294	.064	.02117	.111	.04763	.158	.07965	.205	.11584
.018	.0032	.065	.02165	.112	.04826	.159	.08038	.206	.11665
.019	.00347	.066	.02215	.113	.04889	.16	.08111	.207	.11746
.02	.00375	.067	.02265	.114	.04953	.161	.08185	.208	.11827
.021	.00403	.068	.02315	.115	.05016	.162	.08258	.209	.11908
.022	.00432	.069	.02336	.116	.0508	.163	.08332	.21	.1199
.023	.00462	.07	.02417	.117	.05145	.164	.08406	.211	.12071
.024	.00492	.071	.02468	.118	.05209	.165	.0848	.212	.12153
.025	.00523	.072	.02519	.119	.05274	.166	.08554	.213	.12235
.026	.00555	.073	.02571	.12	.05338	.167	.08629	.214	.12317
.027	.00587	.074	.02624	.121	.05404	.168	.08704	.215	.12399
.028	.00619	.075	.02676	.122	.05469	.169	.08779	.216	.12481
.029	.00653	.076	.02729	.123	.05534	.17	.08853	.217	.12563
.03	.00686	.077	.02782	.124	.056	.171	.08929	.218	.12646
.031	.00721	.078	.02835	.125	.05666	.172	.09004	.219	.12728
.032	.00756	.079	.02889	.126	.05733	.173	.0908	.22	.12811
.033	.00791	.08	.02943	.127	.05799	.174	.09155	.221	.12894
.034	.00827	.081	.02997	.128	.05866	.175	.09231	.222	.12977
.035	.00864	.082	.03052	.129	.05933	.176	.09307	.223	.1306
.036	.00901	.083	.03107	.13	.06	.177	.09384	.224	.13144
.037	.00938	.084	.03162	.131	.06067	.178	.0946	.225	.13227
.038	.00976	.085	.03218	.132	.06135	.179	.09537	.226	.13311
.039	.01015	.086	.03274	.133	.06203	.18	.09613	.227	.13394
.04	.01054	.087	.0333	.134	.06271	.181	.0969	.228	.13478
.041	.01093	.088	.03387	.135	.06339	.182	.09767	.229	.13562
.042	.01133	.089	.03444	.136	.06407	.183	.09845	.23	.13646
.043	.01173	.09	.03501	.137	.06476	.184	.09922	.231	.13731
.044	.01214	.091	.03558	.138	.06545	.185	.1	.232	.13815
.045	.01255	.092	.03616	.139	.06614	.186	.10077	.233	.139
.046	.01297	.093	.03674	.14	.06683	.187	.10155	.234	.13984
.047	.01339	.094	.03732	.141	.06753	.188	.10233	.235	.14069

TABLE.—*(Continued.)*

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.
.236	.14154	.289	.18814	.342	.23737	.395	.28848	.448	.34079
.237	.14239	.29	.18905	.343	.23832	.396	.28945	.449	.34179
.238	.14324	.291	.18995	.344	.23927	.397	.29043	.45	.34278
.239	.14409	.292	.19086	.345	.24022	.398	.29141	.451	.34378
.24	.14494	.293	.19177	.346	.24117	.399	.29239	.452	.34477
.241	.1458	.294	.19268	.347	.24212	.4	.29337	.453	.34557
.242	.14665	.295	.1936	.348	.24307	.401	.29435	.454	.34676
.243	.14751	.296	.19451	.349	.24403	.402	.29533	.455	.34776
.244	.14837	.297	.19542	35	.24498	.403	.29631	.456	.34875
.245	.14923	.298	.19634	.351	.24593	.404	.29729	.457	.34975
.246	.15009	.299	.19725	.352	.24689	.405	.29827	.458	.35075
.247	.15095	.3	.19817	.353	.24784	.406	.29925	.459	.35174
.248	.15182	.301	.19908	.354	.2488	.407	.30024	.46	.35274
.249	.15268	.302	.2	.355	.24976	.408	.30122	.461	.35374
.25	.15355	.303	.20092	.356	.25071	.409	.3022	.462	.35474
.251	.15441	.304	.20184	.357	.25167	.41	.30319	.463	.35573
.252	.15528	.305	.20276	.358	.25263	.411	.30417	.464	.35673
.253	.15615	.306	.20368	.359	.25359	.412	.30515	.465	.35773
.254	.15702	.307	.2046	.36	.25455	.413	.30614	.466	.35872
.255	.15789	.308	.20553	.361	.25551	.414	.30712	.467	.35972
.256	.15876	.309	.20645	.362	.25647	.415	.30811	.468	.36072
.257	.15964	.31	.20738	.363	.25743	.416	.30909	.469	.36172
.258	.16051	.311	.2083	.364	.25839	.417	.31008	.47	.36272
.259	.16139	.312	.20923	.365	.25936	.418	.31107	.471	.36371
.26	.16226	.313	.21015	.366	.26032	.419	.31205	.472	.36471
.261	.16314	.314	.21108	.367	.26128	.42	.31304	.473	.36571
.262	.16402	.315	.21201	.368	.26225	.421	.31403	.474	.36671
.263	.1649	.316	.21294	.369	.26321	.422	.31502	.475	.36771
.264	.16578	.317	.21387	.37	.26418	.423	.316	.476	.36871
.265	.16666	.318	.2148	.371	.26514	.424	.31699	.477	.36971
.266	.16755	.319	.21573	.372	.26611	.425	.31798	.478	.37071
.267	.16844	.32	.21667	.373	.26708	.426	.31897	.479	.3717
.268	.16931	.321	.2176	.374	.26804	.427	.31996	.48	.3727
.269	.1702	.322	.21853	.375	.26901	.428	.32095	.481	.3737
.27	.17109	.323	.21947	.376	.26998	.429	.32194	.482	.3747
.271	.17197	.324	.2204	.377	.27095	.43	.32293	.483	.3757
.272	.17287	.325	.22134	.378	.27192	.431	.32391	.484	.3767
.273	.17376	.326	.22228	.379	.27289	.432	.3249	.485	.3777
.274	.17465	.327	.22321	.38	.27386	.433	.3259	.486	.3787
.275	.17554	.328	.22415	.381	.27483	.434	.32689	.487	.3797
.276	.17643	.329	.22509	.382	.27580	.435	.32788	.488	.3807
.277	.17733	.33	.22603	.383	.27677	.436	.32887	.489	.3817
.278	.17822	.331	.22697	.384	.27775	.437	.32987	.49	.3827
.279	.17912	.332	.22791	.385	.27872	.438	.33086	.491	.3837
.28	.18002	.333	.22886	.386	.27969	.439	.33185	.492	.3847
.281	.18092	.334	.2298	.387	.28067	.44	.33284	.493	.3857
.282	.18182	.335	.23074	.388	.28164	.441	.33384	.494	.3867
.283	.18272	.336	.23169	.389	.28262	.442	.33483	.495	.3677
.284	.18361	.337	.23263	.39	.28359	.443	.33582	.496	.3887
.285	.18452	.338	.23358	.391	.28457	.444	.33682	.497	.3897
.286	.18542	.339	.23453	.392	.28554	.445	.33781	.498	.3907
.287	.18633	.34	.23547	.393	.28652	.446	.3388	.499	.3917
.288	.18723	.341	.23642	.394	.2875	.447	.3398	.5	.3927

To Ascertain the Area of a Segment of a Circle by the preceding Table.

Rule.—Divide the height or versed sine by the diameter of the circle; find the quotient in the column of versed sines. Take the area noted in the next column, multiply it by the square of the diameter, and it will give the area.

Example.—Required the area of a segment, its height being 10, and the diameter of the circle 50 feet.

10÷50 =.2, and .2, per table, =.11182; then .11182 × 50^2=279.55 *feet.*

Note.—If, in the division of a height by the base, the quotient has remainder after the third place of decimals, and great accuracy is required,

Take the area for the first three figures, subtract it from the next following area, multiply the remainder by the said fraction, and add the product to the first area; the sum will be the area for the whole quotient.

2. What is the area of a segment of a circle, the diameter of which is 10 feet, and the height of it 1.575 feet?

1.575÷10=.1575; the tabular area for .157=.07892, and for .158=.07965, the difference between which is .00073.

Then .5 × .00073=.000365.

Hence .157 =.07892
.0005=.000365

.079275, the sum by which the square of the diameter of the circle is to be multiplied; and .079285 × 10^2=7.9286 *feet.*

TABLE IX.

TABLE OF THE AREAS OF THE ZONES OF A CIRCLE.

The Diameter of a Circle assumed to be Unity, and divided into 1000 *equal Parts.*

H'ght.	Area.	H'ght.	Area.	H'ght.	Area	H'ght.	Area.	H'ght.	Area.
.001	.001	.029	.02898	.057	.05688	.085	.08459	.113	.11203
.002	.002	.03	.02998	.058	.05787	.086	.08557	.114	.113
.003	.003	.031	.03093	.059	.05886	.087	.08656	.115	.11398
.004	.004	.032	.03198	.06	.05986	.088	.08754	.116	.11495
.005	.005	.033	.03298	.061	.06085	.089	.08853	.117	.11592
.006	.006	.034	.03397	.062	.06184	.09	.08951	.118	.1169
.007	.007	.035	.03497	.063	.06283	.091	.0905	.119	.11787
.008	.008	.036	.03597	.064	.06382	.092	.09148	.12	.11884
.009	.009	.037	.03697	.065	.06482	.093	.09246	.121	.11981
.01	.01	.038	.03796	.066	.0658	.094	.09344	.122	.12078
.011	.011	.039	.03896	.067	.0668	.095	.09443	.123	.12175
.012	.012	.04	.03996	.068	.0678	.096	.0954	.124	.12272
.013	.013	.041	.04095	.069	.06878	.097	.09639	.125	.12369
.014	.014	.042	.04195	.07	.06977	.098	.09737	.126	.12469
.015	.015	.043	.04295	.071	.07076	.099	.09835	.127	.12562
.016	.016	.044	.04394	.072	.07175	.1	.09933	.128	.12659
.017	.017	.045	.04494	.073	.07274	.101	.10031	.129	.12755
.018	.018	.046	.04593	.074	.07373	.102	.10129	.13	.12852
.019	.019	.047	.04693	.075	.07472	.103	.10227	.131	.12949
.02	.02	.048	.04793	.076	.0755	.104	.10325	.132	.13045
.021	.021	.049	.04892	.077	.07669	.105	.10422	.133	.13141
.022	.022	.05	.04992	.078	.07768	.106	.1052	.134	.13238
.023	.023	.051	.05091	.079	.07867	.107	.10618	.135	.13334
.024	.024	.052	.0519	.08	.07966	.108	.10715	.136	.1343
.025	.025	.053	.0529	.081	.08064	.109	.10813	.137	.13527
.026	.02599	.054	.05389	.082	.08163	.11	.10911	.138	.13623
.027	.02699	.055	.05489	.083	.08262	.111	.11008	.139	.13719
.028	.02799	.056	.05588	.084	.0836	.112	.11106	.14	.13815

TABLE.—(*Continued.*)

H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.
.141	.13911	.197	.19178	.253	.24175	.309	.28801	.365	.32931
.142	.14007	.198	.1927	.254	.24261	.31	.2888	.366	.32999
.143	.14103	.199	.19361	.255	.24347	.311	.28958	.367	.33067
.144	.14198	.2	.19453	.256	.24433	.312	.29036	.368	.33135
.145	.14294	.201	.19545	.257	.24519	.313	.29115	.369	.33203
.146	.1439	.202	.19636	.258	.24604	.314	.29192	.37	.3327
.147	.14485	.203	.19728	.259	.2469	.315	.2927	.371	.33337
.148	.14581	.204	.19819	.26	.24775	.316	.29348	.372	.33404
.149	.14677	.205	.1991	.261	.24861	.317	.29425	.373	.33471
.15	.14772	.206	.20001	.262	.24946	.318	.29502	.374	.33537
.151	.14867	.207	.20092	.263	.25021	.319	.2958	.375	.33604
.152	.14962	.208	.20183	.264	.25116	.32	.29656	.376	.3367
.153	.15058	.209	.20274	.265	.25201	.321	.29733	.377	.33735
.154	.15153	.21	.20365	.266	.25285	.322	.2981	.378	.33801
.155	.15248	.211	.20156	.267	.2537	.323	.29886	.379	.33866
.156	.15343	.212	.20546	.268	.25455	.324	.29962	.38	.33931
.157	.15438	.213	.20637	.269	.25539	.325	.30039	.381	.33996
.158	.15533	.214	.20727	.27	.25623	.326	.30114	.382	.34061
.159	.15628	.215	.20818	.271	.25707	.327	.3019	.383	.34125
.16	.15723	.216	.20908	.272	.25791	.328	.30266	.384	.3419
.161	.15817	.217	.20998	.273	.25875	.329	.30341	.385	.34253
.162	.15912	.218	.21088	.274	.25959	.33	.30416	.386	.34317
.163	.16006	.219	.21178	.275	.26043	.331	.30491	.387	.3438
.164	.16101	.22	.21268	.276	.26126	.332	.30566	.388	.34444
.165	.16195	.221	.21358	.277	.26209	.333	.30641	.389	.34507
.166	.1629	.222	.21447	.278	.26293	.334	.30715	.39	.34569
.167	.16384	.223	.21537	.279	.26376	.335	.3079	.391	.34632
.168	.16478	.224	.21626	.28	.26459	.336	.30864	.392	.34694
.169	.16572	.225	.21716	.281	.26541	.337	.30938	.393	.34756
.17	.16667	.226	.21805	.282	.26624	.338	.31012	.394	.34818
.171	.16761	.227	.21894	.283	.26706	.339	.31085	.395	.34879
.172	.16855	.228	.21983	.284	.26789	.34	.31159	.396	.3494
.173	.16948	.229	.22072	.285	.26871	.341	.31232	.397	.35001
.174	.17042	.23	.22161	.286	.26953	.342	.31305	.398	.35062
.175	.17136	.231	.2225	.287	.27035	.343	.31378	.399	.35122
.176	.1723	.232	.22335	.288	.27117	.344	.3145	.4	.35182
.177	.17323	.233	.22427	.289	.27199	.345	.31523	.401	.35242
.178	.17417	.234	.22515	.29	.2728	.346	.31595	.402	.35302
.179	.1751	.235	.22604	.291	.27362	.347	.31667	.403	.35361
.18	.17603	.236	.22692	.292	.27443	.348	.31739	.404	.3542
.181	.17697	.237	.2278	.293	.27524	.349	.31811	.405	.35479
.182	.1779	.238	.22868	.294	.27605	.35	.31882	.406	.35538
.183	.17883	.239	.22956	.295	.27686	.351	.31954	.407	.35596
.184	.17976	.24	.23044	.296	.27766	.352	.32025	.408	.35654
.185	.18069	.241	.23131	.297	.27847	.353	.32096	.409	.35711
.186	.18162	.242	.23219	.298	.27927	.354	.32167	.41	.35769
.187	.18254	.243	.23306	.299	.28007	.355	.32237	.411	.35826
.188	.18347	.244	.23394	.3	.28088	.356	.32307	.412	.35883
.189	.1844	.245	.23481	.301	.28167	.357	.32377	.413	.35939
.19	.18532	.246	.23568	.302	.28247	.358	.32447	.414	.35995
.191	.18625	.247	.23655	.303	.28327	.359	.32517	.415	.36051
.192	.18717	.248	.23742	.304	.28406	.36	.32587	.416	.36107
.193	.18809	.249	.23829	.305	.28486	.361	.32656	.417	.36162
.194	.18902	.25	.23915	.306	.28565	.362	.32725	.418	.36217
.195	.18994	.251	.24002	.307	.28644	.363	.32794	.419	.36272
.196	.19086	.252	.24089	.308	.28723	.364	.32862	.42	.36326

TABLE.—*(Continued.)*

H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.
.421	.3638	.437	.37202	.453	.37931	.469	.38549	.485	.39026
.422	.36434	.438	.3725	.454	.37973	.47	.38583	.486	.3905
.423	.36488	.439	.37298	.455	.38014	.471	.38617	.487	.39073
.424	.36541	.44	.37346	.456	.38056	.472	.3865	.488	.39095
.425	.36594	.441	.37393	.457	.38096	.473	.38683	.489	.39117
.426	.36646	.442	.3744	.458	.38137	.474	.38715	.49	.39137
.427	.36698	.443	.37487	.459	.38177	.475	.38747	.491	.39156
.428	.3675	.444	.37533	.46	.38216	.476	.38778	.492	.39175
.429	.36802	.445	.37579	.461	.38255	.477	.38808	.493	.35192
.43	.36853	.446	.37624	.462	.38294	.478	.38838	.494	.39208
.431	.36904	.447	.37669	.463	.38332	.479	.38867	.495	.39223
.432	.36954	.448	.37714	.464	.38369	.48	.38895	.496	.39236
.433	.37005	.449	.37758	.465	.38406	.481	.38923	.497	.39248
.434	.37054	.45	.37802	.466	.38443	.482	.3895	.498	.39258
.435	.37104	.451	.37845	.467	.38479	.483	.38976	.499	.39266
.436	.37153	.452	.37888	.468	.38514	.484	.39001	.5	.3927

This Table is computed only for Zones, the longest chord of which is diameter.

To Ascertain the Area of a Zone by the preceding Table.

Rule 1.—*When the Zone is Less than a Semicircle,* Divide the height by the diameter, and find quotient in the column of height. Take out the area opposite to it in the next column on the right nd, and multiply it by the quare of the longest chord; the product will be the area of the zone.

Example.—Required the area of a zone the diameter of which is 50, and its height 15.

$15 \div 50 = .3$; and .3, as per table, $= .28088$.

Hence $.28088 \times 50^2 = 702.2$ *area.*

Rule 2.—*When the Zone is Greater than a Semicircle:* Take the height on each side of the diaeter of the circle, and ascertain, by Rule I, their respective areas; add the areas of these two porns together, and the sum will be the area of the zone.

Example.—Required the area of a zone, the diameter of the circle being 50, and the heights of zone on each side of the diameter of the circle 20 and 15 respectively.

$20 \div 50 = .4$; .4, as per table, $= .35182$; and $.35182 \times 50^2 = 879.55$.

$15 \div 50 = .3$; .3, as per table $= .28088$; and $.28088 \times 50^2 = 702.2$.

Hence $879.55 + 702.2 = 1581.75$ *area.*

Rule 3.—*When the longest chord of the zone is less than diameter,* Take the height or distance om the diam. to each of the chords respectively; find the area corresponding to each height and duct the lesser from the greater area; the result will be the area required.

Note.—When, in the division of a height by the chord, the quotient has a remainder after the rd place of decimals, and great accuracy is required,

Take the area for the first three figures, subtract it from the next following area, multiply the mainder by the said fraction, and add the product to the first area; the sum will be the area for whole quotient.

Example.—What is the area of a zone of a circle, the greater chord being 100 feet, and the eadth of it 14 feet 3 inches?

feet 3 inches $= 14.25$ and $14.25 \div 100 = .1425$; the tabular area for $.142 = .14007$, and for .143 $= .14103$, the difference between which is .00096.

Then $.5 \times .00096 = .00048$.

Hence $.142 = .14007$

$.0005 = .00048$

$\overline{.14055}$, the sum by which the square of the greater chord is to be multiplied; and 4055 $\times 100^2 = 1405.5$ *feet.*

TABLE X.

SPECIFIC GRAVITIES.

The Specific Gravity of a body is the proportion it bears to the weight of another body of known density.

If a body float on a fluid, the part immersed is to the whole body as the specific gravity of the body is to the specific gravity of the fluid.

When a body is immersed in a fluid, it loses such a portion of its own weight as is equal to that of the fluid it displaces.

An immersed body, ascending or descending in a fluid, has a force equal to the difference between its own weight and the weight of its bulk of the fluid, less the resistance of the fluid to its passage.

Water is well adapted for the standard of gravity ; and as a cubic foot of it weights 1000 ounces avoirdupois, its weight is taken as the unit, *viz* : 1000.

To Ascertain the Specific Gravity of a Body heavier than Water.

Rule.—Weigh it both in and out of water, and note the dtfference; then, as the weight lost in water is to tne whole weight, so is 1000 to the specific gravity of the body. Or, $\frac{W \times 1000}{W-w} = G$, *$w$ representing the weight in water, and G the specific gravity.*

Example.—What is the specific gravity of a stone which weighs in air 15 lbs., in water 10 lbs. ?

15—10= 5 ; then 5 : 15 : : 1000 : : 3000 *spec. grav.*

To Ascertain the Specific Gravity of a Body lighter than Water.

Rule.—Annex to the lighter body another that is heavier than water, or the fluid used ; weigh the piece added and the compound mass separarately, both in and out of the water, or the fluid ; ascertain how much each loses in water, or the fluid, by subtracting its weight in water, or the fluid, from its weight in air, and subtract the less of these differences from the greater ; then,

As the last remainder is to the weight of the light body in air, so is 1000 to the specific gravity of the body.

Example.—What is the specific gravity of a piece of wood that weighs 20 lbs. in air ; annexed to it is a piece of metal that weighs 24 lbs. in air and 21 lbs. in water, and the two pieces in water weigh 8 lbs. ?

20+24—8=44—8=36=*loss of compound mass in water ;*
24—21 = 3=*loss of heavy body in water.*
$\overline{33}$: 20 : : 1000 : 606=24 *spec. grav.*

To Ascertain the Specific Gravity of Fluid.

Rule.—Take a body of known specific gravity, weigh it in and out of the fluid ; then, as the weight of the body is to the loss of weight, so is the specific gravity of the body to that of the fluid.

Example.—What is the specific gravity of a fluid in which a piece of copper (*spec. grav.*=9000) weighs 70 lbs. in, and 80 lbs. out of it?

$$80 : 80-70=10 : : 9000 : 1125 \textit{ spec. grav.}$$

To Compute the Proportions of two Ingredients in a Compound, or to discover Adulteration in Metals.

Rule.—Take the differences of each specific gravity of the ingredients and the specific gravity of the compound, then multiply the gravity of the one by the difference of the other; and, as the sum of the products is to the respective products, so is the specific gravity of the body to the proportions of the ingredients.

Example.—A compound of gold (*spec. grav.*=18.888) and silver (*spec. grav.*= 10.535) has a specific gravity of 14; what is the proportion of each metal?

$$18.888-14=4.888\times 10.535=51.495$$

$$14.-10.535=3.465\times 18.888=65.447$$

$$65.447+51.495 : 65.447 : : 14 : 7.835 \textit{ gold,}$$

$$65.447+51.495 : 51.495 : : 14 : 6.165 \textit{ silver.}$$

To compute the Weights of the Ingredients, that of the compound being given.

Rule.—As the specific gravity of the compound is to the weight of the compound, so are each of the proportions to the weight of its material.

Example.—The weight, as above, being 28 lbs., what are the weights of the ingredients?

$$14 : 28 : : \begin{cases} 7.835 : 15.67 \textit{ gold,} \\ 6.165 : 12.33 \textit{ silver.} \end{cases}$$

Proof of Spirituous Liquors.

A cubic inch of *proof spirits* weighs 234 grains; then, if an immersed cubic inch of any heavy body weighs 234 grains less in spirits than air, it shows that the spirit in which it was weighed is *proof.*

If it lose less of its weight, the spirit is above proof; and if it lose more, it is below proof.

Illustration.—A cubic inch of glass weighing 700 grains weighs 500 grains when weighed in a certain spirit; what is the proof of it?

$$700-500=200=\textit{grains}=\textit{weight lost in the spirit.}$$

Then 200 : 234 : : 1. : 1.17=*ratio of proof of spirits compared to proof spirits, or* 1. =.17 *above proof.*

Solids.

Rule.—Divide the specific gravity of the substance by 16, and the quotient will give the weight of a cubic foot of it in pounds.

OF DIFFERENT BODIES AND SUBSTANCES.

METALS.	Specific gravity.	Weight of a cubic inch.
Alluminum	2560	.0926
Antimony	6712	.2428
Arsenic	5763	.2084
Barium	470	.017
Bismuth	9823	.3553
Brass, copper 84 " tin 16	8832	.3194
" copper 67 " zinc 33	7820	.2828
" plate	8380	.3031
" wire	8214	.2972
Bronze, gun metal	8700	.3147
Boron	2000	.0723
Bromine	3000	.1085
Cadmium	8650	3129
Calcium	1580	.057
Chromium	5900	2134
Cinnabar	8098	.2929
Cobalt	8600	.3111
Columbium	6000	.217
Gold, pure, cast	19258	.6965
" hammered	19361	.7003
" 22 carats fine	17486	.6325
" 20 carats fine	15709	.5682
Copper, cast	8788	.3179
" plates	8698	.3146
" wire	8880	.3212
Iridium	18680	.6756
" hammered	23000	.8319
Iron, cast	7207	.2607
" " gun metal	7308	.264
" hot blast	7065	.2555
" cold "	7218	.2611
" wrought bars	7788	.2817
" " wire	7774	.2811
" rolled plates	7704	.2787
Lead, cast	11352	.4106
" rolled	11388	.4119
Lithium	590	.0213
Manganese	8000	.2894
Magnesium	1750	.0633
Mercury—40°	15632	.5661
" +32°	13598	.4918
" 60°	13580	.4912
" 212°	13370	.4836
Molybdenum	8600	.3111
Nickel	8800	.3183
" cast	8279	.2994
Osmium	10000	.3613

METALS.	Specific gravity.	Weight of a cubic inch.
Palladium	11350	.4105
Platinum, hammered	20337	.7356
" native	16000	.5787
" rolled	22069	.7982
Potassium, 59°	865	.0313
Red-lead	8940	.3241
Rhodium	10650	.3852
Ruthenium	8600	.3111
Selenium	4500	.1627
Silicium		
Silver, pure, cast	10474	.3788
" " hammered	10511	.3902
Sodium	970	.0351
Steel, plates	7806	.2823
" soft	7833	2833
" tempered and hardened	7818	.2828
" wire	7847	.2838
Strontium	2540	.0918
Tin, Cornish, hammerd	7390	.2673
" " pure	7291	.2637
Tellurium	6110	.221
Thalium	11850	.4286
Titanium	5300	.1917
Tungsten	17000	.6149
Uranium	10150	.3671
Wolfram	7119	.2575
Zinc, cast	6861	.2482
" rolled	7191	.26

WOODS (Dry).		Cubic foot.
Alder	800	50
Apple	793	49.562
Ash	845 600	52.812 43 125
Bamboo	400	25
Bay	822	51.375
Beech	852	53 25
"	690	43.125
Birch	567	35.437
Box, Brazilian	1031	64.437
" Dutch	912	57.
" French	1328	83.
Bullet-wood	928	58.
Butternut	376	23.5
Campeachy	913	57.062
Cedar	561	35.062
" Indian	1315	82.157

WOODS, (Dry.) (*Continued.*)	Specific gravity.	Weight of a cubic foot.
Charcoal, pine	441	27.562
" fresh burned	380	23.75
" oak	1573	98.312
" soft wood	280	17.5
" triturated	1380	86.25
Cherry	715	44.687
Chesnut, sweet	610	38.125
Citron	726	45.375
Cocoa	1040	65.
Cork	240	15.
Cypress, Spanish	644	40.25
Dog-wood	756	47.25
Ebony, American	1331	83.187
" Indian	1209	75.562
Elder	695	43 437
Elm	570	35 625
	671	41.937
Filbert	600	37.5
Fir (Norway Space)	512	32.
Gum, blue	843	52.687
" water	1000	62.5
Hackmatack	592	37.
Hazel	860	53.75
Hawthorn	910	56.875
Hemlock	368	23.
Hickory, pig-nut	792	49.5
" shell-bark	690	43.125
Holly	760	47.5
Jasmine	770	48 125
Juniper	566	35 375
Lance-wood	720	45.
Larch	544	34.
	560	35
Lemon	703	43.937
Lignum-vitæ	1333	83.312
Lime	804	50.25
Linden	604	37.75
Locust	728	45.5
Logwood	913	57.062
Mahogany	720	45.
	1063	66.437
" Honduras	560	35.
" Spanish	852	53.25
Maple	750	46.875
" bird's eye	576	36.
Mastic	849	53.062
Mulberry	561	35.062
	897	56.062
Oak, African	823	51.437
" Canadian	872	54.5

WOODS, (Dry), (*Continued.*)	Specific gravity.	Waight of a cubic foot.
Oak, Dantzic	759	47.437
" English	932	58.25
" green	1446	71.625
" heart, 60 years	1170	73.125
" live, green	1260	78.75
" " seasoned	1068	66.75
" white	860	53.75
Orange	705	44.062
Pear	661	41 312
Persimmon	710	44.375
Plum	785	49.062
Pine, pitch	660	41.25
" red	590	36.875
" white	554	34.625
" yellow	461	28.812
Pomegranate	1354	84.625
Poon	580	36.25
Poplar	383	23.937
" white	529	33.062
Quince	705	44.062
Rose-wood	728	45.5
Sassafras	482	30.125
Satin-wood	885	55.312
Spruce	500	31.25
Sycamore	623	38.937
Tamarack	383	23.937
Teak (African oak)	657	41.062
	745	46.562
Walnut	671	41.937
" black	500	31.25
Willow	486	30.375
	585	36.562
Yew, Dutch	788	49.25
" Spanish	807	50.437
(Well Seasoned.)*		
Ash	722	45.125
Beech	624	39.
Cherry	606	37.875
Cypress	441	27.562
Hickory, red	838	52.375
Mahogany, St. Domg.	720	45.
Pine, white	473	29.562
" yellow	541	33.812
Poplar	587	36.687
White Oak, upland	687	42.937
" James River	759	42 437

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* Orduance manual 1841.

Stones, Earths, &c.	Specific gravity.	Weight of a cubic foot.
Agate	2590	—
Alabaster, white	2730	170 625
" yellow	2699	168.687
Alum	1714	107.125
Amber	1078	67.375
Ambergris	866	—
Asbestos, starry	3073	192 062
Asphaltum	905	56.562
	1650	103.125
Barytes, sulphate	4000	250.
	4865	304.062
Basalts	2740	171.25
	2864	179.
Borax	1714	107.125
Brick	1900	118.75
	1367	85.437
" fire	2201	137.562
" work in cement	1800	112.50
" " " mortar	1600	100.
	2000	125.
Carbon	3500	218 75
Cement, Portland	1300	81 25
" Roman	1560	97.25
Chalk	1520	95.
	2784	174.
Chrysolite	2782	—
Clay	1930	120.625
" with gravel	2480	155.
Coal, Antracite	1436	89.75
	1640	102.5
" Borneo	1290	80.625
" Cannel	1238	77.375
	1318	82 375
" Caking	1277	79.812
" Cherry	1276	79.75
" Chili	1290	80.625
" Derbyshire	1292	80 75
" Lancaster	1273	79.562
" Maryland	1355	84.687
" Newcastle	1270	79.375
" Rive de Gier	1300	81.25
" Scotch	1259	78.687
	1300	81.25
" Splint	1302	81.375
" Wales, mean	1315	82.187
Coke	1000	62.5
" Nat'l, Va	746	46.64
Concrete, mean	2000	125.
Copal	1045	65.312
Coral, red	2700	—
" white	2550	—
Cornelian	2613	—
Diamond, Oriental	3521	—
" Brazilian	3444	—
Earth, † common soil	2194	137.125
" loose	1500	93.75
" moist sand	2050	128 125
" mould, fresh	2050	128.125
" rammed	1600	100.
" rough sand	1920	120.
" with gravel	2020	126.25
Emery	4000	250.
Flint, black	2582	161.375
" white	2594	162.125
Fluorine	1320	82.5
Glass, bottle	2732	170.75
" Crown	2487	155.437
" flint	2933	183.312
	3200	196.
" green	2642	165 125
" optical	3450	215.625
" white	2892	180.75
" window	2642	165.125
Garnet	4189	—
" black	3750	—
Granite, Egyptian red	2654	165 875
" Patapsco	2640	165.
" Quincy	2652	165 75
" Scotch	2625	164.062
" Susquehanna	2704	169.
Gravel, common	1749	109.312
Grindstone	2143	133.937
Gypsum, opaque	2168	135.5
Hone, white, razor	2876	179.75
Hornblende	3540	221.25
Iodine	4940	—
Jet	1300	—
Lime, hydraulic	2745	171.562
" quick	804	50.25
Limestone, green	3180	198.75
" white	3156	197.25
Magnesia, carbonate	2400	150.
Marble, Adelaide	2715	169.687
" African	2708	169.25
" Biscayan, black,	2695	168.437
" Carara	2716	169.75
" common	2686	167.875
" Egyptian	2668	166.75
" French	2649	165.562
" Italian, white	2708	169.25

† Spec. grav. of the earth is variously estimated at from 5.450 to 5.600.

Stones, Earths, &c.	Specific gravity.	Weight of a Cubic Foot.
Marble Parian	2838	177.375
" Vermont, white	2650	165.57
Marl, mean	1750	109.375
Mica	2800	175.
Mortar	1384	86.5
	1750	109.375
Millstone	2484	155 25
Mud	1630	101.875
Nitre	1900	118.75
Opal	2114	—
Oyster-shell	2092	130.75
Paving-stone	2416	151.
Peal, Oriental	2650	—
Peat	600	37.5
	1329	83.062
Phosphorus	1770	110.625
Plaster of Paris	1176	73.5
Plumbago	2100	131.25
Porphyry, red	2765	172.812
Porcelain, China	2300	143.75
Pumice-stone	915	57.187
Quartz	2660	166.25
Rotten-stone	1981	123.812
Red-lead	8940	558 75
Resin	1089	68.062
Rock, crystal	2735	170.937
Ruby	4283	—
Salt, common	2130	133.125
Saltpetre	2090	130.625
Sand, coarse	1800	112 5
" common	1670	104.375
" damp and loose	1392	87.
" dried and loose	1560	97 5
" dry	1420	88.75
" mortar, Ft. Rich.	1659	103 66
" " Brooklyn	1716	107.25
" sillicious	1701	106.33
Sapphire	3994	—
Shale	2600	162.5
Slate	2900	181.25
	2672	167.
Slate, purple	2784	174.
Smalt	2440	152.5
Stone, Bath..... Engl.	1961	122.562
" Blue Hill	2640	165.
" Bluestone (basalt)	2625	164 062
" Breakneck..N.Y.	2704	169.
" Bristol.....Engl.	2510	156.875
" Caen, Normandy	2076	129 75
" Common	2520	157.5

Stones, Farths, &c.	Specific gravity.	Weight of a Cubic Foot.
Stone, Craigleth..Engl.	2316	144.75
" Kentish rag "	2651	165.687
" Kip's Bay..N.Y.	2759	172.
" Norfolk (Parliament House).	2304	144.
" Portland ..Engl.	2368	148.
" Sandstone, mean	2200	137.5
" " Sydney	2237	139.812
" Staten Isl'd. N Y.	2976	186.
" Sullivan Co. "	2688	168.
Schorl	3170	198.125
Spar, calcareous	2735	170 937
" Feld, blue	2693	168.312
" " green	2704	169.
" " Fluor	3400	215.5
Stalactite	2415	150.937
Sulphur, native	2033	127.062
Talc, mean	2500	156 25
Tale, black	2900	181.25
Tile	1815	113.437
Topaz, Oriental	4011	—
Trap	2720	170.
Turquoise	2750	—
MISCELLANEOUS.		
Asphaltum	905	56.562
	1650	103.125
Atmospheric Air	*	.07529
Beeswax	965	60.312
Butter	942	58.875
Camphor	988	61.75
Caoutchouc	903	56.437
Egg	1090	—
Fat of Beef	923	57.687
" Hogs	936	58.5
" Mutton	923	57.687
Gamboge	1222	—
Gum Arabic	1452	90.75
Gunpowder, loose	900	56.25
" shaken	1000	62.5
" solid	1550	96.875
	1800	112.5
Gutta-percha	980	61.25
Horn	1689	105.562
Ice, at 32°	920	57.5
Indigo	1009	63.062
Isinglass	1111	69.437
Ivory	1825	114 062
Lard	947	59.187

(*) .001205.

MISCELLANEOUS.	Specific gravity.	Weight of a Cubic Foot.	LIQUIDS.	Specific gravity.	Weight of a Cubic Foot.
Mastic	1074	67.125	Aquafortis, double	1300	81.25
Myrrh	1360	85.	“ single	1200	75.
Opium	1336	83.5	Beer.	1034	64.625
Soap, Castile	1071	56.937	Bitumen, liquid	848	53.
Spermaceti	943	58.937	Blood (human).	1054	65.875
Starch	950	59 375	Brandy, ⅝ or 5 of spirit	924	57.75
Sugar	1606	100.375	Cider	1018	63.625
“ .66	1326	82 875	Ether, acetic	866	54.125
	972	60.25	“ muriatic	845	52 812
Tallow	941	58 812	“ sulphuric	715	44.687
Wax	964	60.25	Honey	1450	90 625
	970	60.625	Milk	1032	64.5
			Oil, Anise-seed	986	61.625
LIQUIDS.			“ Codfish	923	57.687
			“ Cotton-seed	—	—
Acid, Acetic	1062	66.375	“ Linseed	940	58.75
“ Benzoic	667	41.687	“ Naphta	848	53.
“ Citric	1034	64.625	“ Olive	915	57.187
“ Concentrated	1521	95.062	“ Palm	969	60.562
“ Fluoric	1500	93 75	“ Petroleum	878	54.875
“ Muriatic	1200	75.	“ Rape	914	57 125
“ Nitric	1217	76.062	“ Sunflower	926	57.875
“ Phosphoric	1558	97.375	“ Turpentine	870	54.375
“ “ solid	2800	175	“ Whale	925	57 687
“ Sulphuric	1849	115.562	Spirit, rectified	824	51.5
Alcohol, pure, 60°	794	49.622	Tar	1015	63.437
“ 95 per cent	816	51.	Vinegar	1080	67.5
“ 80 “	863	53 937	Water, Dead Sea	1240	77.5
“ 50 “	934	58.375	“ 60°	999	62.449
“ 40 “	951	59 437	“ 212°	957	59.812
“ 25 “	970	60 625	“ distilled, 39°[†]	998	62 379
“ 10 “	986	61.625	“ Mediterranean	1029	64 312
“ 5 “	992	62.	“ rain	1009	62 5
“ proof spirit,[*]50 per cent 60°	934	58.375	“ sea	1026	64 125
			Wine, Burgundy	992	62.
“ proof spirit, 50 per cent. 80°	875	54 687	“ Champagne	997	64 375
			“ Madeira	1 38	62.312
Ammonia, 27.9 per ct.	891	55.687	“ Port	997	62.312

Compression of the following fluids under a pressure of 15 lbs. per square inch :

Alcohol	.0000216	Mercury	.00000265
Ether	.0000158	Water	.00004663

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[*] Specific gravity of proof spirit according to Ure's Table for Sykes's Hydrometer, 920.

[†] 1 cubic inch =.252.69 Troy grains.

Elastic Fluids.

1‡ *Cubic Foot of Atmospheric Air weighs* 527.04 *Troy Grains.*
Its assumed Gravity of 1 *is the Unit for Elastic Fluids.*

Atmospheric air, 34°	1.	Phosphureted hydrogen	1.77
Ammonia	.589	Sulphureted "	1 17
Azote	976	Sulphurous acid	2.21
Carbonic acid.	1.52	Steam,* 212°	.4883
" oxyd	.972	Smoke, of bituminous coal	.102
Carbureted hydrogen	559	" coke	.105
Chlorine.	2 47	" wood	.09
Chloro-carbonic	3.389	Vapor of alcohol	1.613
Cyanogen.	1.815	" bisulphuret of carbon	2.64
Gas, coal	4	Vapor of bromine	5.1
	.752	" chloric ether	3.44
Hydrogen	.07	" ether.	2.586
Hydrochloric acid	1.278	" hydrochloric ether	2.255
Hydrocyanic "	.942	" iodine	8.675
Muriatic acid	1.247	" nitric acid	3 75
Nitrogen	.972	" spirits of turpentine	4.763
Nitric oxyd	1.094	" sulphuric acid	2.7
Nitrous acid	2.638	" " ether	2.586
Nitrous oxyd.	1.527	" sulphur	2.214
Oxygen	1.102	" water	.623

Weights and Volumes of various Substances in Ordinary Use.

SUBSTANCES.	Cubic Foot.	Cub. Inch.	SUBSTANCES.	Cubic Foot.	Cub. Inch.
METALS.	**Lbs.**	**Lbs.**	**METALS.**	**Lbs.**	**Lbs.**
Brass { copper 67 zinc 33 }	488.75	.2829	Tin	455.687	.2637
			Zinc, cast	428 812	.2482
" gun metal	543.75	.3147	" rolled.	449.437	.2601
" sheets	513.6	.297			
" wire	524 16	.3033	**WOODS.**		Cub. Feet in a Ton.
Copper, cast	547.25	.3179			
" plates.	543.625	.3167	Ash	52.812	42.414
Iron, cast	450.437	.2607	Bay	51.375	43.601
" gun metal	466 5	.27	Cork	15.	149.333
" heavy forging	479 5	.2775	Cedar	35.062	63.886
" plates	481.5	.2787	Chestnut	38.125	58.754
" wrought bars.	486.75	.2816	Hickory, pig nut	49 5	45.252
Lead, cast	709.5	4106	" shell-bark.	43.125	51.942
" rolled.	711.75	.4119	Lignum-vitæ	83 312	26 886
Mercury, 60°	848.7487	491174	Logwood	57.062	39.255
Steel, plates	487 75	.2823	Mahogany, Honduras	35.	64.
" soft	489.562	.2833		66.437	33 714

‡ Equal to .07529143 lbs. avoirdupois.

* Weight of a cubic foot, 257.353 Troy grains.

SUBSTANCES.	Cubic Foot.	Cub. Feet in a Ton.	SUBSTANCES.	Cubic Foot.	Cub. Feet in a Ton.
Oak, Canadian...	54.5	41.101	Coal, Welsh, mean	81.25	27.569
“ English.....	58.25	38 455	Coke	62.5	35.84
“ live, seasoned	66.75	33.558	Cotton, bale, mean	14.5	154.48
“ white, dry..	53.75	41.674	“ “ pressed	20.	114.
“ “ upland	42.937	52.169		25.	89.6
Pine, pitch..	41.25	54.303	Earth, clay	120.625	18 569
“ red	36.875	60.745	“ common soil	137.125	16.335
“ white.	34.625	64.693	“ “ gravel	109.312	20 49
“ well seasoned	29.562	75.773	“ dry, sand...	120.	18.667
“ yellow	33.812	66.248	“ loose.	93.75	23.893
Spruce..........	31.25	71.68	“ moist, sand	128.125	17 482
Walnut, black, dry	31.25	71.68	“ mold.......	128.125	17.482
Willow....	36.562	61.265	“ mud........	101.875	21.987
“ dry.......	30.375	73.744	“ with gravel.	126.25	17.742
			Granite, Quincy..	165.75	13.514
Miscellaneous.			“ Susqueh'na	169.	13.254
			Hay, bale........	9.525	235.17
Air.............	.075291	—	“ pressed.....	25.	89.6
Basalt, mean.....	175.	12.8	India rubber.	56.437	39.69
Brick, fire.......	137.562	16.284	“ vulcanized	—	—
“ mean	102.	21 961	Limestone..	197.25	11.355
Coal, anthracite	89.75	24.958	Marble, mean....	167.875	13.343
	102.5	21.854	Mortar, dry, mean.	97.98	22.862
“ bitumin., mean	80.	28.	Water, fresh......	62.5	35.84
“ Cannel.	94.875	23.609	“ salt.......	64.125	34.931
“ Cumberland..	84.687	26 451	Steam	.036747	—

Application of the Tables.

When the Weight of a Substance is required. RULE.—Ascertain the volume of the substance in cubic feet; multiply it by the unit in the second column of tables, and divide the product by 16; the quotient will give the weight in pounds.

When the Volume is given or ascertained in Inches. RULE.—Multiply it by the unit in the third column of the tables, and the product will be the weight in pounds.

EXAMPLE.—What is the weight of a cube of Italian marble, the sides being 3 feet?

$3^3 \times 2708 = 73116$ *oz.*, which $\div\ 16 = 4569.75$ *lbs.*

Or of a sphere of cast iron 2 inches in diameter?

$2^3 \times .5236 \times .26$ *weight of a cubic inch* $= 1.089$ *lbs.*

Comparative Weight of Timber in a Green and Seasoned State.

Timber.	Weight of a Cub. Ft.		Timber.	Weight of a Cub. Ft.	
	Green.	Seasoned		Green.	Seasoned
	Lbs. Oz.	Lbs. Oz.		Lbs. Oz.	Lbs. Oz.
American Pine..........	44.12	30.11	Cedar....................	32.	28. 4
Ash.....................	58. 3	50	English Oak............	71.10	43. 8
Beech..................	60.	53.6	Riga Fir..............	48.12	35. 8

To Compute the Capacity of a Balloon.

Rule.—From specific gravity of the air in grains par cubic foot substract that of the gaz with which it is inflated ; multiply the remainder by the volume of the balloon in cubic feet ; divide the product by 7000, and from the quotient substract the weight of the balloon and its attachments.

Example.—The diameter of a balloon is 26.6 feet, its weight is 100 lbs., and the specific gravity of the gaz with which it is inflated is .06 (air beieg assumed at 1) ; what is its capacity ?

$$\frac{527.04-31.62\times\overline{26.6^3\times.5236}}{7000}-100=\frac{495.42\times9854.726}{7000}-100=597.461\ \mathit{lbs.}$$

To Compute the Diameter of a Balloon, the Weight to be raised being given.

By inversion of the preceding rule,

$\sqrt[3]{\dfrac{W\times700\div\overline{s-s'}}{.5236}}=d$, *s* and *s′ representing the weight of air and gas in grins per cubic foot, and d the diameter of the balloon in feet.*

Example.—Given the elements in the preceding case.

$$\text{Then}\quad\frac{\sqrt[3]{597.46+100\times7000\div\overline{527.04-31.62}}}{.5236}=\sqrt[3]{18821.09}=26.6\ \mathit{feet.}$$

To Compute the Weight of Cast Metal by the Weight of the Pattern.

When the Pattern is of White Pine.

Rule.—Multiply the weight of the pattern in pounds by the following multiplier, and the product will give the weight of the casting :

Iron, 14 ; Brass, 15 ; Lead, 22 ; Tin, 14 ; Zinc, 13.5.

When there are Circular Cores or Prints.—Multiply the square of the diameter of the core or print by its length in inches, the product by .0175, and the result is the weight of the pattern of the core or print to be deducted from the weight of the pattern.

It is customary, in the making of patterns for castings, to allow for shrinkage per lineal foot of pattern.

Iron and Lead $\frac{1}{8}$th of an inch, Brass and Zinc $\frac{3}{16}$ths, and Tin $\frac{1}{12}$th.

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